2.11 Summary

In this chapter, we introduced some important and fundamental concepts regarding the characterization of discrete-time signals and systems in the time domain. Certain basic discrete-time signals that play important roles in discrete-time signal processing have been defined, along with basic mathematical operations used for generating more complex signals and systems. The relation between a continuous-time signal and the discrete-time signal generated by sampling the former at uniform time intervals has been examined.

This text deals almost exclusively with linear, time-invariant (LTI) discrete-time systems that find numerous applications in practice. These systems are defined, and their convolution sum representation in the time domain is derived. The concepts of causality and stability of LTI systems are introduced. Also discussed is an important class of LTI systems characterized by an input–output relation composed of a linear constant coefficient difference equation and the procedure for computing its output for a given input and initial conditions. Several types of classification of LTI discrete-time systems are then considered, of which the more common one is in terms of the length of the impulse response. Finally, the concepts of the autocorrelation of a sequence and the cross-correlation between a pair of sequences are introduced.

Further insights can often be obtained by considering the frequency-domain representations of discrete-time signals and LTI discrete-time systems. These are discussed in Chapters 3, 5, 6, and 7.

2.12 Problems

2.1 Determine the $\mathcal{L}_1$-, $\mathcal{L}_2$-, and $\mathcal{L}_\infty$-norms of the following finite-length sequences:

(a) $\{x_1[n]\} = \{4.50 \quad -2.68 \quad -0.14 \quad 3.91 \quad 2.62 \quad -0.43 \quad -4.81 \quad 3.21 \quad -0.55\}$,
(b) $\{x_2[n]\} = \{0.92 \quad 2.34 \quad 3.37 \quad 1.90 \quad -2.59 \quad -0.75 \quad 3.48 \quad 3.33\}$.

2.2 Express the sequence $x[n] = 1$, $-\infty < n < \infty$, in terms of the unit step sequence $\mu[n]$.

2.3 Verify the relation between the unit sample sequence $\delta[n]$ and the unit step sequence $\mu[n]$ given in Eq. (2.31).

2.4 Express the length-4 sequence $x[n] = \{1 \quad 3 \quad -2 \quad 4\}$, $0 \leq n \leq 3$, in terms of the unit step sequence $\mu[n]$.

2.5 Consider the following sequences:

$$x[n] = \{-4 \quad 5 \quad 1 \quad -2 \quad -3 \quad 0 \quad 2\}, \quad -3 \leq n \leq 3$$

$$y[n] = \{6 \quad -3 \quad -1 \quad 0 \quad 8 \quad 7 \quad -2\}, \quad -1 \leq n \leq 5$$

$$w[n] = \{3 \quad 2 \quad 2 \quad -1 \quad 0 \quad -2 \quad 5\}, \quad 2 \leq n \leq 8.$$

The sample values of each of the above sequences outside the ranges specified are all zeros. Generate the following sequences: (a) $c[n] = x[-n + 2]$, (b) $d[n] = y[-n - 3]$, (c) $e[n] = w[-n]$, (d) $u[n] = x[n] + y[n - 2]$, (e) $v[n] = x[n] \cdot w[n + 4]$, (f) $s[n] = y[n] - w[n + 4]$, and (g) $r[n] = 3.5y[n]$.

2.6 (a) Express the sequences $x[n]$, $y[n]$, and $w[n]$ of Problem 2.5 as a linear combination of delayed unit sample sequences.

(b) Express the sequences $x[n]$, $y[n]$, and $w[n]$ of Problem 2.5 as a linear combination of delayed unit step sequences.

2.7 Analyze the block diagrams of Figure P2.1 and develop the relation between $y[n]$ and $x[n]$.
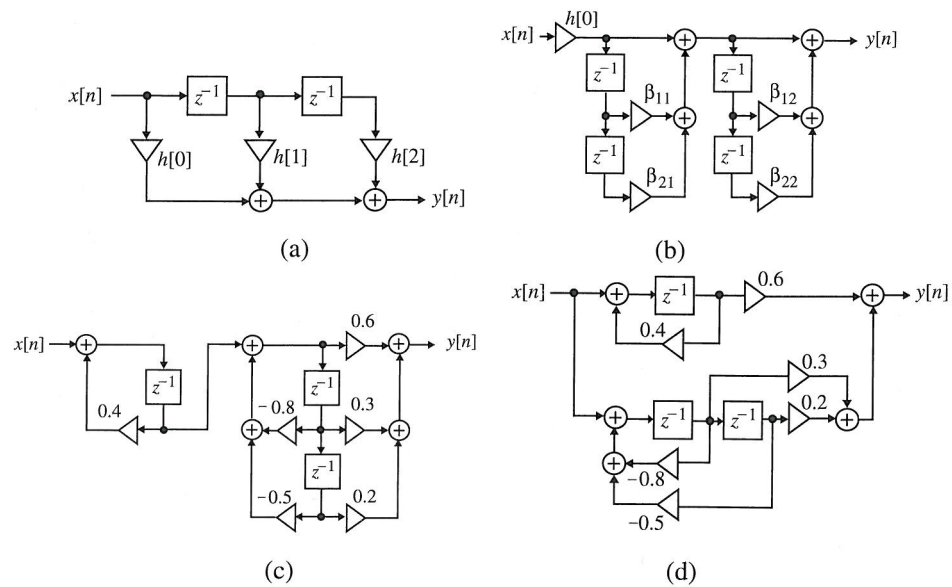
(a)

(b)

(c)

(d)

Figure P2.1

2.8 Determine the conjugate symmetric and conjugate antisymmetric parts of the following sequences:
(a) $x_1[n] = \{1 + j4 \quad -2 + j5 \quad 3 - j2 \quad -7 + j3 \quad -1 + j\}, -2 \leq n \leq 2$, (b) $x_2[n] = e^{j\pi n/3}$,
(c) $x_3[n] = je^{-j\pi n/5}$.

2.9 Determine the even and odd parts of the sequences $x[n]$, $y[n]$, and $w[n]$ of Problem 2.5.

2.10 Determine the even and odd parts of the following real sequences:
(a) $x_1[n] = \mu[n + 2]$, (b) $x_2[n] = \alpha^n \mu[n - 3]$, (c) $x_3[n] = n\alpha^n \mu[n]$, (d) $x_4[n] = \alpha^{|n|}$.

2.11 Show that the even and odd parts of a real sequence are, respectively, even and odd sequences.

2.12 Let $x_{ev}[n]$ and $x_{od}[n]$ be even and odd real sequences, respectively. Which one of the following sequences is an even sequence, and which one is an odd sequence?
(a) $g[n] = x_{ev}[n]x_{ev}[n]$, (b) $u[n] = x_{ev}[n]x_{od}[n]$, (c) $v[n] = x_{od}[n]x_{od}[n]$.

2.13 (a) Show that a causal real sequence $x[n]$ can be fully recovered from its even part $x_{ev}[n]$ for all $n \geq 0$, whereas it can be recovered from its odd part $x_{od}[n]$ only for all $n > 0$.
(b) Is it possible to fully recover a causal complex sequence $y[n]$ from its conjugate antisymmetric part $y_{ca}[n]$? Can $y[n]$ be fully recovered from its conjugate symmetric part $y_{cs}[n]$? Justify your answers.

2.14 Determine the causal sequence $x[n]$ whose even part is given by $x_{ev}[n] = \cos(\omega_o n)$.

2.15 Which ones of the following sequences are bounded sequences?
(a) $x[n] = A\alpha^n$, where A and α are complex numbers, and $|\alpha| < 1$,
(b) $y[n] = A\alpha^n \mu[n]$, where A and α are complex numbers, and $|\alpha| < 1$,
(c) $h[n] = C\beta^n \mu[n]$, where C and β are complex numbers, and $|\beta| > 1$,
(d) $g[n] = 4\cos(\omega_a n)$, (e) $v[n] = \left(1 - \frac{1}{n^2}\right)\mu[n - 1]$.

2.16 Show that the sequence $x[n] = \frac{(-1)^{n+1}}{n}\mu[n-1]$ is not absolutely summable even though $\sum_{n=1}^{\infty} x[n] = \ln 2$.

2.17 Show that the following sequences are absolutely summable: (a) $x_1[n] = \alpha^n \mu[n-1]$, (b) $x_2[n] = n\alpha^n \mu[n-1]$, (c) $x_3[n] = n^2\alpha^n \mu[n-1]$, where $|\alpha| < 1$.

2.18 Show that the following sequences are absolutely summable. (a) $x_a[n] = \frac{1}{2^n}\mu[n]$, (b) $x_b[n] = \frac{1}{(n+1)(n+2)}\mu[n]$.

2.19 Show that an absolutely summable sequence has finite energy, but a finite energy sequence may not be absolutely summable.

2.20 Show that the square-summable sequence $x_1[n] = \frac{1}{n}$ of Eq. (2.29) is not absolutely summable.

2.21 Show that the sequence $x_2[n] = \frac{\cos \omega_c n}{\pi n}\mu[n-1]$ is square-summable but not absolutely summable.

2.22 The odd part of a real-valued sequence $x[n]$ is given by $x_{od} = \left(\frac{1}{3}\right)^3 \mu[n]$. If the average power of $x[n]$ is $\mathcal{P}_x = 10$, determine the average power of its even part x_{ev}.

2.23 Compute the energy of the length-N sequence $x[n] = \sin(2\pi kn/N)$, $0 \le n \le N-1$.

2.24 Compute the energy of the following sequences:
(a) $x_a[n] = A\alpha^n \mu[n]$, $|\alpha| < 1$, (b) $x_b[n] = \frac{1}{n^2}\mu[n-1]$.

2.25 Determine the average power and the energy of the following sequences:
(a) $x_1[n] = (-1)^n$, (b) $x_2[n] = \mu[n]$, (c) $x_3[n] = n\mu[n]$, (d) $x_4[n] = A_o e^{j\omega_o n}$, (e) $x_5[n] = A\cos(\frac{2\pi n}{M} + \phi)$.

2.26 Determine the period and the average power of the following periodic sequences:
(a) $\tilde{x}_1[n] = 4\cos(2\pi n/5)$, (b) $\tilde{x}_2[n] = 3\cos(3\pi n/5)$, (c) $\tilde{x}_3[n] = 2\cos(3\pi n/7)$, (d) $\tilde{x}_4[n] = 4\cos(5\pi n/3)$,
(e) $\tilde{x}_5[n] = 4\cos(2\pi n/5) + 3\cos(3\pi n/5)$, (f) $\tilde{x}_6[n] = 4\cos(5\pi n/3) + 3\cos(3\pi n/5)$.

2.27 Let $x[n]$ be an absolutely summable sequence. Show that the sequence $\tilde{y}[n]$ formed by an N-periodic extension according to Eq. (2.38) is a periodic sequence with a period N.

2.28 Determine the samples of one period of the periodic sequences obtained by an N-periodic extension of the sequences of Problem 2.5 for the following values of N: (a) $N = 5$, and $N = 7$.

2.29 The following sequences represent one period of a sinusoidal sequence of the form $\tilde{x}[n] = A\cos(\omega_o n + \phi)$:
(a) $\{1 \quad -1 \quad -1 \quad 1 \quad 1 \quad -1 \quad -1 \quad 1\}$, (b) $\{0 \quad -\sqrt{3} \quad 0 \quad \sqrt{3} \quad 0 - \sqrt{3} \quad 0 \quad \sqrt{3}\}$,
(c) $\{1 \quad -0.366 \quad -1.366 \quad -1 \quad 0.366 \quad 1.366\}$, (d) $\{2 \quad 0 \quad -2 \quad 0 \quad 2 \quad 0-2 \quad 0\}$.
Determine the values of the parameters A, ω_o, and ϕ for each case.

2.30 Determine the fundamental period of the following periodic sequences:
(a) $\tilde{x}_a[n] = e^{j0.5\pi n}$, (b) $\tilde{x}_b[n] = \sin(0.8\pi n + 0.8\pi)$, (c) $\tilde{x}_c[n] = \text{Re}\left(e^{j\pi n/5}\right) + \text{Im}\left(e^{j\pi n/10}\right)$,
(d) $\tilde{x}_4[n] = 3\cos(1.3\pi n) - 4\sin(0.5\pi n + 0.5\pi)$, (e) $\tilde{x}_5[n] = 5\cos(1.5\pi n + 0.75\pi) + 4\cos(0.6\pi n) - \sin(0.5\pi n)$.

2.31 Determine the fundamental period of the sinusoidal sequence $x[n] = A\sin(\omega_o n)$ for the following values of the angular frequency ω_o:

(a) 0.6π, (b) 0.28π, (c) 0.45π, (d) 0.55π, (e) 0.65π.

2.32 Determine the period of the sinusoidal sequence $x_1[n] = \sin(0.08\pi n)$. Determine at least two other distinct sinusoidal sequences having the same period as $x_1[n]$.

2.33 Show that the continuous-time signal $x_a(t) = A\cos(\Omega_o t + \phi)$ can be uniquely recovered from its sampled version $x[n] = x_a(nT)$, $-\infty < n < \infty$, if the sampling frequency $\Omega_T = 2\pi/T > 2\Omega_o$.

2.34 A continuous-time sinusoidal signal $x_a(t) = \cos \Omega_o t$ is sampled at $t = nT$, $-\infty < n < \infty$, generating the discrete-time sequence $x[n] = x_a(nT) = \cos(\Omega_o nT)$. For what values of T is $x[n]$ a periodic sequence? What is the fundamental period of $x[n]$ if $\Omega_o = 20$ radians and $T = \pi/8$ seconds?

2.35 Show that the discrete-time systems described by the following equations are linear systems:
   (a) Eq. (2.18),   (b) Eq. (2.20),   (c) Eq. (2.21),   (d) Eq. (2.61),   (e) Eq. (2.65),   and   (f) Eq. (2.66).

2.36 For each of the following discrete-time systems, where $y[n]$ and $x[n]$ are, respectively, the output and the input sequences, determine whether or not the system is (1) linear, (2) causal, (3) stable, and (4) shift-invariant:
   (a) $y[n] = n^3 x[n]$;   (b) $y[n] = (x[n])^5$;   (c) $y[n] = \beta + \sum_{\ell=0}^{3} x[n-\ell]$ β is a nonzero constant;
   (d) $y[n] = \ln(2 + |x[n]|)$;   (e) $y[n] = \alpha x[-n+2]$; α is a nonzero constant;   (f) $y[n] = x[n-4]$.

2.37 Show that the median filter defined by Eq. (2.67) is a time-invariant system.

2.38 The second derivative $y[n]$ of a sequence $x[n]$ at time instant n is usually approximated by

$$y[n] = x[n+1] - 2x[n] + x[n-1].$$

If $y[n]$ and $x[n]$ denote the output and input of a discrete-time system, is the system linear? Is it time-invariant? Is it causal?

2.39 A discrete-time system is characterized by [Kai80]

$$y[n] = x^2[n] - x[n-1]x[n+1],$$

where $y[n]$ and $x[n]$ are the output and input sequences. Is the above system linear? Is it time-invariant? Is it causal?

2.40 Consider the discrete-time system characterized by the input–output relation [Cad87]

$$y[n] = \frac{1}{2}\left(y[n-1] + \frac{x[n]}{y[n-1]}\right), \tag{2.138}$$

where $x[n]$ and $y[n]$ are, respectively, the input and output sequences. Show that the output $y[n]$ of the above system for an input $x[n] = \alpha\mu[n]$ with $y[-1] = 1$ converges to $\sqrt{\alpha}$ as $n \to \infty$ when α is a positive number. Is the above system linear or nonlinear? Is it time-invariant? Justify your answer.

2.41 An algorithm for the calculation of the square root of a number α is given by [Mik92]

$$y[n] = x[n] - y^2[n-1] + y[n-1], \tag{2.139}$$

where $x[n] = \alpha\mu[n]$ with $0 < \alpha < 1$. If $x[n]$ and $y[n]$ are considered as the input and output of a discrete-time system, is the system linear or nonlinear? Is it time-invariant? As $n \to \infty$, show that $y[n] \to \sqrt{\alpha}$. Note that $y[-1]$ is a suitable initial approximation to $\sqrt{\alpha}$.

2.42 Determine the expression for the impulse response of the factor-of-3 linear interpolator of Eq. (2.66).

2.43 Determine the expression for the impulse response of the factor-of-L linear interpolator.

2.44 Derive Eq. (2.111) by induction by first evaluating Eq. (2.108) for $n = 0, 1, 2, \ldots$, and then solving for $h[0], h[1], h[2]$, etc.

2.45 Let $y[n] = h[n] \circledast x[n]$, where $h[n]$ and $x[n]$ are right-sided sequences. Show that $\sum y[n] = \left(\sum h[n] \right) \left(\sum x[n] \right)$.

2.46 Develop closed-form expressions for the following convolution sums: (a) $\alpha^n \mu[n] \circledast \mu[n]$, (b) $n\alpha^n \mu[n] \circledast \mu[n]$.

2.47 Develop a general expression for the output $y[n]$ of an LTI discrete-time system in terms of its input $x[n]$ and the unit step response $s[n]$ of the system.

2.48 A periodic sequence $\tilde{x}[n]$ with a period N is applied as an input to an LTI discrete-time system characterized by an impulse response $h[n]$ generating an output $y[n]$. Is $y[n]$ a periodic sequence? If it is, what is its period?

2.49 Consider the following sequences:
(i) $x_1[n] = 3\delta[n-2] - 2\delta[n+1]$, (ii) $x_2[n] = 5\delta[n-3] + 2\delta[n+1]$, (iii) $h_1[n] = -\delta[n+2] + 4\delta[n] - 2\delta[n-1]$,
(iv) $h_2[n] = 3\delta[n-4] + 1.5\delta[n-2] - \delta[n+1]$.
Determine the following sequences obtained by a linear convolution of a pair of the above sequences:
(a) $y_1[n] = x_1[n] \circledast h_1[n]$, (b) $y_2[n] = x_2[n] \circledast h_2[n]$, (c) $y_3[n] = x_1[n] \circledast h_2[n]$, (d) $y_4[n] = x_2[n] \circledast h_1[n]$.

2.50 Determine the following sequences obtained by a linear convolution of the sequences given in Problem 2.5:
(a) $u[n] = x[n] \circledast y[n]$,   (b) $v[n] = x[n] \circledast w[n]$,   (c) $g[n] = w[n] \circledast y[n]$.

2.51 Let $g[n]$ be a finite-length sequence defined for $-3 \le n \le 4$, and let $h[n]$ be a finite-length sequence defined for $2 \le n \le 6$. Define $y[n] = g[n] \circledast h[n]$. (a) What is the length of $y[n]$? (b) What is the range of the index n for which $y[n]$ is defined?

2.52 Let $y[n] = x_1[n] \circledast x_2[n]$ and $v[n] = x_1[n - N_1] \circledast x_2[n - N_2]$. Express $v[n]$ in terms of $y[n]$.

2.53 Let $g[n] = x_1[n] \circledast x_2[n] \circledast x_3[n]$ and $h[n] = x_1[n - N_1] \circledast x_2[n - N_2] \circledast x_3[n - N_3]$. Express $h[n]$ in terms of $g[n]$.

2.54 Prove that the convolution sum operation is commutative and distributive.

2.55 Consider the following three sequences:

$$x_1[n] = A \text{ (a constant)}, \qquad x_2[n] = \mu[n], \qquad x_3[n] = \begin{cases} 1, & \text{for } n = 0, \\ -1, & \text{for } n = 1, \\ 0, & \text{otherwise.} \end{cases}$$

Show that $x_3[n] \circledast x_2[n] \circledast x_1[n] \ne x_2[n] \circledast x_3[n] \circledast x_1[n]$.

2.56 Prove that the convolution operation is associative for stable and single-sided sequences.

2.57 Show that the convolution of a length-M sequence with a length-N sequence leads to a sequence of length $(M + N - 1)$.

2.58 Let $\{x[n]\}$ be a length-N sequence defined for $0 \le n \le N - 1$, with its n-th sample given by a_n. If all samples of $x[n]$ are non-negative, determine the location and the value of the largest positive sample of $y[n] = x[n] \circledast x[n]$ without performing the convolution operation.

2.59 Let $\{x[n]\}$ and $\{h[n]\}$ be two length-N sequences defined for $0 \le n \le N - 1$, with their n-th samples given by a_n and b_n, respectively. If all samples of $x[n]$ and $h[n]$ are non-negative, determine the location and the value of the largest positive sample of $y[n] = x[n] \circledast h[n]$ without performing the convolution operation.

2.60 Consider two real sequences $h[n]$ and $g[n]$ expressed as a sum of their respective even and odd parts, i.e., $h[n] = h_{\text{ev}}[n] + h_{\text{od}}[n]$, and $g[n] = g_{\text{ev}}[n] + g_{\text{od}}[n]$. For each of the following sequences, determine if it is even or odd.

    (a) $h_{\text{ev}}[n] \circledast g_{\text{ev}}[n]$,   (b) $h_{\text{od}}[n] \circledast g_{\text{ev}}[n]$,   (c) $h_{\text{od}}[n] \circledast g_{\text{od}}[n]$.

2.61 Consider a cascade of two causal stable LTI systems characterized by impulse responses $\alpha^n \mu[n]$ and $\beta^n \mu[n]$, where $0 < \alpha < 1$ and $0 < \beta < 1$. Determine the expression for the impulse response $h[n]$ of the cascade.

2.62 Determine the impulse response $g[n]$ of the inverse system of the LTI discrete-time system of Example 2.31.

2.63 Determine the impulse response $g[n]$ characterizing the inverse system of the cascaded LTI discrete-time system of Problem 2.61.

2.64 Determine the expression for the impulse response of each of the LTI systems shown in Figure P2.2.
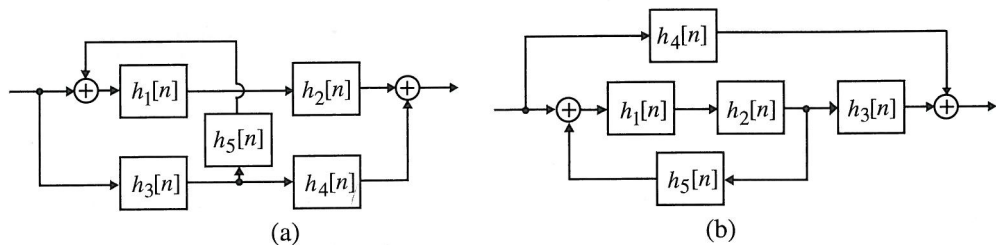

Figure P2.2

2.65 Determine the overall impulse response of the system of Figure P2.3, where the impulse responses of the component systems are: $h_1[n] = 2\delta[n-2] - 3\delta[n+1]$, $h_2[n] = \delta[n-1] + 2\delta[n+2]$, and $h_3[n] = 5\delta[n-5] + 7\delta[n-3] + 2\delta[n-1] - \delta[n] + 3\delta[n+1]$.
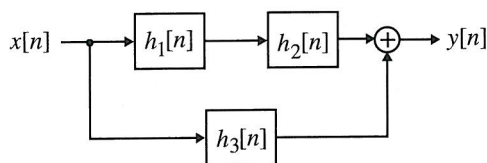

Figure P2.3

2.66 Let $y[n]$ be the sequence obtained by a linear convolution of two causal finite-length sequences $h[n]$ and $x[n]$. For each pair of $y[n]$ and $h[n]$ listed below, determine $x[n]$. The first sample in each sequence is at time index $n = 0$.

    (a) $\{y[n]\} = \{6, 11, -13, 16, 1, 9, 2, 8\}$, $\{h[n]\} = \{2, 5, -1, 4\}$,

    (b) $\{y[n]\} = \{2, 1, 2, 6, 0\ 17\ 12\}$, $\{h[n]\} = \{2, -3, 2, 3\}$,

    (c) $\{y[n]\} = \{-4, 10, -15, 8, 0, 1, -3, 1\}$, $\{h[n]\} = \{-4, 2, 1, 0, -1\}$.

2.67 Consider a causal discrete-time system characterized by a first-order linear, constant-coefficient difference equation given by

$$y[n] = ay[n-1] + bx[n], \quad n \geq 0,$$

where $y[n]$ and $x[n]$ are, respectively, the output and input sequences. Compute the expression for the output sample $y[n]$ in terms of the initial condition $y[-1]$ and the input samples.

(a) Is the system time-invariant if $y[-1] = 1$? Is the system linear if $y[-1] = 1$?

(b) Repeat part (a) if $y[-1] = 0$.

(c) Generalize the results of parts (a) and (b) to the case of an Nth-order causal discrete-time system given by Eq. (2.82).

2.68 Consider the causal LTI system described by the difference equation

$$y[n] = p_0 x[n] + p_1 x[n-1] - d_1 y[n-1],$$

where $x[n]$ and $y[n]$ denote, respectively, its input and output. Determine the difference equation representation of its inverse system.

2.69 A causal LTI discrete-time system is said to have an *overshoot* in its step response if the response exhibits an oscillatory behavior with decaying amplitudes around a final constant value. Show that the system has no overshoot in its step response if the impulse response $h[n]$ of the system is nonnegative for all $n \geq 0$.

2.70 The sequence of Fibonacci numbers $f[n]$ is a causal sequence defined by

$$f[n] = f[n-1] + f[n-2], \quad n \geq 2,$$

with $f[0] = 0$ and $f[1] = 1$.

(a) Develop an exact formula to calculate $f[n]$ directly for any n.

(b) Show that $f[n]$ is the impulse response of a causal LTI system described by the difference equation [Joh89]

$$y[n] = y[n-1] + y[n-2] + x[n-1].$$

2.71 Consider a first-order complex digital filter characterized by a difference equation

$$y[n] = \alpha y[n-1] + x[n],$$

where $x[n]$ is the real input sequence, $y[n] = y_{re}[n] + j y_{im}[n]$ is the complex output sequence with $y_{re}[n]$ and $y_{im}[n]$ denoting its real and imaginary parts, and $\alpha = a + jb$ is a complex constant. Develop an equivalent two-output, single-input real difference equation representation of the above complex digital filter. Show that the single-input, single-output digital filter relating $y_{re}[n]$ to $x[n]$ is described by a second-order difference equation.

2.72 Let $h[0]$, $h[1]$, and $h[2]$ denote the first three impulse response samples of the first-order causal LTI system of Problem 2.68. Show that the coefficients of the difference equation characterizing this system can be uniquely determined from these impulse response samples.

2.73 Let a causal IIR digital filter be described by the difference equation

$$\sum_{k=0}^{N} d_k y[n-k] = \sum_{k=0}^{M} p_k x[n-k], \tag{2.140}$$

where $y[n]$ and $x[n]$ denote the output and the input sequences, respectively. If $h[n]$ denotes its impulse response, show that

$$p_k = \sum_{n=0}^{k} h[n] d_{k-n}, \quad k = 0, 1, \ldots, M.$$

From the above result, show that $p_n = h[n] \circledast d_n$.

2.74 Prove that the BIBO stability condition of Eq. (2.74) also holds for an LTI digital filter with a complex impulse response.

2.75 Is the cascade connection of two stable LTI systems also stable? Justify your answer.

2.76 Is the parallel connection of two stable LTI systems also stable? Justify your answer.

2.77 Prove that the cascade connection of two passive (lossless) LTI systems is also passive (lossless).

2.78 Is the parallel connection of two passive (lossless) LTI systems also passive (lossless)? Justify your answer.

2.79 Consider a causal FIR filter of length $L+1$ with an impulse response given by $\{g[n]\}$, $n = 0, 1, \ldots, L$. Develop the difference equation representation of the form of Eq. (2.90) where $M + N = L$ of a causal finite-dimensional IIR digital filter with an impulse response $\{h[n]\}$ such that $h[n] = g[n]$ for $n = 0, 1, \ldots, L$.

2.80 Compute the output of the accumulator of Eq. (2.60) for an input $x[n] = n\mu[n]$ and the following initial conditions: (a) $y[-1] = 0$, and (b) $y[-1] = -2$.

2.81 In the rectangular method of numerical integration, the integral on the right-hand side of Eq. (2.85) is expressed as

$$\int_{(n-1)T}^{nT} x(\tau)d\tau = T \cdot x\left((n-1)T\right). \tag{2.141}$$

Develop the difference equation representation of the rectangular method of numerical integration.

2.82 Develop a recursive implementation of the time-varying linear discrete-time system characterized by

$$y[n] = \begin{cases} \frac{1}{n}\sum_{\ell=1}^{n} x[\ell], & n > 0, \\ 0, & n \leq 0. \end{cases}$$

2.83 Determine the total solution for $n \geq 0$ of the difference equation

$$y[n] - 0.35y[n-1] = 2.4\mu[n],$$

with the initial condition $y[-1] = 3$.

2.84 Determine the total solution for $n \geq 0$ of the difference equation

$$y[n] - 0.3y[n-1] - 0.04y[n-2] = 3^n\mu[n],$$

with the initial condition $y[-1] = 2$, and $y[-2] = 1$.

2.85 Determine the total solution for $n \geq 0$ of the difference equation

$$y[n] - 0.3y[n-1] - 0.04y[n-2] = x[n] + 2x[n-1],$$

with the initial condition $y[-1] = 2$, and $y[-2] = 1$, when the forcing function is $x[n] = 3^n\mu[n]$.

2.86 Determine the impulse response $h[n]$ of the LTI system described by the difference equation

$$y[n] - 0.35y[n-1] = x[n].$$

2.87 Determine the impulse response $h[n]$ of the LTI system described by the difference equation

$$y[n] - 0.3y[n-1] - 0.04y[n-2] = x[n] + 2x[n-1].$$

2.88 Determine the step response of an LTI discrete-time system characterized by an impulse response $h[n] = (-\alpha)^n\mu[n]$, $0 < \alpha < 1$.

2.89 Show that the sum $\sum_{n=0}^{\infty} |n^K (\lambda_i)^n|$ converges if $|\lambda_i| < 1$.

2.90 (a) Evaluate the autocorrelation sequence of each of the sequences of Problem 2.5.
  (b) Evaluate the cross-correlation sequence $r_{xy}[\ell]$ between the sequences $x[n]$ and $y[n]$, and the cross-correlation sequence $r_{xw}[\ell]$ between the sequences $x[n]$ and $w[n]$ of Problem 2.5.

2.91 Determine the autocorrelation sequence of each of the following sequences, and show that it is an even sequence in each case. What is the location of the maximum value of the autocorrelation sequence in each case?
  (a) $x_1[n] = \alpha^n \mu[n]$, (b) $x_2[n] = \begin{cases} 1, & 0 \le n \le N-1, \\ 0, & \text{otherwise.} \end{cases}$

2.92 Determine the autocorrelation sequence and its period of each of the following periodic sequences.
  (a) $\tilde{x}_1[n] = \cos(\pi n/M)$, where M is a positive integer, (b) $\tilde{x}_2[n] = n$ modulo 6, (c) $\tilde{x}_3[n] = (-1)^n$.

2.13 MATLAB Exercises

M 2.1 (a) Using Program 2.2, generate the sequences shown in Figures 2.17 and 2.18.
  (b) Generate and plot the complex exponential sequence $-3.6e^{(-0.5+j\pi/4)n}$ for $0 \le n \le 82$ using Program 2.2.

M 2.2 Generate the sequences of Problem 2.30(b) to 2.30(e) using MATLAB.

M 2.3 (a) Write a MATLAB program to generate a sinusoidal sequence $x[n] = A \sin(\omega_o n + \phi)$, and plot the sequence using the stem function. The input data specified by the user are the desired length L, amplitude A, the angular frequency ω_o, and the phase ϕ where $0 < \omega_o < \pi$ and $0 \le \phi \le 2\pi$. Using this program, generate the sinusoidal sequences shown in Figure 2.16.
  (b) Generate sinsusoidal sequences with the angular frequencies given in Problem 2.31. Determine the period of each sequence from the plot, and verify the result theoretically.

M 2.4 Write a MATLAB program to plot a continuous-time sinusoidal signal and its sampled version, and verify Figure 2.22. You need to use the hold function to keep both plots.

M 2.5 Using the program developed in the previous problem, verify experimentally that the family of continuous-time sinusoids given by Eq. (2.57) lead to identical sampled signals.

M 2.6 Using Program 2.4, investigate the effect of signal smoothing by a moving-average filter of lengths 5, 7, and 9. Does the signal smoothing improve with an increase in the length? What is the effect of the length on the delay between the smoothed output and the noisy input?

M 2.7 Write a MATLAB program implementing the discrete-time system of Eq. (2.138) in Problem 2.40, and show that the output $y[n]$ of this system for an input $x[n] = \alpha\mu[n]$ with $y[-1] = 1$ converges to $\sqrt{\alpha}$ as $n \to \infty$.

M 2.8 Write a MATLAB program to compute the square root using the algorithm of Eq. (2.139) in Problem 2.41, and show that the output $y[n]$ of this system for an input $x[n] = \alpha\mu[n]$ with $y[-1] = 1$ converges to $\sqrt{\alpha}$ as $n \to \infty$. Plot the error as a function of n for several different values of α. How would you compute the square-root of a number α with a value greater than one?

M 2.9 Using Program 2.7, determine the autocorrelation and the cross-correlation sequences of Problem 2.90. Are your results the same as those determined in Problem 2.90?

M 2.10 Modify Program 2.7 to determine the autocorrelation sequence of a sequence corrupted with a uniformly distributed random signal generated using the M-function rand. Using the modified program, demonstrate that the autocorrelation sequence of a noise-corrupted signal exhibits a peak at zero lag.