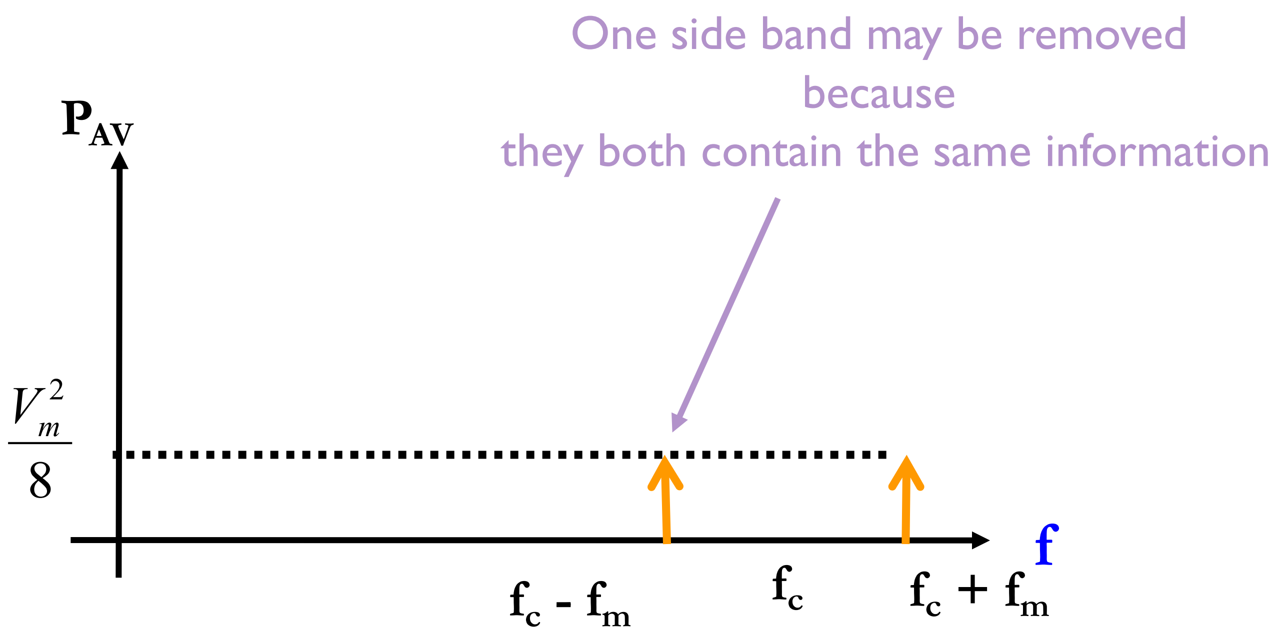
Amplitude Modulation:
Single Sideband (SSB)

Single Side Band AM

Q: What is Single Side Band AM?

Looking at the power spectrum of an DSB we notice that:

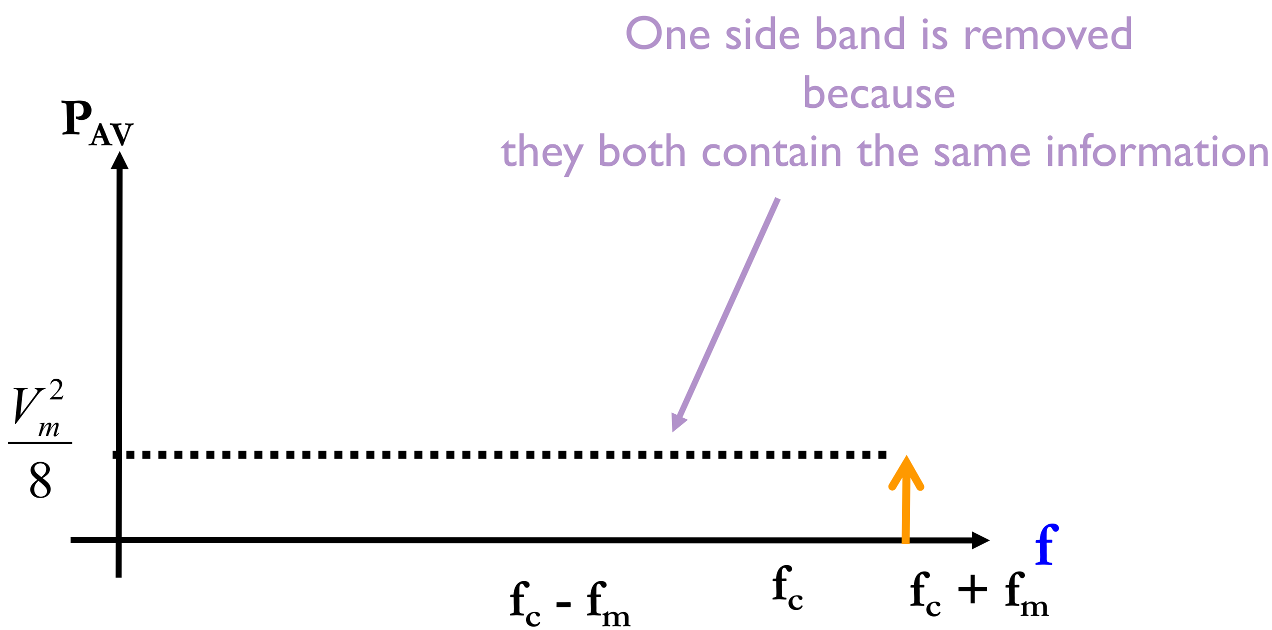


Power Spectrum

Single Side Band AM

Q: What is Single Side Band AM?

This is the power spectrum of an AM SSB signal:



Power Spectrum

Single Side Band AM

Q: What is SSB in the Time Domain?

In the Frequency Domain:

$$V_{DSB-SC}(f) = \frac{V_m}{4}\left[\delta(f - f_{USB}) + \delta(f + f_{USB})\right] + \frac{V_m}{4}\left[\delta(f - f_{LSB}) + \delta(f + f_{LSB})\right]$$

$$V_{SSB}(f) = \frac{V_m}{4}\left[\delta(f - f_{USB}) + \delta(f + f_{USB})\right]$$

In the time domain:

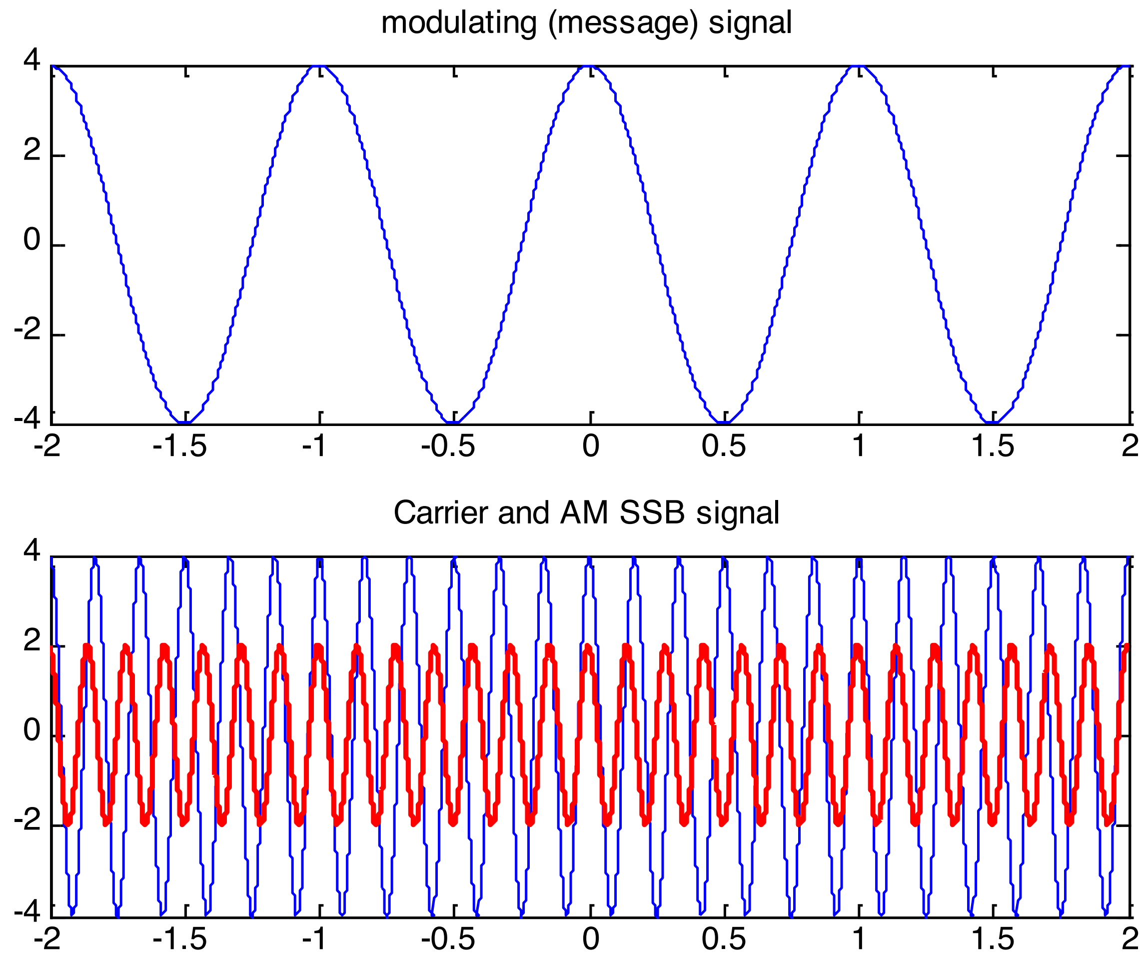
$$v_{SSB}(t) = \frac{V_m}{2}\cos(2\pi\ f_{USB}\ t)$$

$$v_{SSB}(t) = \frac{V_m}{2}\cos(2\pi\ (f_c + f_m)\ t)$$

Single Side Band AM

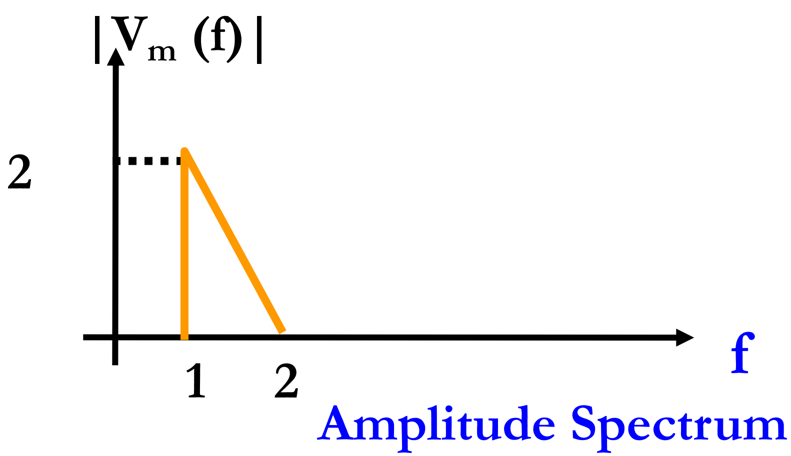
Q: What is Single Side Band AM in the Time Domain?

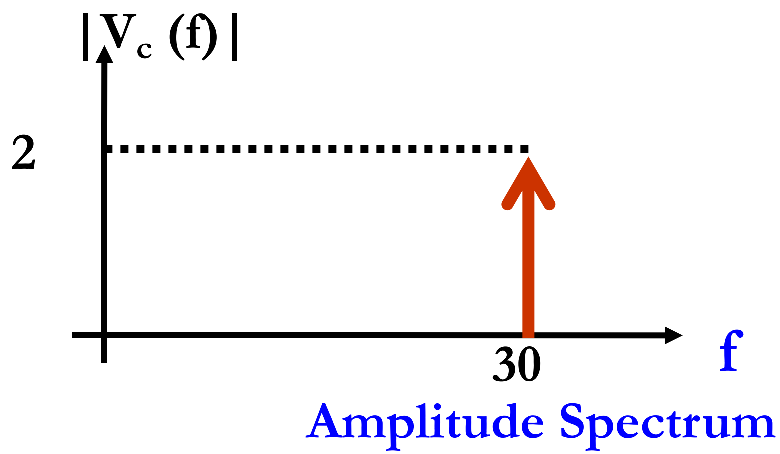
For tone modulation:

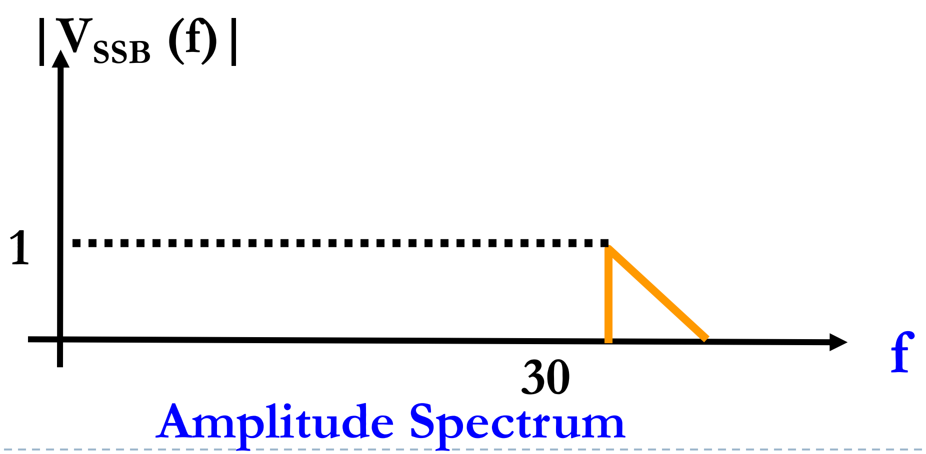


SSB spectrum

Exercise: Find the spectrum of the following DSB-SC signal.





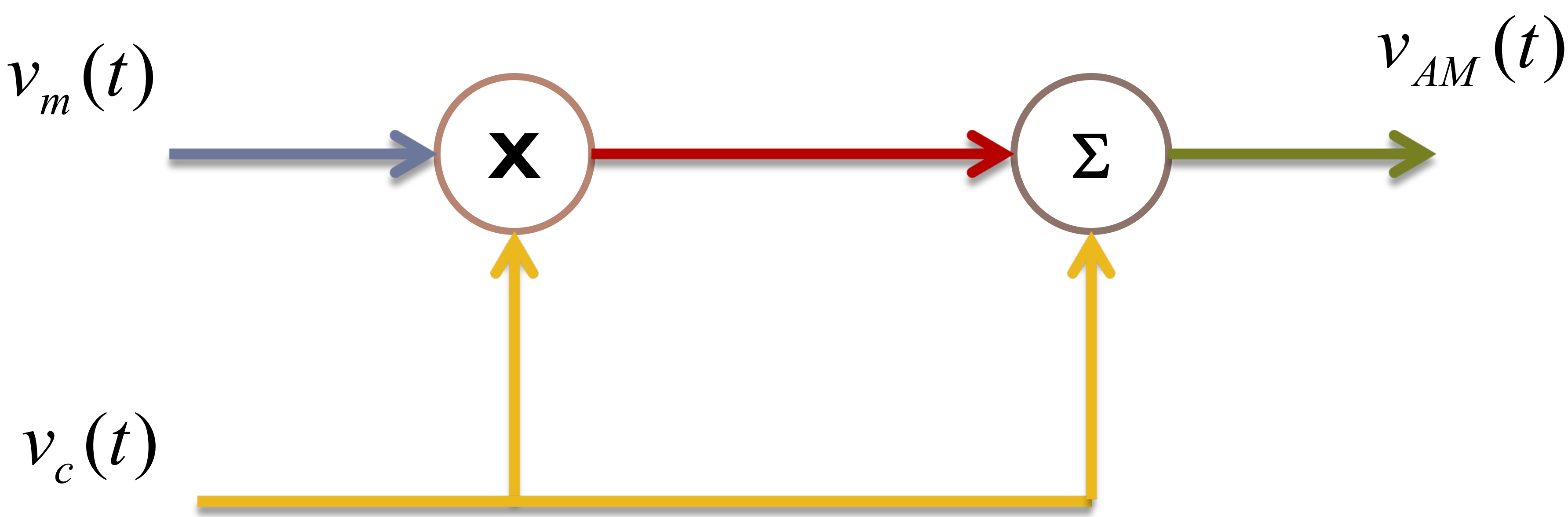


$BW = \ldots\ldots\ldots\ldots$

Amplitude Modulation: Generation DSB-TC

Generation DSB-TC

$$v_{AM}(t) = V_c \cos(2\pi\ f_c\ t) + V_m(t)\cos(2\pi\ f_c\ t)$$



Generation DSB-TC

Q: How is a DSB-TC signal generated?

Using a modulator system.

In general to modulate mean mixing frequencies to get a different frequency.

A linear system will not do that for us:

We need nonlinear systems to modulate

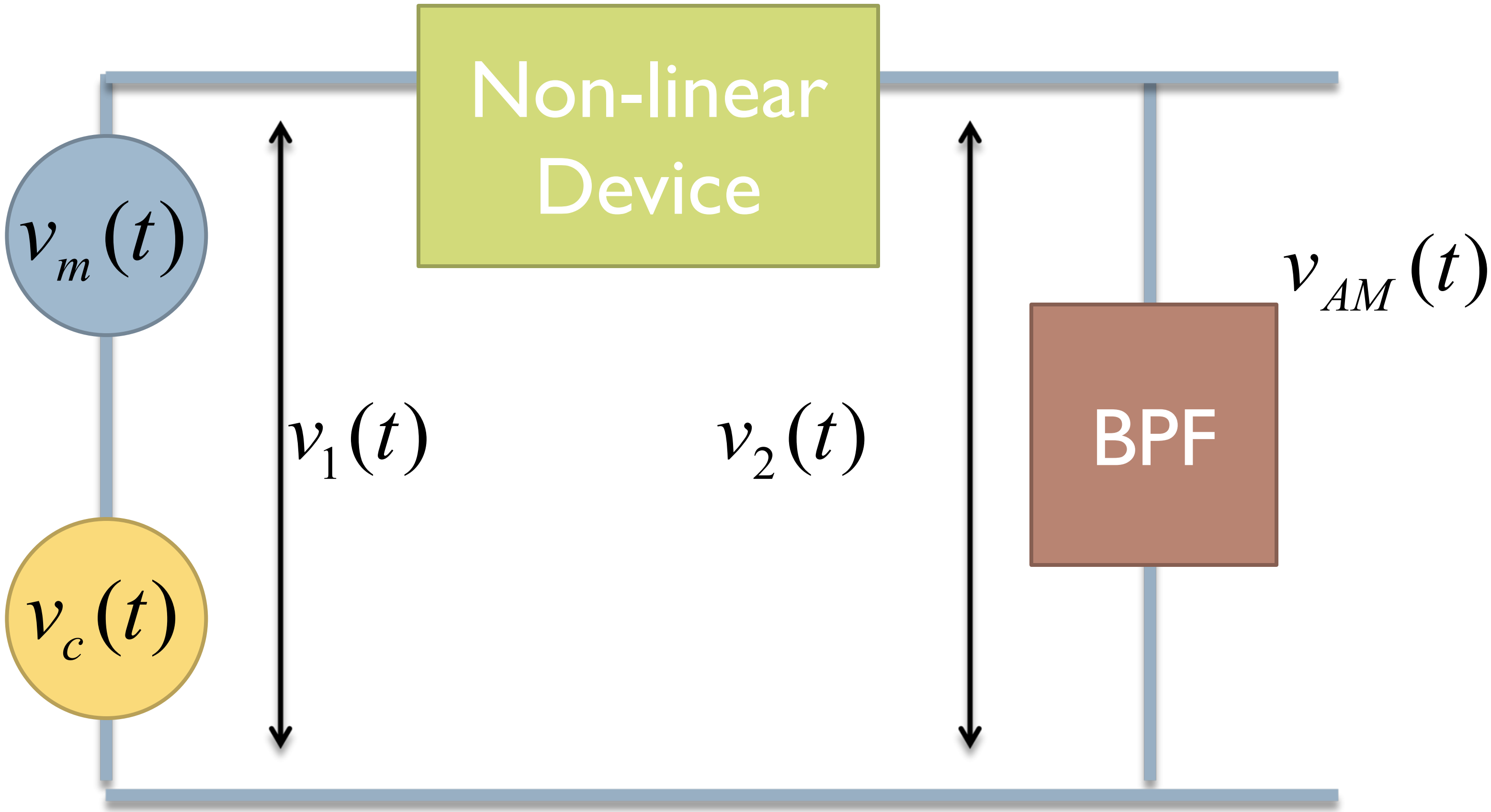
Generation DSB-LC

Q: What is an AM modulator?

Square Law Modulator

Square Law Modulator

Q: What is a square law modulator?



Square Law Modulator

Q: What is a square law modulator?

The input to the Nonlinear Devices (NLD) is equal to:

$$v_1(t) = v_m(t) + v_c(t)$$

$$v_1(t) = v_m(t) + V_c \cos(2\pi\, f_c t)$$

The output of the NLD is:

$$v_2(t) = a_0 + a_1 v_1(t) + a_2 v_1^{\,2}(t) + a_3 v_1^{\,3}(t) + \ldots..$$

We consider only second order terms:

$$v_2(t) \approx a_0 + a_1 v_1(t) + a_2 v_1^{\,2}(t)$$

Square Law Modulator

Q: What is output of the nonlinear device?

Substituting:

$$v_2(t) \approx a_0 + a_1\left[v_m(t) + V_c\cos(2\pi\ f_c t)\right] + a_2\left[v_m(t) + V_c\cos(2\pi\ f_c t)\right]^2$$

$$v_2(t) \approx a_0 + a_1 v_m(t) + a_1 V_c\cos(2\pi\ f_c t)$$
$$+ 2a_2 v_m(t) V_c\cos(2\pi\ f_c t) + a_2\left[v_m(t)\right]^2 + a_2 V_c^2\cos^2(2\pi\ f_c t)$$

$$v_2(t) \approx a_0 + a_1 v_m(t) + a_1 V_c\cos(2\pi\ f_c t)$$
$$+ 2a_2 v_m(t) V_c\cos(2\pi\ f_c t) + a_2\left[v_m(t)\right]^2 + \frac{a_2 V_c^2}{2}\left[1 + \cos(4\pi\ f_c t)\right]$$

Square Law Modulator

Q: What is output of the nonlinear device in the frequency domain?

Taking the Fourier Transform:

$$V_2(f) = \left[a_0 + \frac{a_2 V_c^2}{2}\right]\delta(f) + a_1 V_m(f) + \frac{a_1 V_c}{2}[\delta(f - f_c) + \delta(f + f_c)]$$

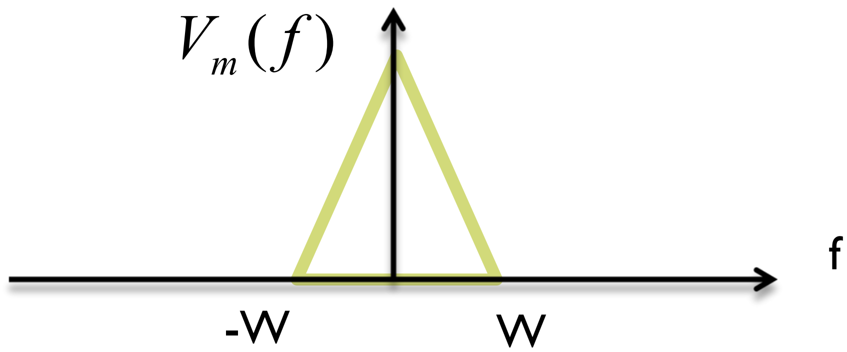
$$+ 2a_2 V_m(f) * \frac{V_c}{2}[\delta(f - f_c) + \delta(f + f_c)] + a_2(V_m(f) * V_m(f)) + \frac{a_2 V_c^2}{4}[\delta(f - 2f_c) + \delta(f + 2f_c)]$$

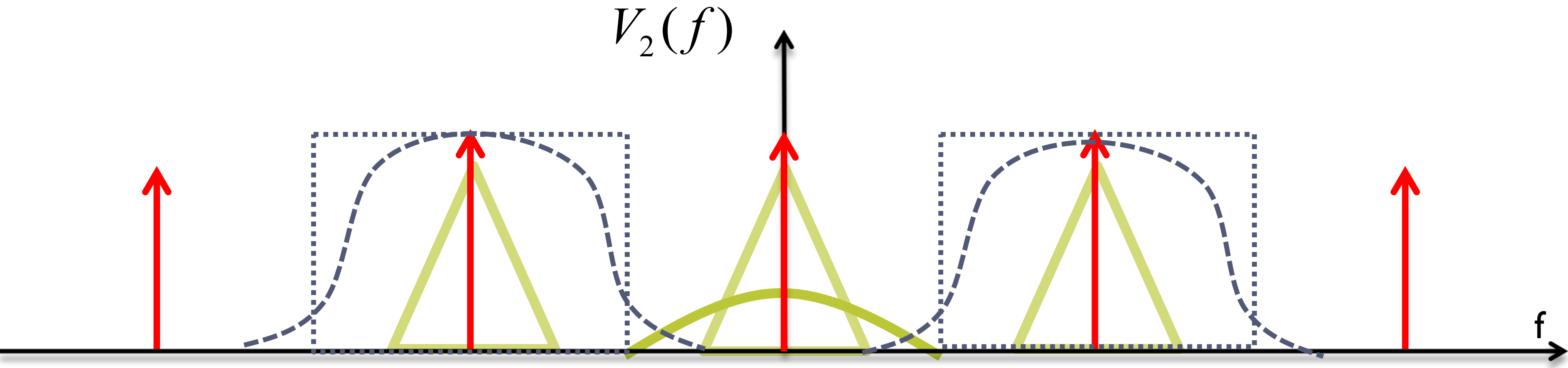
$$V_2(f) = \left[a_0 + \frac{a_2 V_c^2}{2}\right]\delta(f) + a_1 V_m(f) + \boxed{\frac{a_1 V_c}{2}[\delta(f - f_c) + \delta(f + f_c)]}$$

$$\boxed{+ a_2 V_c[V_m(f - f_c) + V_m(f + f_c)]} + a_2(V_m(f) * V_m(f)) + \frac{a_2 V_c^2}{4}[\delta(f - 2f_c) + \delta(f + 2f_c)]$$

Square Law Modulator

Q: Assuming the spectrum of the information signal, draw the $V_2(f)$.





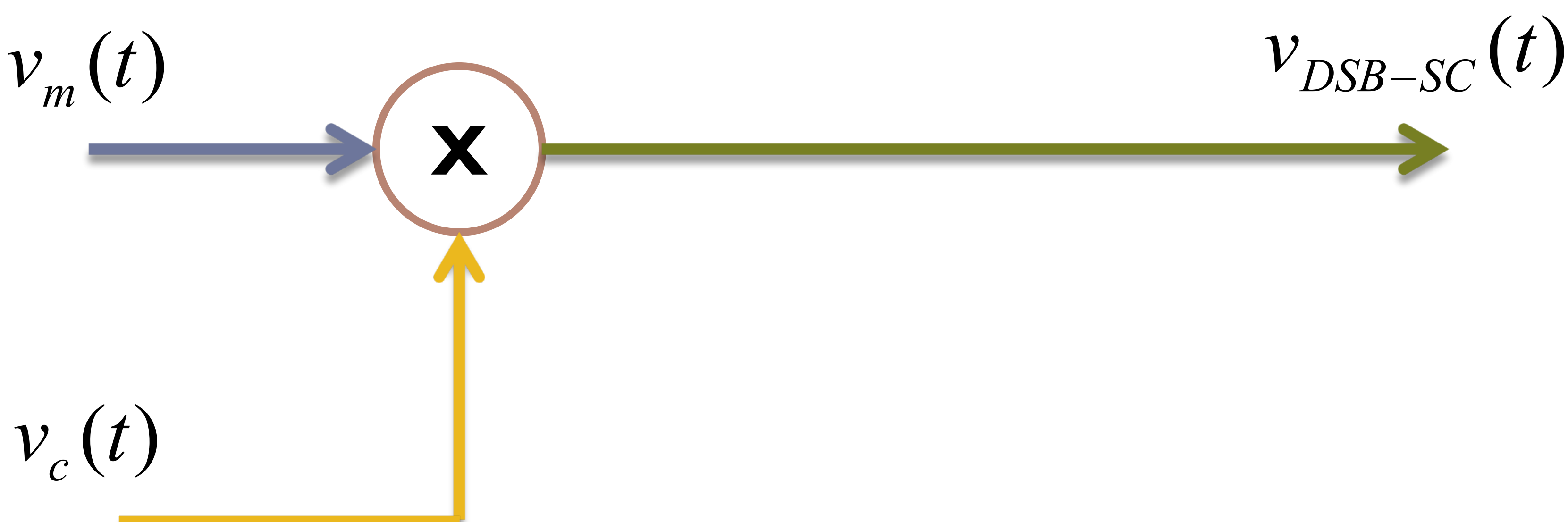
Amplitude Modulation: Generation DSB-SC

Generation DSB-SC

$$v_{DSB-SC}(t) = v_m(t)\cos(2\pi f_c t)$$

$$V_{DSB-SC}(f) = V_m(f) * \frac{1}{2}\left[\delta(f - f_c) + \delta(f - f_c)\right]$$

$$V_{DSB-SC}(f) = \frac{1}{2}\left[V_m(f - f_c) + V_m(f - f_c)\right]$$



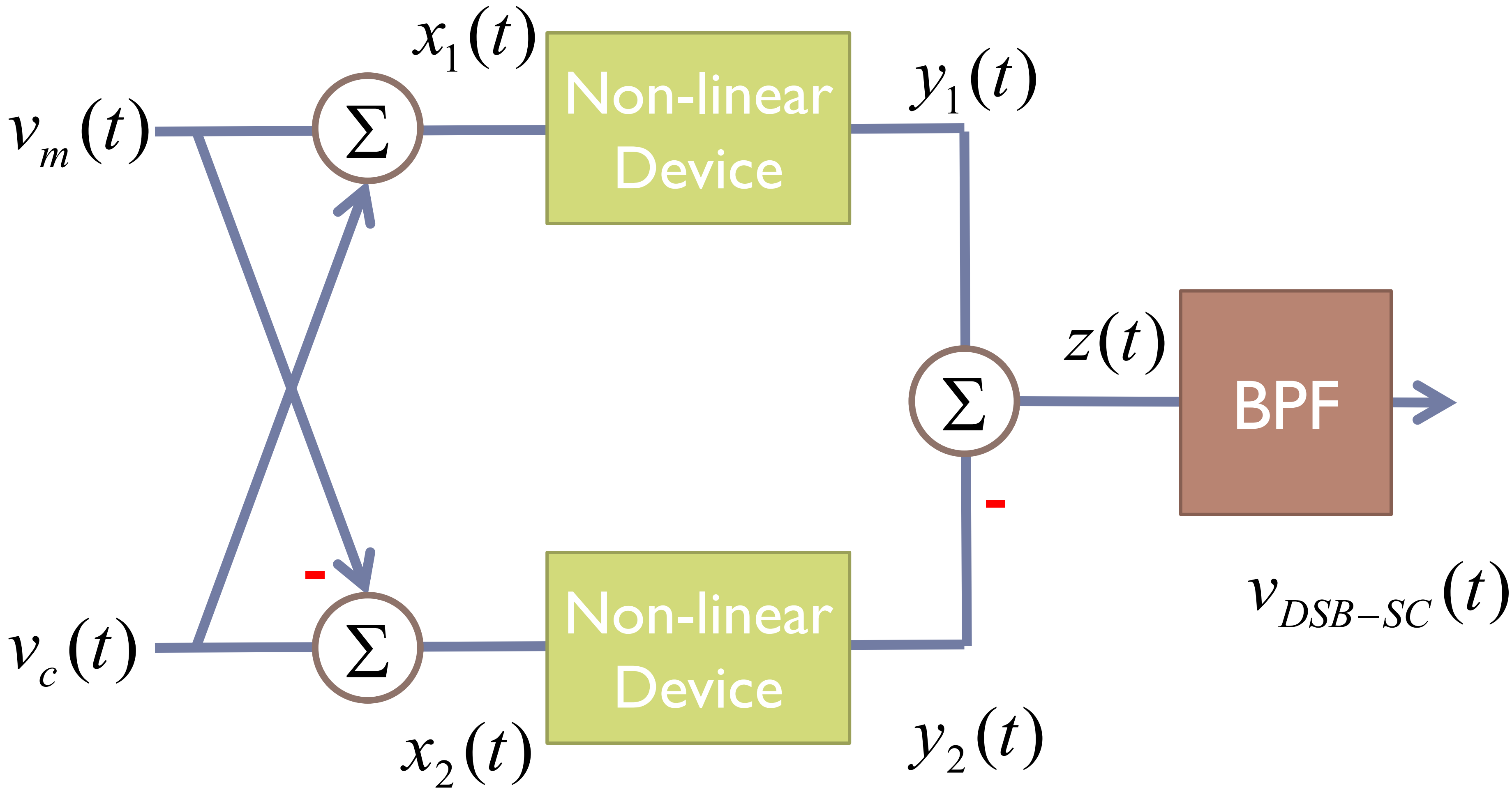
Generation DSB-SC

Q: What is a DSB-SC modulator?

Balanced Bridge Modulator

Balanced Bridge Modulator

Q: What is a square law modulator for DSB-SC?



Balanced Bridge Modulator

Q: What is a square law modulator?

The inputs to the Nonlinear Devices (NLD) are:

$$x_1(t) = v_c(t) + v_m(t) = V_c \cos(2\pi f_c t) + v_m(t)$$

$$x_2(t) = v_c(t) - v_m(t) = V_c \cos(2\pi f_c t) - v_m(t)$$

The output of the NLD is:

$$y_1(t) = a_0 + a_1 x_1(t) + a_2 x_1^2(t) + a_3 x_1^3(t) + \ldots..$$

$$y_2(t) = a_0 + a_1 x_2(t) + a_2 x_2^2(t) + a_3 x_2^3(t) + \ldots..$$

We consider only second order terms:

$$y_1(t) \approx a_0 + a_1 x_1(t) + a_2 x_1^2(t) \qquad y_2(t) \approx a_0 + a_1 x_2(t) + a_2 x_2^2(t)$$

Balanced Bridge Modulator

Q: What is output of the nonlinear device?

Substituting:

$$y_{1,2}(t) \approx a_0 + a_1\left[V_c \cos(2\pi\, f_c t) \pm v_m(t)\right] + a_2\left[V_c \cos(2\pi\, f_c t) \pm v_m(t)\right]^2$$

$$y_{1,2}(t) \approx a_0 \pm a_1 v_m(t) + a_1 V_c \cos(2\pi\, f_c t)$$
$$\pm 2a_2 v_m(t) V_c \cos(2\pi\, f_c t) + a_2\left[v_m(t)\right]^2 + a_2 V_c^2 \cos^2(2\pi\, f_c t)$$

$$y_{1,2}(t) \approx a_0 \pm a_1 v_m(t) + a_1 V_c \cos(2\pi\, f_c t)$$
$$\pm 2a_2 v_m(t) V_c \cos(2\pi\, f_c t) + a_2\left[v_m(t)\right]^2 + \frac{a_2 V_c^2}{2}\left[1 + \cos(4\pi\, f_c t)\right]$$

Balanced Bridge Modulator

Q: What is output of the nonlinear device?

Subtracting: $$z(t) = y_1(t) - y_2(t)$$

$$z(t) \approx a_0 \pm a_1 v_m(t) + a_1 V_c \cos(2\pi f_c t)$$

$$\pm 2a_2 v_m(t) V_c \cos(2\pi f_c t) + a_2 [v_m(t)]^2 + \frac{a_2 V_c^2}{2}[1 + \cos(4\pi f_c t)]$$

$$z(t) = 2a_1 v_m(t) + 4a_2 v_m(t) V_c \cos(2\pi f_c t)$$

After BPF:

$$V_{DSB-SC}(t) = 4a_2 v_m(t) V_c \cos(2\pi f_c t)$$

This is a single balanced bridge circuit

Amplitude Modulation: Generation SSB

Generation SSB

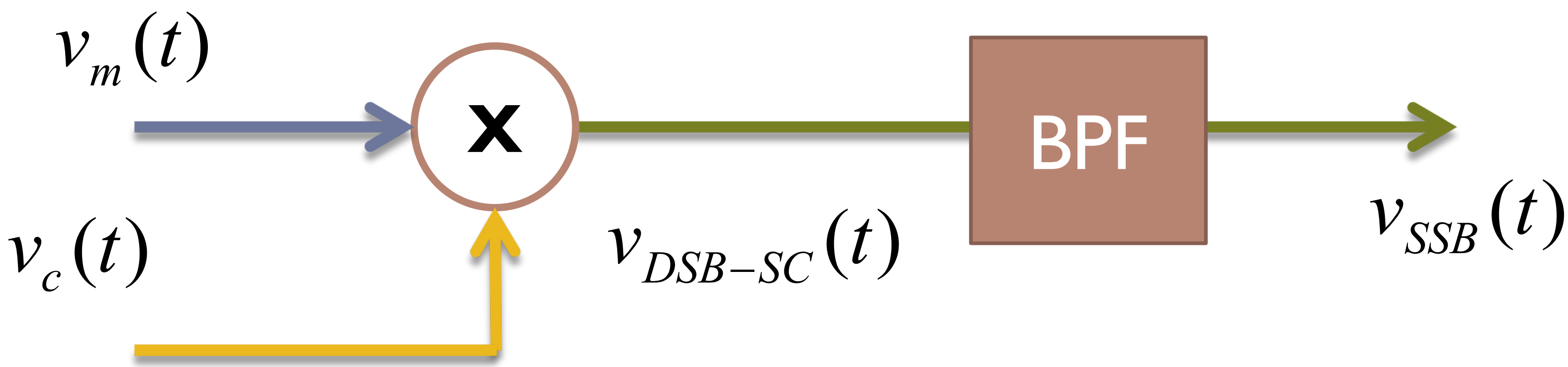
Q: What is a SSB modulator?

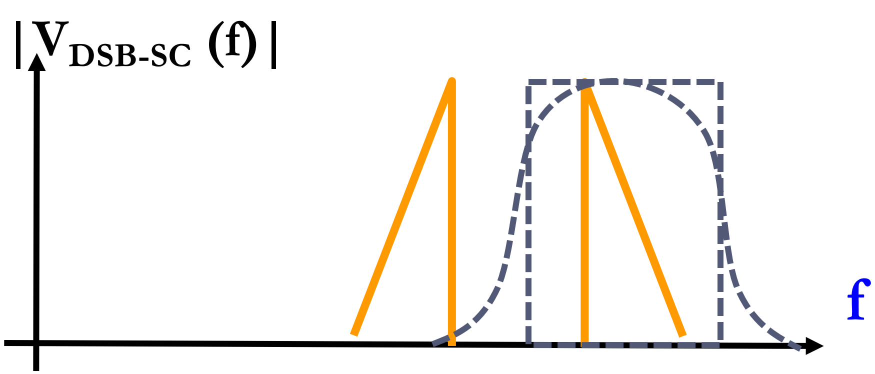
We will study two types:

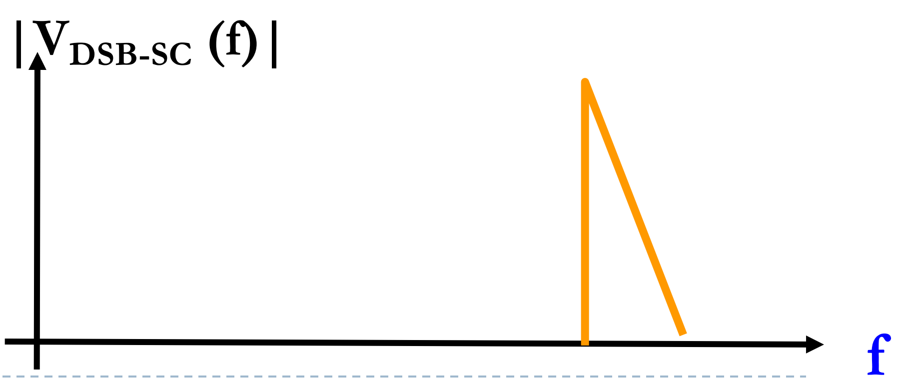
1. Filtering Method
2. Phase shift method

Filtering Method

$$v_{SSB}(t) = \cos(2\pi\left(f_c + f_m\right)\ t) = \cos(\left(\omega_c + \omega_m\right)\ t)$$

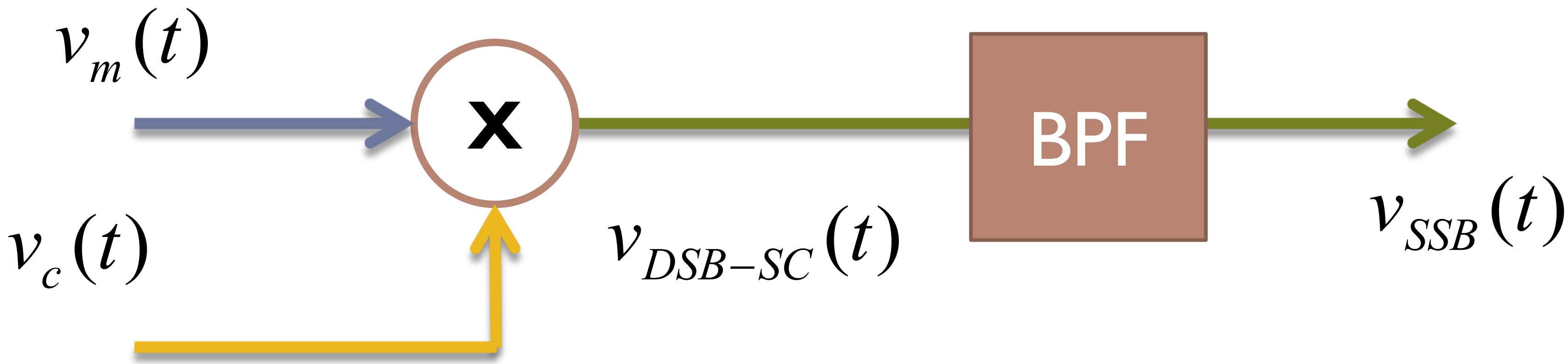


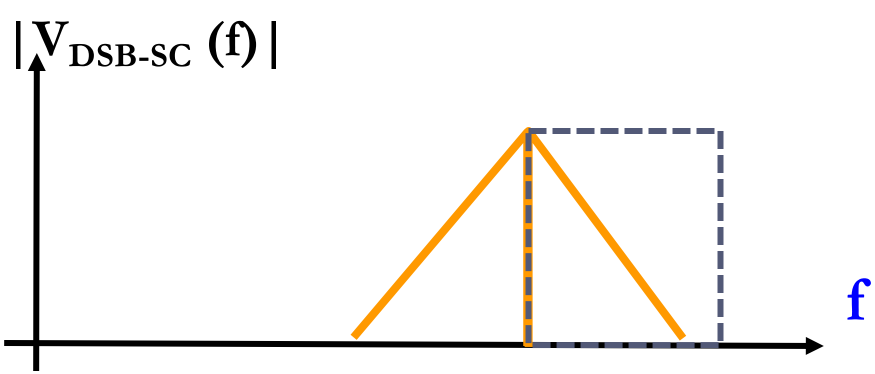


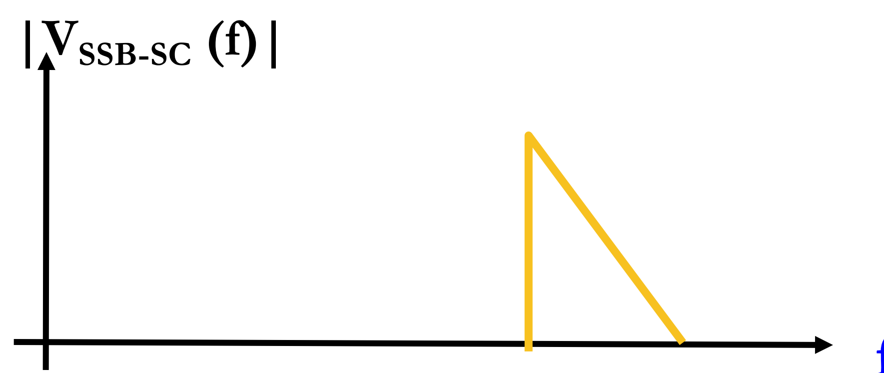


Limitations of Filtering Method

$$v_{SSB}(t) = \cos(2\pi\left(f_c + f_m\right)\ t) = \cos(\left(\omega_c + \omega_m\right)\ t)$$







Phase shift method

Q: Phase shift method generation SSB?

Starting with:

$$v_{SSB}(t) = \cos(2\pi \left(f_c + f_m\right) t) = \cos(\left(\omega_c + \omega_m\right) t)$$

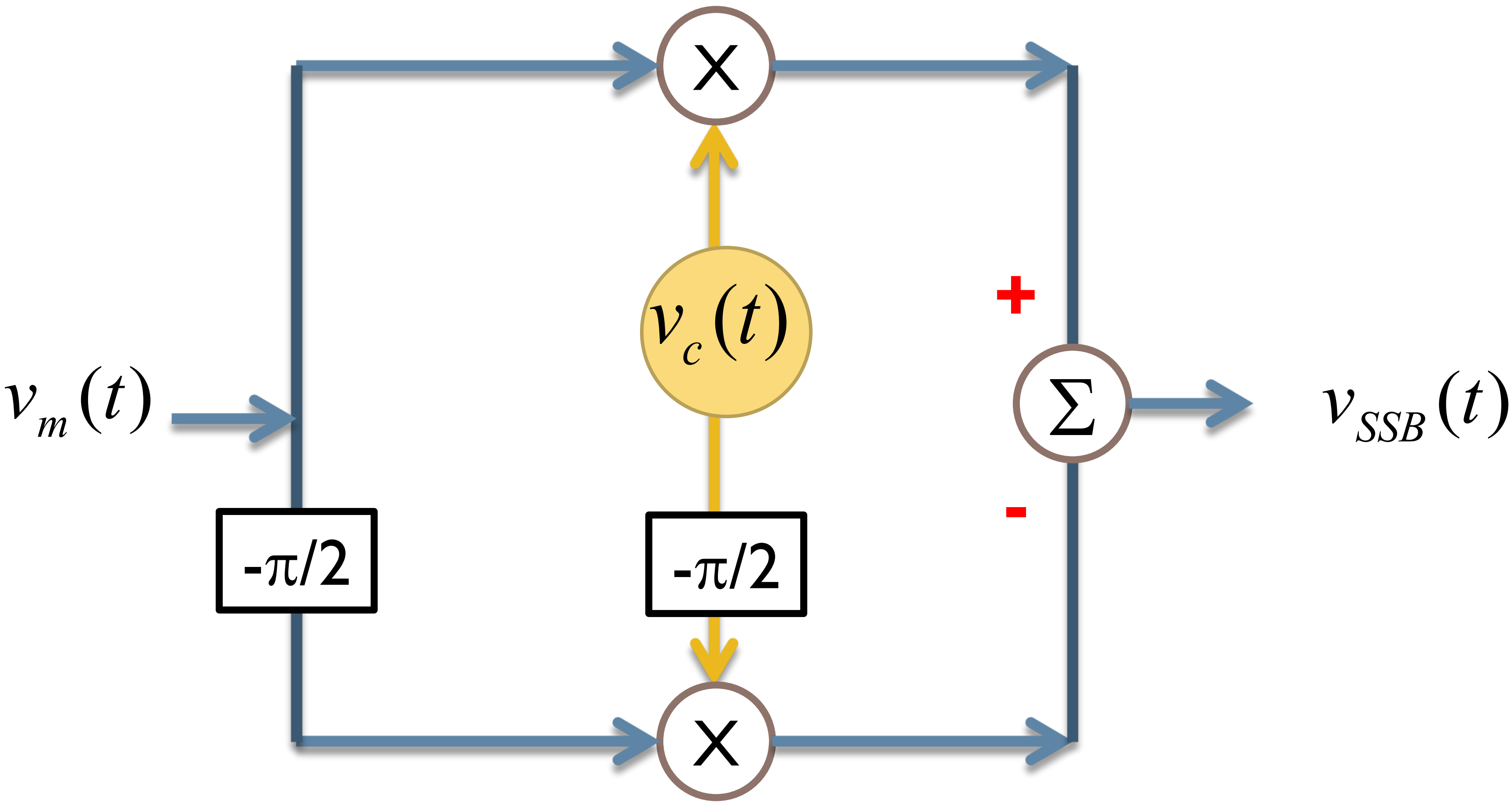
Using:

$\cos(\alpha + \beta) = \cos(\alpha) \cos(\beta) - \sin(\alpha) \sin(\beta)$

We get:

$$v_{SSB}(t) = \cos\left(\omega_c t\right) \cos\left(\omega_m t\right) - \sin\left(\omega_c t\right) \sin\left(\omega_m t\right)$$

Phase shift method



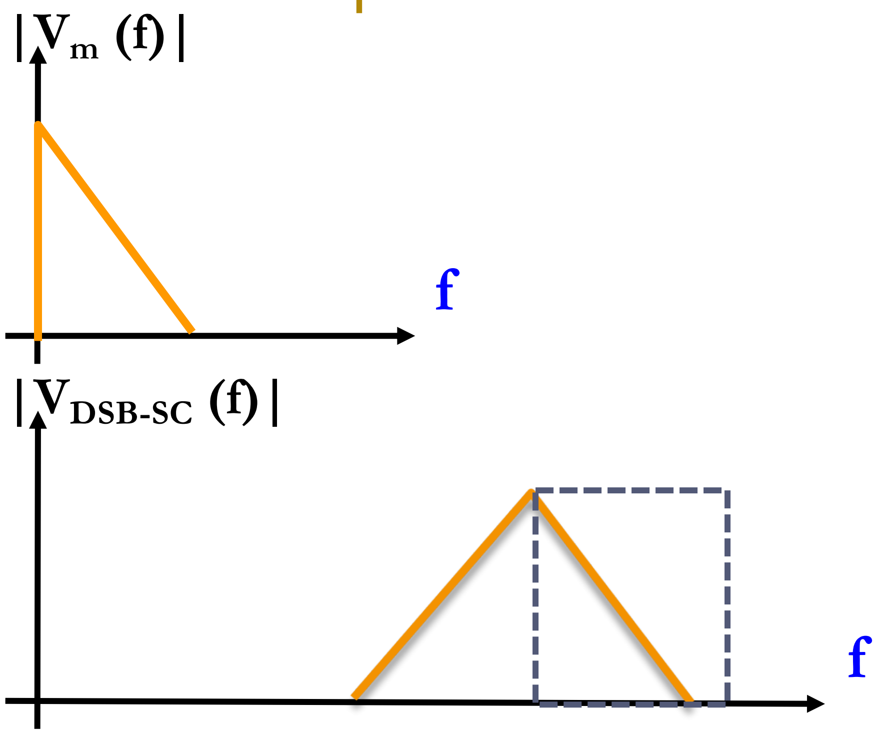
Amplitude Modulation: VSB

Vestigial Side Band (VSB

Q: What is Vestigial Side Band Modulation?

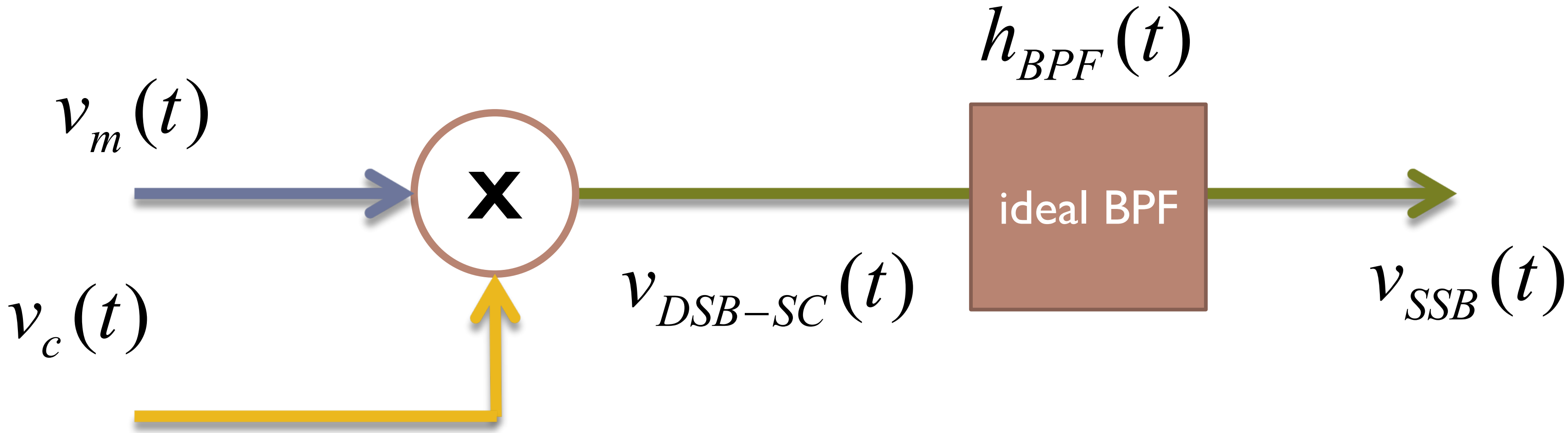
It is between DSB and SSB

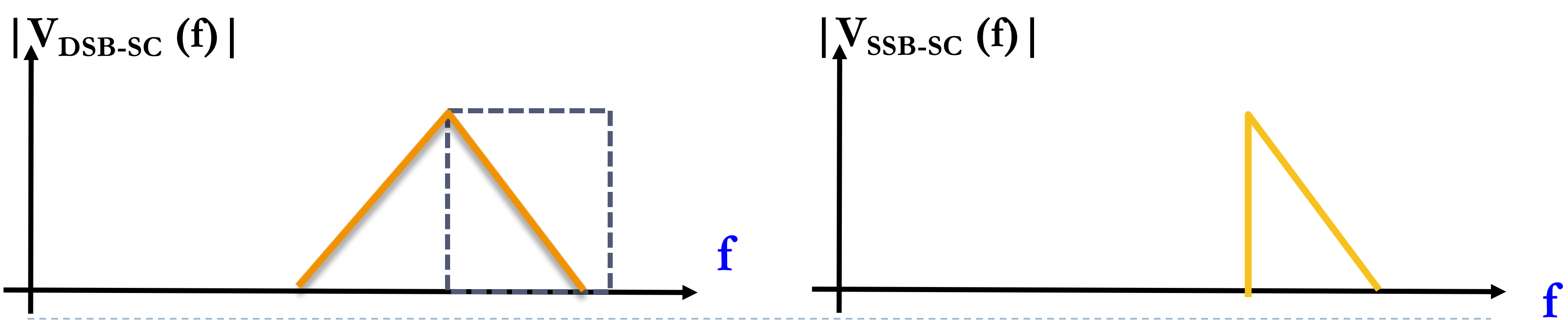
It is used for examples where it is difficult to filter out the LSB because the message signal contains frequencies down to DC



Vestigial Side Band (VSB)

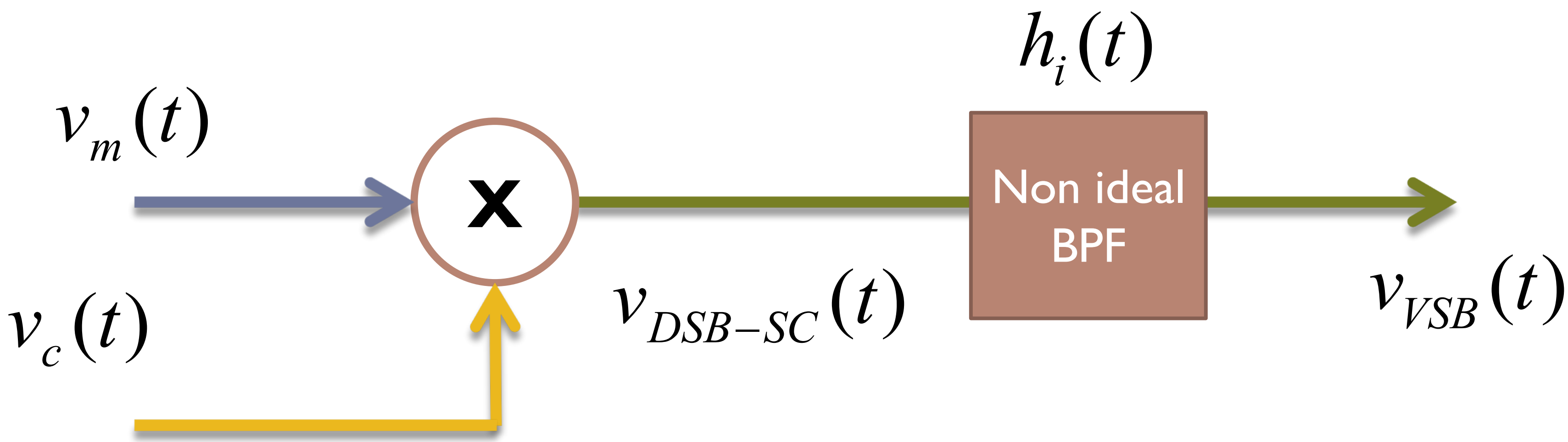
$$v_{DSB-SC}(t) = v_m(t)\cos(2\pi f_c t)$$

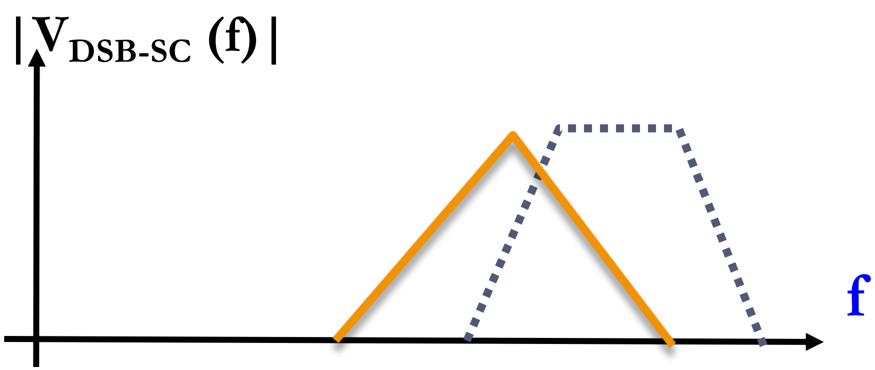


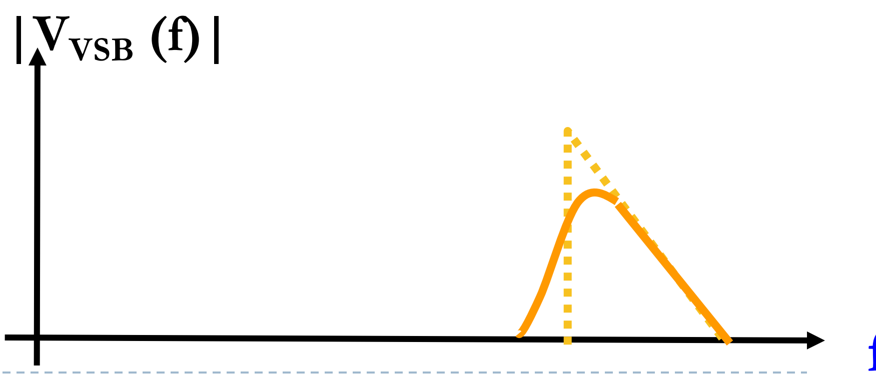


Vestigial Side Band (VSB)

$$v_{DSB-SC}(t) = v_m(t)\cos(2\pi f_c t)$$







Vestigial Side Band (VSB)

For SSB: $$v_{SSB}(t) = v_{DSB-SC}(t) * h_{BPF}(t)$$

$$V_{SSB}(f) = V_{DSB-SC}(f) \bullet H_{BPF}(f)$$

$$H_{BPF}(f) = \begin{cases} 1 & f_c \le f \le f_c + f_m \\ 0 & otherwise \end{cases}$$

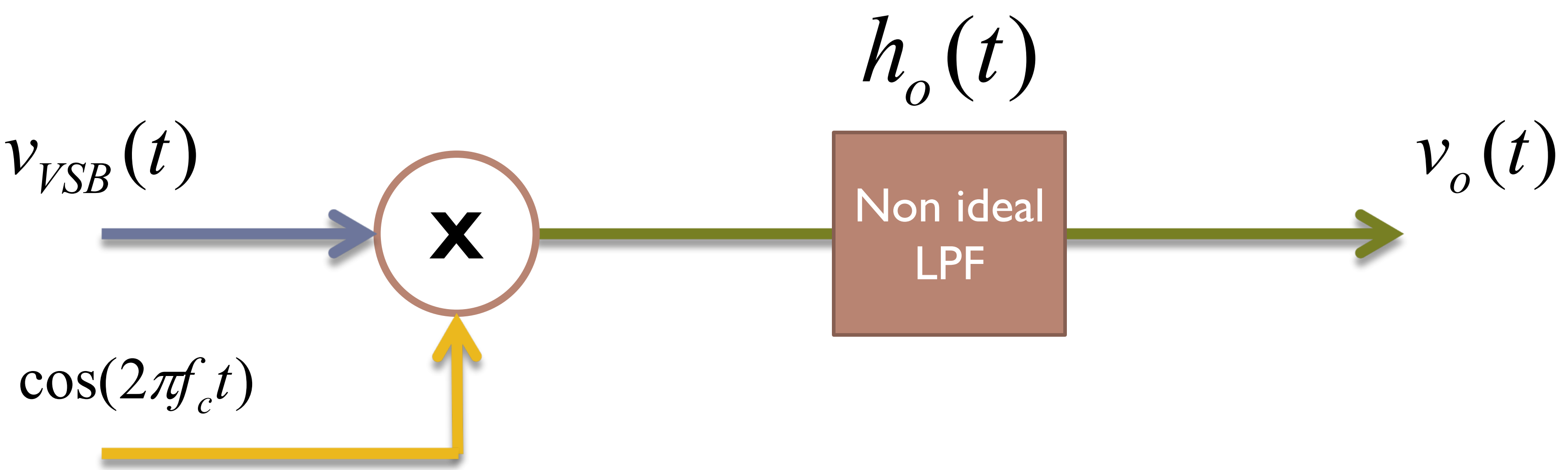
For VSB:

$$v_{VSB}(t) = v_{DSB-SC}(t) * h_i(t)$$

$$V_{VSB}(f) = V_{DSB-SC}(f) \bullet H_i(f) = \frac{1}{2}\left[V_m(f + f_c) + V_m(f - f_c)\right] \bullet H_i(f)$$

Vestigial Side Band (VSB)

We need to know the value of the filter such that we can recover the message when we demodulate.



$$v_o(t) = v_{VSB}(t)\cos(2\pi f_c t) * h_o(t)$$

$$V_o(f) = \frac{1}{2}\left[V_{VSB}(f - f_c) + V_{VSB}(f + f_c)\right] \bullet H_o(f)$$

Vestigial Side Band (VSB)

Q: What is the relationship between $H_o(f)$ and $H_i(f)$?

We will assume the output of the LPF is:

$$V_o(f) = \frac{1}{2}\left[V_{VSB}(f - f_c) + V_{VSB}(f + f_c)\right] \bullet H_o(f)$$

We know that:

$$V_{VSB}(f) = \frac{1}{2}\left[V_m(f + f_c) + V_m(f - f_c)\right] \bullet H_i(f)$$

Substituting:

$$V_o(f) = \frac{1}{4}$$

$$\begin{bmatrix} \left[V_m(f + f_c - f_c) + V_m(f - f_c - f_c)\right] \bullet H_i(f - f_c) \\ + \left[V_m(f + f_c + f_c) + V_m(f - f_c + f_c)\right] \bullet H_i(f + f_c) \end{bmatrix} \bullet H_o(f)$$

Vestigial Side Band (VSB)

Q: What is $H_V(f)$?

$$V_o(f)=\frac{1}{4}\begin{bmatrix}\left[V_m(f_c)+V_m(f-2f_c)\right]\bullet H_i(f-f_c)\\+\left[V_m(f+2f_c)+V_m(f_c)\right]\bullet H_i(f+f_c)\end{bmatrix}\bullet H_o(f)$$

$$V_o(f)=\frac{1}{4}\left[V_m(f)\bullet H_i(f-f_c)+V_m(f)\bullet H_i(f+f_c)\right]\bullet H_o(f)$$

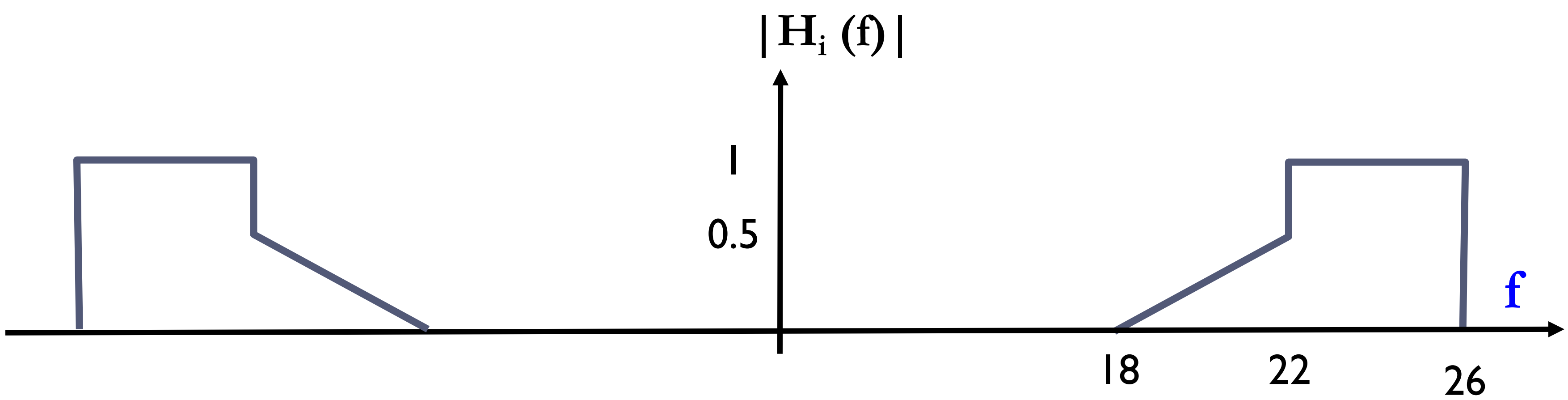
$$V_o(f)=\frac{1}{4}V_m(f)\left[H_i(f-f_c)+H_i(f+f_c)\right]\bullet H_o(f)$$

For $V_o(f) = k \,.V_m(f)$:

$$H_o(f)=\frac{1}{\left[H_i(f-f_c)+H_i(f+f_c)\right]}$$

Vestigial Side Band (VSB)

Exercise: The carrier for a VSB signal is 20kHz and the message signal is 6kHz. The VSB shaping filter $H_i(f)$if shown below. Find the output low pass filter $H_o(f)$.



Amplitude Modulation: QAM

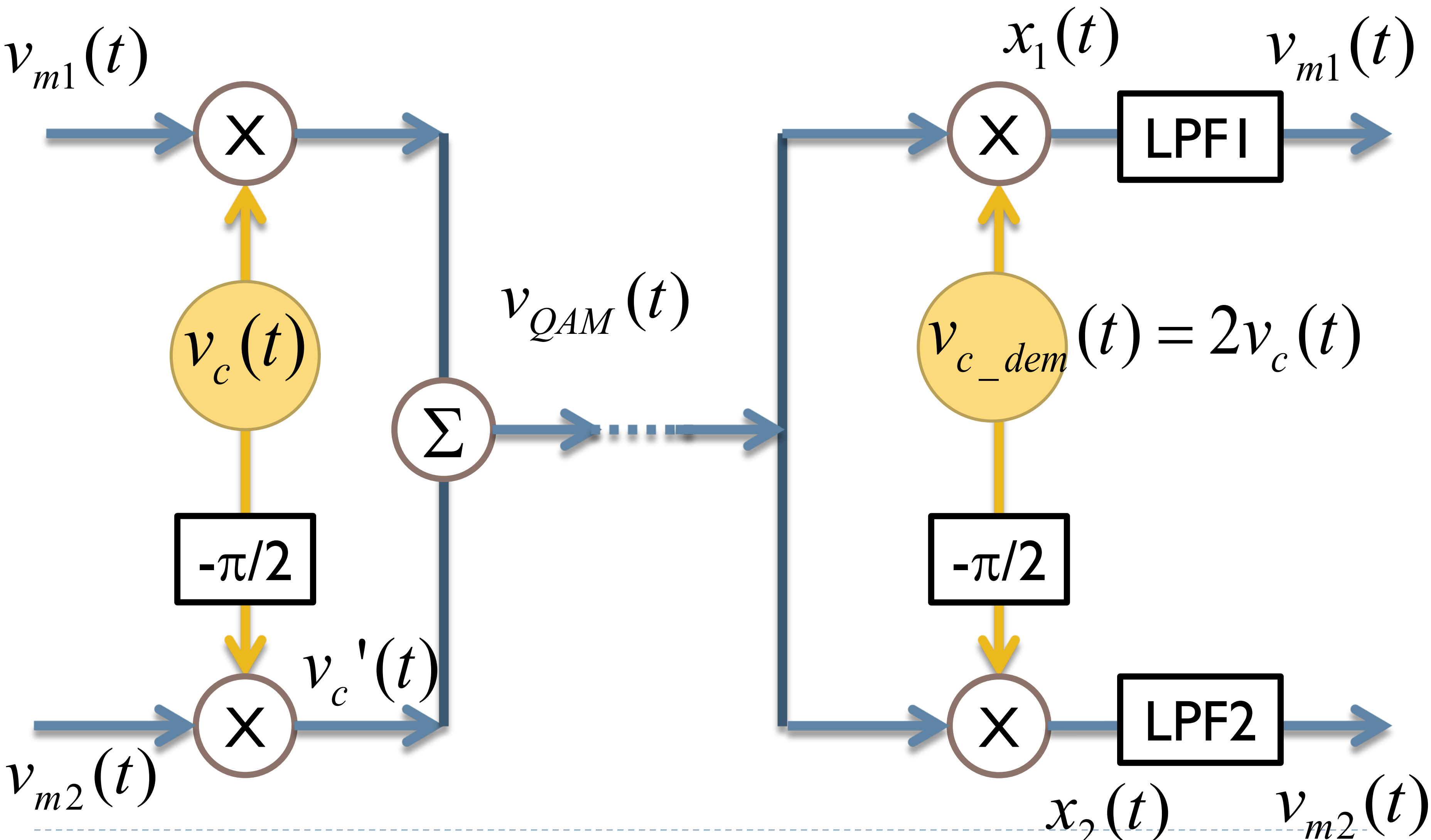
Quadrature Amplitude Modulation (QAM)

Q: What is QAM?

It is an AM technique that overcomes the BW issue of DSB-SC.

It is basically transmitting 2 information signals on the same carrier frequency.

Quadrature Amplitude Modulation (QAM)



Quadrature Amplitude Modulation (QAM)

The carrier frequency

$$v_c(t) = \cos(\omega_c t)$$

After the -90 phase shift

$$v_c'(t) = \sin(\omega_c t)$$

Upper arm (in-phase or I channel):

$$v_{m1}(t)\cos(\omega_c t)$$

Lower arm (quadrature or Q channel)

$$v_{m2}(t)\sin(\omega_c t)$$

Output of the transmitter:

$$v_{QAM}(t) = v_{m1}(t)\cos(\omega_c t) + v_{m2}(t)\sin(\omega_c t)$$

Quadrature Amplitude Modulation (QAM)

At the receiver:

Upper arm:

$$v_{c\_dem}(t) = 2\cos(\omega_c t)$$

$$x_1(t) = v_{QAM}(t).v_{c\_dem}(t) = v_{QAM}(t).2\cos(\omega_c t)$$

$$x_1(t) = [v_{m1}(t)\cos(\omega_c t) + v_{m2}(t)\sin(\omega_c t)].2\cos(\omega_c t)$$

$$x_1(t) = v_{m1}(t) + v_{m1}(t)\cos(2\omega_c t) + v_{m2}(t)\sin(2\omega_c t)$$

After LPF 1:

$$x_{LPF1}(t) = v_{m1}(t)$$

Quadrature Amplitude Modulation (QAM)

Lower arm:

$$v'_{c\_dem}(t) = 2\sin(\omega_c t)$$

$$x_2(t) = v_{QAM}(t).v'_{c\_dem}(t) = v_{QAM}(t).2\sin(\omega_c t)$$

$$x_2(t) = \left[v_{m1}(t)\cos(\omega_c t) + v_{m2}(t)\sin(\omega_c t)\right].2\sin(\omega_c t)$$

After LPF2:

$$x_{LPF2}(t) = v_{m2}(t)$$

Quadrature Amplitude Modulation (QAM)

Exercise: QAM must be total synchronous. To see this, check what happens if the carrier in the demodulator is out of phase from the modulator by θ:

$$v_c(t) = \cos(\omega_c t)$$

$$v_{c\_dem}(t) = 2\cos(\omega_c t + \theta)$$