The *Filter Design Toolbox* of MATLAB has a large number of M-files to aid in the design of FIR digital filters. This toolbox may be useful in many practical applications. The article by Losada in the Mathworks website is recommended to the reader interested in finding out about some of the practical aspects of FIR filter design.[14]

10.8 Problems

10.1 Verify the FIR filter orders given in Table 10.1.

10.2 A lowpass FIR filter of order $N = 75$ is to be designed with a transition band given by $\omega_s - \omega_p = 0.05\pi$ using the Parks–McClellan method. Determine the approximate value of the stopband attenuation α_s in dB and the corresponding stopband ripple δ_s of the designed filter if the filter order is estimated using each of the following formulas: (a) Kaiser's formula of Eq. (10.3), (b) Bellanger's formula of Eq. (10.4), and (c) Hermann's formula of Eq. (10.5). Assume the passband and stopband ripples to be the same.

10.3 Repeat Problem 10.2 if the filter is designed using the Kaiser's window-based method.

10.4 Verify the expression for the impulse response coefficients $h_{ML}[n]$ given in Eq. (10.20) for the zero-phase multiband filter with a frequency response $H_{ML}(e^{j\omega})$ defined in Eq. (10.19) and shown in Figure 10.1.

10.5 Show that the ideal Hilbert transformer with a frequency response $H_{HT}(e^{j\omega})$ defined in Eq. (10.21) has an impulse response $h_{HT}[n]$ as given in Eq. (10.22). Since the impulse response is doubly infinite, the ideal discrete-time Hilbert transformer is not realizable. To make it realizable, the impulse response has to be truncated to $|n| \le M$. What type of linear-phase FIR filter is the truncated impulse response? Plot the frequency response of the truncated approximation for various values of M. Comment on your results.

10.6 Let $\mathcal{H}\{\cdot\}$ denote the ideal operation of Hilbert transformation defined by

$$\mathcal{H}\{x[n]\} = \sum_{\ell=-\infty}^{\infty} h_{HT}[n-\ell]x[\ell],$$

where $h_{HT}[n]$ is as given in Eq. (10.22). Evaluate the following quantities:

$$\text{(a) } \mathcal{H}\{\mathcal{H}\{\mathcal{H}\{\mathcal{H}\{x[n]\}\}\}\}, \quad \text{(b) } \sum_{\ell=-\infty}^{\infty} x[\ell]\mathcal{H}\{x[\ell]\}.$$

10.7 Let $h_{LP}[n]$ denote the impulse response of an ideal lowpass filter with a cutoff at $\omega_c = \pi/2$. Show that

$$h_{HT}[n] = (-1)^n 2h_{LP}[2n]$$

is the impulse response of an ideal Hilbert transformer [Che2001]. If $h_{LP}[n]$ is the impulse response of a causal Type 1 FIR lowpass filter of order N with $M = N/2$ odd, then show that the Hilbert transformer obtained using the above relation is a causal Type 3 FIR filter of order M.

10.8 Show that the ideal differentiator with a frequency response $H_{DIF}(e^{j\omega})$ defined in Eq. (10.23) has an impulse response $h_{DIF}[n]$ as given in Eq. (10.24). Since the impulse response is doubly infinite, the ideal discrete-time differentiator is not realizable. To make it realizable, the impulse response has to be truncated to $|n| \le M$. What type of linear-phase FIR filter is the truncated impulse response? Plot the frequency response of the truncated approximation for various values of M. Comment on your results.

[14]See the white paper by R. Losada in the website **www.mathworks.com/products/filterdesign/**.

10.9 Develop the expression for the impulse response $\hat{h}_{HP}[n]$ of a causal highpass FIR filter of length $N = 2M + 1$ obtained by truncating and shifting the impulse response $h_{HP}[n]$ of the ideal highpass filter given by Eq. (10.16). Show that the causal lowpass FIR filter $\hat{h}_{LP}[n]$ of Eq. (10.15) and $\hat{h}_{HP}[n]$ are a delay-complementary pair.

10.10 Determine the impulse response $h_{LLP}[n]$ of a zero-phase ideal linear passband lowpass filter characterized by a frequency response shown in Figure P10.1(a).

10.11 Determine the impulse response $h_{BLDIF}[n]$ of a zero-phase ideal band-limited differentiator characterized by a frequency response shown in Figure P10.1(b).

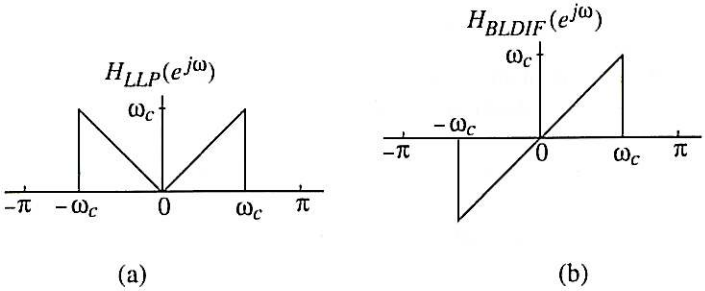

Figure P10.1

10.12 The magnitude response of an ideal *notch filter* $H_{\text{notch}}(z)$ is defined by

$$|H_{\text{notch}}(e^{j\omega})| = \begin{cases} 0, & |\omega| = \omega_o, \\ 1, & \text{otherwise,} \end{cases} \tag{10.115}$$

where ω_o is the *notch frequency*. Determine its impulse response $h_{\text{notch}}[n]$ [Yu90].

10.13 Let $h_d[n], -\infty < n < \infty$, denote the impulse response samples of a zero-phase filter with a frequency response $H_d(e^{j\omega})$. We have shown in Section 10.2.1 that the frequency response $H_t(e^{j\omega})$ of the zero-phase FIR filter $h_t[n], -M \leq n \leq M$, obtained by multiplying $h_d[n]$ with a rectangular window $w_R[n], -M \leq n \leq M$, has the least integral-squared error Φ_R defined in Eq. (10.9). Let Φ_{Hann} denote the integral-squared error if a length-$2M+1$ Hann window is used to develop the FIR filter. Determine an expression for the excess error $\Phi_{excess} = \Phi_R - \Phi_{Hann}$.

10.14 Repeat Problem 10.13 if a Hamming window is used instead.

10.15 For each of the lowpass filter specifications given below, design an FIR filter with the smallest length meeting the specifications using the window-based approach, and plot its magnitude response using MATLAB:
(a) $\omega_p = 0.47\pi, \omega_s = 0.59\pi, \delta_p = 0.001, \delta_s = 0.007$, (b) $\omega_p = 0.61\pi, \omega_s = 0.78\pi, \delta_p = 0.001, \delta_s = 0.002$.

10.16 Design a bandpass FIR filter with the smallest length using the window-based approach and meeting the following specifications: $\omega_{p1} = 0.45\pi, \omega_{p2} = 0.65\pi, \omega_{s1} = 0.3\pi, \omega_{s2} = 0.8\pi, \delta_p = 0.01, \delta_{s1} = 0.008$, and $\delta_{s2} = 0.05$, where δ_{s1} and δ_{s2} are, respectively, the ripple in the lower and upper stopbands. Plot the magnitude response of the filter designed using MATLAB.

10.17 Design a bandstop FIR filter with the smallest length using the window-based approach and meeting the following specifications: $\omega_{p1} = 0.3\pi, \omega_{p2} = 0.8\pi, \omega_{s1} = 0.45\pi, \omega_{s2} = 0.65\pi, \delta_{p1} = 0.05, \delta_{p2} = 0.009$, and $\delta_s = 0.02$, where δ_{p1} and δ_{p2} are, respectively, the ripple in the lower and upper passbands. Plot the magnitude response of the filter designed using MATLAB.

10.18 The frequency response of a zero-phase lowpass filter with a passband edge at ω_p, a stopband edge at ω_s, and a raised cosine transition function is given by [Bur92], [Par87]

$$H_{LP}(e^{j\omega}) = \begin{cases} 1, & 0 \le |\omega| < \omega_p, \\ \frac{1}{2}\left(1 + \cos\left(\frac{\pi(\omega - \omega_p)}{\omega_s - \omega_p}\right)\right), & \omega_p \le |\omega| \le \omega_s, \\ 0, & \omega_s < |\omega| < \pi. \end{cases} \tag{10.116}$$

Show that its impulse response is of the form

$$h_{LP}[n] = \left[\frac{\cos(\Delta\omega n/2)}{1 - (\Delta\omega/\pi)^2 n^2}\right] \cdot \frac{\sin(\omega_c n)}{\pi n}, \tag{10.117}$$

where $\Delta\omega = \omega_s - \omega_p$ and $\omega_c = (\omega_p + \omega_s)/2$.

10.19 The length-$(2M+1)$ Hann, Hamming, and Blackman window sequences given in Eqs. (10.30) to (10.32) are all of the form of raised cosine windows and can be expressed as

$$w_{GC}[n] = \left[\alpha + \beta\cos\left(\frac{2\pi n}{2M+1}\right) + \gamma\cos\left(\frac{4\pi n}{2M+1}\right)\right] w_R[n], \tag{10.118}$$

where $w_R[n]$ is a length-$(2M+1)$ rectangular window sequence. Express the Fourier transform of the above generalized cosine window in terms of the Fourier transform of the rectangular window $\Psi_R(e^{j\omega})$. From this expression, determine the Fourier transform of the Hann, Hamming, and Blackman window sequences.

10.20 In this problem, we consider the design of an FIR digital filter approximating a fractional delay z^{-D}:

$$z^{-D} \cong \sum_{n=0}^{N} h[n] z^{-n},$$

where the delay D is a positive real rational number.

(a) Show that the filter coefficients obtained using the Lagrange interpolation method [15] are given by [Laa96]

$$h[n] = \prod_{\substack{k=0 \\ k \ne n}}^{N} \frac{D-k}{n-k}, \qquad 0 \le n \le N.$$

(b) Design a length-21 FIR fractional delay filter with a delay of 90/13 samples. Using MATLAB, plot the group delay response of the designed filter along with that of the ideal fractional delay filter. Comment on your results.

10.21 An ideal zero-phase comb filter with notches at a fundamental frequency ω_o and its harmonics has a frequency response given by

$$H_{\text{comb}}(e^{j\omega}) = \begin{cases} 0, & \omega = k\omega_o, \; 0 \le k \le M, \\ 1, & \text{otherwise}. \end{cases} \tag{10.119}$$

If the input to the comb filter is of the form $x[n] = s[n] + r[n]$, where $s[n]$ is the desired signal and $r[n] = \sum_{k=0}^{M} A_k \sin(k\omega_o n + \phi_k)$ is the harmonic interference with a fundamental frequency ω_o, the comb filter suppresses the interference and generates $s[n]$ as its output. Let $D = 2\pi/\omega_o$ denote the fractional sample delay.

(a) Show that $r[n-D] = r[n]$.

(b) Next, by computing the output $y[n]$ of a filter $H(z) = 1 - z^{-D}$ whose input is $x[n]$, show that $y[n]$ does not contain any harmonic interference.

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[15] See Section 13.5.2.

(c) Even though the filter $H(z) = 1 - z^{-D}$ eliminates the harmonic interference completely, it does not have a unity magnitude at frequencies $\omega \neq k\omega_o$, thus introducing signal distortion at its output. The distortion in the passband of $H(z)$ can be eliminated by modifying the filter according to [Pei98]

$$H_c(z) = \frac{1 - z^{-D}}{1 - \rho^D z^{-D}},$$

where $0 < \rho < 1$. In practice, ρ should be close to 1. Using MATLAB, plot the magnitude response of $H_c(z)$ for $\omega_o = 0.22\pi$ and $\rho = 0.99$.

(d) Develop an efficient realization of the improved comb filter $H_c(z)$.

10.22 Using the method of Problem 10.21 and the FIR fractional delay filter design method of Problem 10.20, design a comb filter of order 18 for $\omega_o = 0.18\pi$ and $\rho = 0.98$. Using MATLAB, plot the magnitude response of the designed filter.

10.23 Using the method of Problem 10.21 and the allpass fractional delay filter design method of Problem 9.29, design an IIR comb filter of order 11 for $\omega_o = 0.18\pi$ and $\rho = 0.98$. Using MATLAB, plot the magnitude response of the designed filter.

10.24 By computing the inverse discrete-time Fourier transform of the frequency response $H_{LP}(e^{j\omega})$ of the zero-phase modified lowpass filter of Figure 10.13(a) with a first-order spline as the transition function, verify the expression for its impulse response $h_{LP}[n]$ as given in Eq. (10.43). Show that $h_{LP}[n]$ of Eq. (10.43) can also be derived by computing the inverse discrete-time Fourier transform of the derivative function $G(e^{j\omega})$ of Figure 10.13(b) and then using the differentiation-in-frequency property of the discrete-time Fourier transform given in Table 3.3.

10.25 Show that the impulse response $h_{LP}[n]$ of the zero-phase modified lowpass filter with a Pth-order spline as the transition function is given by Eq. (10.44).

10.26 Prove Eqs. (10.87a) and (10.87b).

10.27 Many applications require the fitting of a set of $2L+1$ equally spaced data samples $x[n]$ by a smooth polynomial $x_a(t)$ of degree N where $N < 2L$. In the least-squares fitting approach, the polynomial coefficients $\alpha_i, i = 0, 1, \ldots, N$, are determined so that the mean-square error

$$\varepsilon(\alpha_i) = \sum_{k=-L}^{L} \{x[k] - x_a(k)\}^2 \tag{10.120}$$

is a minimum [Ham89]. In smoothing a very long data sequence $x[n]$ based on the least-squares fitting approach, the central sample in a set of consecutive $2L + 1$ data samples is replaced by the polynomial coefficient minimizing the corresponding mean-square error.

(a) Develop the smoothing algorithm for $N = 1$ and $L = 5$, and show that it is a moving average FIR filter of length 5.

(b) Develop the smoothing algorithm for $N = 2$ and $L = 5$. What type of digital filter is represented by this algorithm?

(c) By comparing the frequency responses of the previous two FIR smoothing filters, select the filter that provides better smoothing.

10.28 An improved smoothing algorithm is *Spencer's* 15-point smoothing formula given by [Ham89]

$$\begin{aligned} y[n] = \tfrac{1}{320}\{&-3x[n-7] - 6x[n-6] - 5x[n-5] + 3x[n-4] \\ &+ 21x[n-3] + 46x[n-2] + 67x[n-1] + 74x[n] \\ &+ 67x[n+1] + 46x[n+2] + 21x[n+3] + 3x[n+4] \\ &- 5x[n+5] - 6x[n+6] - 3x[n+7]\}. \end{aligned} \tag{10.121}$$

Evaluate its frequency response and, comparing it with that of the two smoothing filters of Problem 10.27, show why Spencer's formula yields the better result.

10.29 In Problem 9.3, we considered filtering by a cascade of a number of identical filters. While the cascade provides more stopband attenuation than that obtained by a single filter section, it also increases the passband ripple or, in effect, decreases the passband width for a given maximum passband deviation. In the case of an FIR filter $H(z)$ with a symmetric impulse response, improved passband and stopband performances can be achieved by employing the *filter sharpening approach* [Kai77] in which the overall system $G(z)$ is implemented as

$$G(z) = \sum_{\ell=1}^{L} \alpha_\ell [H(z)]^\ell, \tag{10.122}$$

where $\{\alpha_\ell\}$ are real constants. In this problem, we outline the method of selecting the weighting coefficients $\{\alpha_\ell\}$ for a given L. It follows from the above that $G(z)$ is also an FIR filter with a symmetric impulse response. Let x denote a specific value of the amplitude response of $H(z)$ at a given angular frequency ω. If we denote the value of the amplitude response of $G(z)$ at this value of ω as $P(x)$, then it is related to x through

$$P(x) = \sum_{\ell=1}^{L} \alpha_\ell x^\ell. \tag{10.123}$$

$P(x)$ is called the *amplitude change function*. For a BR transfer function $H(z)$, $0 \le x \le 1$, where $x = 0$ is in the stopband and $x = 1$ is in the passband. If we further desire $G(z)$ to be a BR transfer function, then the amplitude change function must satisfy the two basic properties $P(0) = 0$ and $P(1) = 1$. Additional conditions on the amplitude change function are obtained by constraining the behavior of its slope at $x = 0$ and $x = 1$. To improve the performance of $G(z)$ in the stopband, we need to ensure

$$\left.\frac{d^k P(x)}{dx^k}\right|_{x=0} = 0, \quad k = 1, 2, \ldots, n, \tag{10.124}$$

and to improve the performance of $G(z)$ in the passband, we need to ensure

$$\left.\frac{d^k P(x)}{dx^k}\right|_{x=1} = 0, \quad k = 1, 2, \ldots, m, \tag{10.125}$$

where $m + n = L - 1$. Determine the coefficients $\{\alpha_\ell\}$ for $L = 3, 4$, and 5.

10.30 Consider a Type 3 linear-phase FIR filter with an amplitude response as given in Eq. (10.54). Show that if the amplitude response is symmetric, that is, $\breve{H}(\omega) = \breve{H}(\pi - \omega)$, then it is possible to choose the parameters $c[k]$ of Eq. (10.54) so that the even-indexed impulse response samples $h[n]$ are zero.

10.31 In the frequency sampling approach of FIR filter design, the specified frequency response $H_d(e^{j\omega})$ is first uniformly sampled at M equally spaced points $\omega_k = 2\pi k/M$, $0 \le k \le M - 1$, providing M *frequency samples* $H[k] = H_d(e^{j\omega_k})$. These M frequency samples constitute an M-point DFT $H[k]$, whose M-point inverse-DFT thus yields the impulse response coefficients $h[n]$ of the FIR filter of length M [Gol69a]. The basic assumption here is that the specified frequency response is uniquely characterized by the M frequency samples and, hence, can be fully recovered from these samples.

(a) Show that the transfer function $H(z)$ of the FIR filter can be expressed as

$$H(z) = \frac{1 - z^{-M}}{M} \sum_{k=0}^{M-1} \frac{H[k]}{1 - W_M^{-k} z^{-1}}.$$

(b) Develop a realization of the FIR filter based on the above expression.

(c) Show that the frequency response $H(e^{j\omega})$ of the FIR filter designed via the frequency sampling-based approach has exactly the specified frequency samples $H(e^{j\omega_k}) = H[k]$ at $\omega_k = 2\pi k/M$, $0 \le k \le M - 1$.

10.32 Let $|H_d(e^{j\omega})|$ denote the desired magnitude response of a real linear-phase FIR filter of length M.

(a) For M odd (Type 1 FIR filter), show that the DFT samples $H[k]$ needed for a frequency sampling-based design are given by

$$H[k] = \begin{cases} |H_d\left(e^{j2\pi k/M}\right)| e^{-j2\pi k(M-1)/2M}, & k = 0, 1, \ldots, \frac{M-1}{2}, \\ |H_d\left(e^{j2\pi k/M}\right)| e^{j2\pi(M-k)(M-1)/2M}, & k = \frac{M+1}{2}, \ldots, M-1. \end{cases} \quad (10.126)$$

(b) For M even (Type 2 FIR filter), show that the DFT samples $H[k]$ needed for a frequency sampling-based design are given by

$$H[k] = \begin{cases} |H_d\left(e^{j2\pi k/M}\right)| e^{-j2\pi k(M-1)/2M}, & k = 0, 1, \ldots, \frac{M}{2} - 1, \\ 0, & k = \frac{M}{2}, \\ |H_d\left(e^{j2\pi k/M}\right)| e^{j2\pi(M-k)(M-1)/2M}, & k = \frac{M}{2} + 1, \ldots, M-1. \end{cases} \quad (10.127)$$

10.33 Design a linear-phase FIR lowpass filter of length 19 with a passband edge at $\omega_p = 0.55\pi$ using the frequency sampling approach. Assume an ideal brickwall characteristic for the desired magnitude response.

(a) Using Eq. (10.126), develop the exact values for the desired frequency samples.

(b) Using MATLAB, plot the magnitude response of the designed filter.

10.34 Design a linear-phase FIR lowpass filter of length 39 with a passband edge at $\omega_p = 0.35\pi$ using the frequency sampling approach. Assume an ideal brickwall characteristic for the desired magnitude response.

(a) Using Eq. (10.126), develop the exact values for the desired frequency samples.

(b) Using MATLAB, plot the magnitude response of the designed filter.

10.35 By solving Eq. (10.70), derive the value of ε given by Eq. (10.71).

10.36 Determine the weighting function $W(\omega)$ that is to be used to design a Type 1 linear-phase FIR lowpass filter using the Parks–McClellan method to meet the following specifications: $\omega_p = 0.45\pi$, $\omega_s = 0.6\pi$, $\delta_p = 0.2043$, and $\delta_s = 0.0454$.

10.37 Determine the weighting function $W(\omega)$ that is to be used to design a Type 1 linear-phase FIR highpass filter using the Parks–McClellan method to meet the following specifications: $\omega_p = 0.7\pi$, $\omega_s = 0.55\pi$, $\delta_p = 0.03808$, and $\delta_s = 0.0112$.

10.38 Determine the weighting function $W(\omega)$ that is to be used to design a Type 1 linear-phase FIR bandpass filter using the Parks–McClellan method to meet the following specifications: $\omega_{p1} = 0.55\pi$, $\omega_{p2} = 0.7\pi$, $\omega_{s1} = 0.44\pi$, $\omega_{s2} = 0.82\pi$, $\delta_p = 0.01$, $\delta_{s1} = 0.007$, and $\delta_{s2} = 0.002$, where δ_{s1} and δ_{s2} are, respectively, the ripples in the lower and upper stopbands.

10.39 Show that the condition of Eq. (10.90) on the impulse response samples $h_{HT}[n]$ of an ideal Hilbert transformer cannot be met by a Type 4 linear-phase FIR filter.

10.40 The *warped discrete Fourier transform* (WDFT) can be employed to determine the N frequency samples of the z-transform $X(z)$ of a length-N sequence $x[n]$ at a warped frequency scale on the unit circle. The N-point WDFT $\check{X}[k]$ of $x[n]$ is given by the N equally spaced frequency samples on the unit circle of the modified z-transform $X(\check{z})$ obtained by applying an allpass first-order spectral transformation to $X(z)$ [Mak2001]:

$$X(\check{z}) = X(z)\Big|_{z^{-1} = \frac{-\alpha + \check{z}^{-1}}{1 - \alpha\check{z}^{-1}}} = \frac{P(\check{z})}{D(\check{z})}, \quad (10.128)$$

where $|\alpha| < 1$. Thus, the N-point WDFT $\check{X}[k]$ of $x[n]$ is given by

$$\check{X}[k] = X(\check{z})\big|_{\check{z} = e^{j2\pi k/N}}, \quad 0 \le k \le N-1. \quad (10.129)$$

(a) Develop the expressions for $P(\check{z})$ and $D(\check{z})$.

(b) If we denote

$$P(\check{z}) = \sum_{n=0}^{N-1} p[n]\check{z}^{-n} \text{ and } D(\check{z}) = \sum_{n=0}^{N-1} d[n]\check{z}^{-n},$$

show that $\check{X}[k] = P[k]/D[k]$, where $P[k]$ and $D[k]$ are, respectively, the N-point DFTs of the sequences $p[n]$ and $d[n]$.

(c) If we denote $\mathbf{P} = [p[0]\ p[1]\ \cdots p[N-1]]^T$, and $\mathbf{X} = [x[0]\ x[1]\ \cdots x[N-1]]^T$, show that $\mathbf{P} = \mathbf{Q} \cdot \mathbf{X}$, where $\mathbf{Q} = [q_{r,s}]$ is a real $N \times N$ matrix whose first row is given by $q_{0,s} = \alpha^s$, first column is given by $q_{r,0} = {}^{N-1}C_r \alpha^r$, and remaining elements $q_{r,s}$ can be derived using the recursion relation

$$q_{r,s} = q_{r-1,s-1} + \alpha q_{r,s-1} - \alpha q_{r-1,s}.$$

10.9 MATLAB Exercises

M 10.1 Plot the magnitude response of a linear-phase FIR highpass filter by truncating the impulse response $h_{HP}[n]$ of the ideal highpass filter of Eq. (10.16) to length $N = 2M + 1$ for two different values of M, and show that the truncated filter exhibits oscillatory behavior on both sides of the cutoff frequency.

M 10.2 Plot the magnitude response of a linear-phase FIR bandpass filter by truncating the impulse response $h_{BP}[n]$ of the ideal bandpass filter of Eq. (10.17) to length $N = 2M + 1$ for two different values of M, and show that the truncated filter exhibits oscillatory behavior on both sides of the cutoff frequency.

M 10.3 Plot the magnitude response of a linear-phase FIR Hilbert transformer by truncating the impulse response $h_{HT}[n]$ of the ideal Hilbert transformer of Eq. (10.22) to length $N = 2M + 1$ for two different values of M, and show that the truncated filter exhibits oscillatory behavior near $\omega = 0$ and $\omega = \pi$.

M 10.4 The impulse response obtained by convolving K rectangular windows of length N each approximates an ideal zero-mean FIR Gaussian filter. Using the M-file `firgauss`, generate several such filters, and show that the approximation gets better with an increase in K or N.

M 10.5 Write a MATLAB program to design a linear-phase FIR notch filter by windowing the impulse response of the ideal notch filter of Problem 10.12. Using this program, design an FIR notch filter of order 36 operating at a 500-Hz sampling rate with a notch frequency of 50 Hz.

M 10.6 Determine a linear approximation $a_0 + a_1 x$ to the quadratic function $D(x) = 3.2x^2 + 4.05x - 5.5$ defined for the range $-3 \le x \le 2$ by minimizing the peak value of the absolute error $|D(x) - a_0 - a_1 x|$, i.e.,

$$\max_{-3 \le x \le 2} |D(x) - a_0 - a_1 x|,$$

using the Remez algorithm. Plot the error function after convergence of the algorithm.

M 10.7 Determine a quadratic approximation $a_0 + a_1 x + a_2 x^2$ to the cubic function $D(x) = -5x^3 - 0.2x^2 + 8x + 5.5$ defined for the range $-2 \le x \le 2$ by minimizing the peak value of the absolute error $\left|D(x) - a_0 - a_1 x - a_2 x^2\right|$, i.e.,

$$\max_{-2 \le x \le 2} \left|D(x) - a_0 - a_1 x - a_2 x^2\right|,$$

using the Remez algorithm. Plot the error function after convergence of the algorithm.

M 10.8 Using the windowed Fourier series approach, design a linear-phase FIR lowpass filter with the following specifications: passband edge at 4 rad/sec, stopband edge at 6 rad/sec, maximum passband attenuation of 0.2 dB, minimum stopband attenuation of 42 dB, and a sampling frequency of 18 rad/sec. Use each of the following windows for the design: Hamming, Hann, and Blackman. Show the impulse response coefficients, and plot the gain response of the designed filters for each case. Comment on your results. Do not use the M-file `fir1`.

M 10.9 Repeat Exercise M10.8 using the Kaiser window. Do not use the M-file `fir1`.

M 10.10 Using the windowed Fourier series approach, design a linear-phase FIR lowpass filter of lowest order with the following specifications: passband edge at 0.4π, stopband edge at 0.6π, and minimum stopband attenuation of 42 dB. Which window function is appropriate for this design? Show the impulse response coefficients, and plot the gain response of the designed filter. Comment on your results. Do not use the M-file `fir1`.

M 10.11 Repeat Exercise M10.10 using the Dolph-Chebyshev window. Do not use the M-file `fir1`. Compare your results with that obtained in Exercise M10.10.

M 10.12 Repeat Exercise M10.10 using the M-file `fir1`. Compare your results with that obtained in Exercise M10.10.

M 10.13 Design a linear-phase highpass FIR filter of length 36 with a passband edge at $\omega_p = 0.45\pi$ using the frequency sampling approach. Show the impulse response coefficients, and plot the magnitude response of the designed filter using MATLAB.

M 10.14 Design a linear-phase bandpass FIR filter of order 45 with passband edges at $\omega_{p1} = 0.5\pi$ and $\omega_{p2} = 0.7\pi$ using the frequency sampling approach. Show the impulse response coefficients, and plot the magnitude response of the designed filter using MATLAB.

M 10.15 Using the frequency sampling-based approach, redesign the linear-phase lowpass filter of Problem 10.34 by including a transition band with one frequency sample of magnitude 1/2. Plot the magnitude response of the new filter using MATLAB, and compare it with that obtained in Problem 10.34.

M 10.16 Repeat Exercise M10.15 by including a transition band with two frequency samples of magnitude 2/3 and 1/3, respectively.

M 10.17 Design the linear-phase FIR lowpass filter of Exercise M10.8 using the function `fir1` of MATLAB. Use each of the following windows for the design: Hamming, Hann, Blackman, and Kaiser. Show the impulse response coefficients, and plot the gain response of the designed filters for each case. Compare your results with those obtained in Exercises M10.8 and M10.9.

M 10.18 Using the M-file `fir1`, design a linear-phase FIR highpass filter with the following specifications: stopband edge at 0.4π, passband edge at 0.55π, maximum passband attenuation of 0.1 dB, and minimum stopband attenuation of 42 dB. Use each of the following windows for the design: Hamming, Hann, Blackman, and Kaiser. Show the impulse response coefficients, and plot the gain response of the designed filters for each case. Comment on your results.

M 10.19 Using the M-file `fir1`, design a linear-phase FIR bandpass filter with the following specifications: stopband edges at 0.55π and 0.75π, passband edges at 0.65π and 0.85π, maximum passband attenuation of 0.2 dB, and minimum stopband attenuation of 42 dB. Use each of the following windows for the design: Hamming, Hann, Blackman, and Kaiser. Show the impulse response coefficients, and plot the gain response of the designed filters for each case. Comment on your results.

M 10.20 Design a two-channel crossover FIR lowpass and highpass filter pair for digital audio applications. The lowpass and the highpass filters are of length 31 and have a crossover frequency of 15 kHz operating at a sampling rate of 70 kHz. Use the function `fir1` with a Hamming window to design the lowpass filter, while the highpass filter is derived from the lowpass filter using the delay-complementary property. Plot the gain responses of both filters on the same figure. What is the minimum number of delays and multipliers needed to implement the crossover network?

M 10.21 Design a three-channel crossover FIR filter system for digital audio applications. All filters are of length 33 and operate at a sampling rate of 44.1 kHz. The two crossover frequencies are at 5.5 kHz and 12 kHz, respectively. Use the function `fir1` with a Hann window to design the lowpass and the highpass filters, while the bandpass filter is derived from the lowpass and highpass filters using the delay-complementary property. Plot the gain responses of all filters on the same figure. What is the minimum number of delays and multipliers needed to implement the crossover network?

M 10.22 The M-file `fir2` is employed to design FIR filters with arbitrarily shaped magnitude responses. Using this function, design an FIR filter of order 70 with three different constant magnitude levels: 0.2 in the frequency range 0 to 0.35, 1.0 in the frequency range 0.4 to 0.7, and 0.6 in the frequency range 0.72 to 1.0. Plot the gain response of the designed filter.

M 10.23 Design the linear-phase FIR lowpass filter of Problem 10.36 using the function `remez` and plot its magnitude response.

M 10.24 Design the linear-phase FIR highpass filter of Problem 10.37 using the function `remez` and plot its magnitude response.

M 10.25 Design the linear-phase FIR bandpass filter of Problem 10.38 using the function `remez` and plot its magnitude response.

M 10.26 Design a length-30 discrete-time FIR differentiator using the function `remez` and plot its magnitude response.

M 10.27 Design a 30th-order FIR Hilbert transformer using the function `remez`. The passband is from 0.07π to 0.95π. The two stopbands are from 0.02π to 0.05π, and from 0.97π to π. Plot its magnitude response.

M 10.28 Design a minimum-phase lowpass FIR filter with the passband edge at $\omega_p = 0.35\pi$, stopband edge at $\omega_s = 0.5\pi$, passband ripple of $R_p = 1$ dB, and a minimum stopband attenuation of $R_s = 28$ dB.

M 10.29 Determine the minimum-phase spectral factor of the polynomial

$$\begin{aligned} Q(z) = {} & 2.4 + 6.76z^{-1} + 26.15z^{-2} + 68.43z^{-3} + 186.83z^{-4} + 326.51z^{-5} + 565.53z^{-6} + 678.95z^{-7} + 805.24z^{-8} \\ & + 678.95z^{-9} + 565.53z^{-10} + 326.51z^{-11} + 186.83z^{-12} + 68.43z^{-13} + 26.15z^{-14} + 6.76z^{-15} + 2.4z^{-16}. \end{aligned}$$

M 10.30 Design a linear-phase narrow-band FIR lowpass filter using the interpolated FIR filter design approach to meet the following specifications: $\omega_p = 0.1\pi$, $\omega_s = 0.15\pi$, $\delta_p = 0.001$, and $\delta_s = 0.001$.

M 10.31 Design a linear-phase narrow-band FIR highpass filter using the interpolated FIR filter design approach to meet the following specifications: $\omega_p = 0.9\pi$, $\omega_s = 0.95\pi$, $\delta_p = 0.002$, and $\delta_s = 0.004$.

M 10.32 Another approach to the design of a computationally efficient FIR filter is the *prefilter-equalizer method* [Ada83]. In this method, first, a computationally efficient FIR prefilter $H(z)$ with a frequency response reasonably close to the desired response is selected. Next, an FIR equalizer $F(z)$ is designed so that the cascade of the prefilter and the equalizer meets the desired specifications. An attractive prefilter structure for the design of a lowpass FIR filter is the recursive running sum (RRS) FIR filter of order N, which has a transfer function

$$H(z) = \frac{1 - z^{-(N+1)}}{1 - z^{-1}}.$$

The first null of the frequency response of the RRS filter is at $\omega = 2\pi/(N+1)$. Thus, if the desired stopband edge is at ω_s, the order of the RRS filter should be chosen as $N \cong 2\pi/\omega_s$. If N is a fraction, then both the integer values nearest to $2\pi/\omega_s$ are good candidates for the order of the RRS filter. The Park–McClellan algorithm can be modified to incorporate the frequency response of the RRS filter in the weighting function of the error function $W(\omega)$ of Eq. (9.48). Using the prefilter-equalizer approach, design a computationally efficient narrow-band FIR lowpass filter with the following specifications: $\omega_p = 0.05\pi$, $\omega_s = 0.15\pi$, $\alpha_p = 0.15$ dB, and $\alpha_s = 40$ dB.