EXAMPLE 7.27 Stability Testing Using MATLAB

We test the stability of the transfer function of Eq. (7.154) using MATLAB. To this end we make use of Program 7_2. The input data is the vector `den` of the coefficients of the denominator polynomial entered inside a square bracket in descending powers of z as indicated below:

Program 7_2.m

```
den = [4    3    2    1    1]
```

The output data are the stability test parameters $\{k_i\}$. The program also has a logical output, which is `stable = 1` if the transfer function is stable; otherwise, `stable = 0`.

The output data generated for the transfer function of Eq. (7.154) are as follows:

```
The stability test parameters are
      0.2500    0.0667    0.3527    0.5248

stable = 1
```

Note that the stability test parameters are identical to those computed in Example 7.25.

7.10 Summary

The concept of filtering is introduced, and several ideal filters are defined. Several simple approximations to the ideal filters are next introduced. In addition, various special types of transfer functions that are often encountered in practice are reviewed. The concept of complementary transfer functions relating a set of transfer functions is discussed, and several types of complementary conditions are introduced.

The inverse system design is encountered in estimating the unknown input of a discrete-time system from its known output. The determination of the transfer function of the inverse of a causal LTI discrete-time system with a rational transfer function is outlined. The recursive computation of the unknown causal input signal from the impulse response of a causal LTI system and its known output is outlined. Next, two methods are outlined for the system identification problem. In one approach, a recursive algorithm is described for determining the impulse response of a causal initially relaxed system from its known input and output sequences. In the second method, the frequency response of the system is determined from the cross-energy spectrum of the output and the input signal and the energy spectrum of the input. Alternately, the square magnitude function of the system can be determined from the energy spectrum of the output and the input signals.

An important building block in the design of a single-input, single-output LTI discrete-time system is the digital two-pair, which is a two-input, two-output LTI discrete-time system. Characterizations of the digital two-pairs and their interconnections are discussed. A very simple algebraic procedure for testing the stability of a causal LTI transfer function is then introduced.

7.11 Problems

7.1 Show that the transfer function of the M-point moving-average filter of Eq. (3.97) is a BR function.

7.2 Consider the first-order causal and stable allpass transfer function given by

$$A_1(z) = \frac{1 - d_1^* z}{z - d_1}.$$

Determine the expression for $\left(1 - |A_1(z)|^2\right)$, and then show that

$$\left(1 - |A_1(z)|^2\right) \begin{cases} < 0, & \text{for } |z|^2 < 1, \\ = 0, & \text{for } |z|^2 = 1, \\ > 0, & \text{for } |z|^2 > 1. \end{cases}$$

Now, using the above approach, show that Property 2 given by Eq. (7.20) holds for any arbitrary causal stable allpass transfer function.

7.3 Derive Property 3 of a stable allpass transfer function given by Eq. (7.21).

7.4 A noncausal LTI FIR discrete-time system is characterized by an impulse response $h[n] = a_1\delta[n-2] - a_2\delta[n-1] - a_3\delta[n] + a_4\delta[n+1] - a_5\delta[n+2]$. For what values of the impulse response samples will its frequency response $H(e^{j\omega})$ have a zero phase?

7.5 Let a causal LTI discrete-time system be characterized by a real impulse response $h[n]$ with a DTFT $H(e^{j\omega})$. Consider the system of Figure P7.1, where $x[n]$ is a finite-length sequence. Determine the frequency response of the overall system $G(e^{j\omega})$ in terms of $H(e^{j\omega})$, and show that it has a zero-phase response.

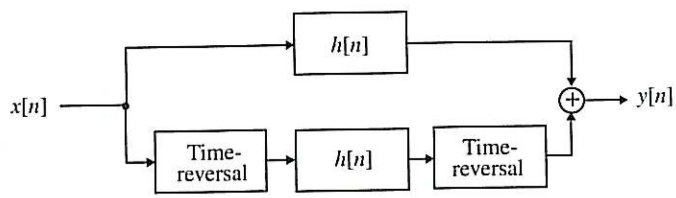

Figure P7.1

7.6 Show that an Mth-order causal complex coefficient allpass transfer function is of the form

$$A_M(z) = \pm \frac{d_M^* + d_{M-1}^* z^{-1} + \cdots + d_1^* z^{-M+1} + z^{-M}}{1 + d_1 z^{-1} + \cdots + d_{M-1} z^{-M+1} + d_M z^{-M}}.$$

7.7 Determine all possible causal stable transfer functions $H(z)$ with a square-magnitude function given by

$$|H(e^{j\omega})|^2 = \frac{9(1.0625 + 0.5\cos\omega)(1.49 - 1.4\cos\omega)}{(1.36 + 1.2\cos\omega)(1.64 + 1.6\cos\omega)}.$$

7.8 Consider the following five FIR transfer functions:

(i) $H_1(z) = 2 + 1.4z^{-1} - 0.9z^{-2} - 0.158z^{-3} + 0.4104z^{-4} + 0.0294z^{-5} - 0.0668z^{-6}$,

(ii) $H_2(z) = 1 + 2.7z^{-1} - 12.61z^{-2} - 24.757z^{-3} + 66.301z^{-4} + 62.072z^{-5} - 126.786z^{-6}$,

(iii) $H_3(z) = 0.2 - 0.26z^{-1} + 1.934z^{-2} + 10.413z^{-3} + 1.934z^{-4} - 0.26z^{-5} + 0.2z^{-6}$,

(iv) $H_4(z) = 1.25 + 0.5z^{-1} - 2.1z^{-2} - 2.1z^{-4} + 0.5z^{-5} + 1.25z^{-6}$,

(v) $H_5(z) = 1.1 + 3.12z^{-1} - 2.5z^{-2} + 0.6z^{-3} + 0.5z^{-4} + 0.06z^{-5} + z^{-6}$.

Using the M-file `zplane`, determine the zero locations of each, and then answer the following questions:

(a) Which one of the above FIR filters have a linear-phase response?

(b) Which one of the above FIR filters have a minimum-phase response?

(c) Which one of the above FIR filters have a maximum-phase response?

7.9 A third-order FIR filter has a transfer function given by

$$G_1(z) = (2 + 3.4z^{-1} - 4z^{-2})(3 - 1.5z^{-1}).$$

(a) Determine the transfer functions of all other FIR filters whose magnitude responses are identical to that of $G_1(z)$.

(b) Which one of these filters has a minimum-phase transfer function, and which one has a maximum-phase transfer function?

(c) If $g_k[n]$ denotes the impulse response of the kth FIR filter determined in Part (a), compute the partial energy of the impulse response given by

$$\mathcal{E}_k[n] = \sum_{m=0}^{n} g_k[m]^2, \qquad 0 \leq n \leq 3,$$

for all values of k, and show that

$$\sum_{m=0}^{n} |g_k[m]|^2 \leq \sum_{m=0}^{n} |g_{\min}[m]|^2,$$

and

$$\sum_{m=0}^{\infty} |g_k[m]|^2 = \sum_{m=0}^{\infty} |g_{\min}[m]|^2,$$

for all values of k, and where $g_{\min}[n]$ is the impulse response of the minimum-phase FIR filter determined in Part (a).

7.10 The transfer functions of five FIR filters with identical magnitude responses are given below:

$$H_1(z) = 1 - 0.5z^{-1} + 0.8z^{-2} - 0.4z^{-3} + 0.25z^{-4} - 0.125z^{-5} + 0.2z^{-6} - 0.1z^{-7},$$
$$H_2(z) = 0.5 + 0.25z^{-1} + 0.4z^{-2} - 0.425z^{-3} + 0.75z^{-4} - 0.75z^{-5} + 0.6z^{-6} - 0.2z^{-7},$$
$$H_3(z) = -0.25 + 0.25z^{-1} + 0.175z^{-2} + 0.7z^{-3} - 0.45z^{-4} + 0.9z^{-5} - 0.6z^{-6} + 0.4z^{-7},$$
$$H_4(z) = -0.5 + z^{-1} - 0.4z^{-2} + 0.8z^{-3} - 0.125z^{-4} + 0.25z^{-5} - 0.1z^{-6} + 0.2z^{-7},$$
$$H_5(z) = -0.1 + 0.2z^{-1} - 0.125z^{-2} + 0.25z^{-3} - 0.4z^{-4} + 0.8z^{-5} - 0.5z^{-6} + z^{-7}.$$

Which transfer has all its zeros outside the unit circle? Which one has all its zeros inside the unit circle? How many other length-8 FIR filters exist that have the same magnitude response as that of the above transfer functions?

7.11 A causal LTI FIR discrete-time system is characterized by an impulse response $h[n] = a_1\delta[n] + a_2\delta[n-1] + a_3\delta[n-2] + a_4\delta[n-3] + a_5\delta[n-4] + a_6\delta[n-5]$. For what values of the impulse response samples will its frequency response $H(e^{j\omega})$ have a constant group delay?

7.12 An FIR LTI discrete-time system is described by the difference equation

$$y[n] = a_1x[n+k] - a_2x[n+k-1] + a_2x[n+k-3] - a_1x[n+k-4],$$

where $y[n]$ and $x[n]$ denote, respectively, the output and the input sequences. Determine the expression for its frequency response $H(e^{j\omega})$. For what values of the constant k will the system have a frequency response $H(e^{j\omega})$ that is a real function of ω?

7.13 Consider the cascade of two causal LTI systems: $h_1[n] = \alpha\delta[n] + \beta\delta[n-1]$ and $h_2[n] = \gamma^n\mu[n]$, $|\beta| < 1$. Determine the frequency response $H(e^{j\omega})$ of the overall system. For what values of α, β and γ will $|H(e^{j\omega})| = K$, where K is a real constant?

7.14 The input–output relation of a nonlinear discrete-time system in the frequency domain is given by

$$Y(e^{j\omega}) = |X(e^{j\omega})|^{\alpha} e^{j\arg X(e^{j\omega})}, \tag{7.157}$$

where $0 < \alpha \leq 1$ and $X(e^{j\omega})$ and $Y(e^{j\omega})$ denote the DTFTs of the input and output sequences, respectively. Determine the expression for its frequency response $H(e^{j\omega}) = Y(e^{j\omega})/X(e^{j\omega})$, and show that it has zero phase. The nonlinear algorithm described by Eq. (7.157) is known as the *alpha-rooting method* and has been used in image enhancement [Jai89].

7.15 An FIR filter of length 3 is defined by a symmetric impulse response; i.e., $h[0] = h[2]$. Let the input to this filter be a sum of two cosine sequences of angular frequencies 0.3 rad/samples and 0.6 rad/samples, respectively. Determine the impulse response coefficients so that the filter passes only the low-frequency component of the input.

7.16 (a) Design a length-5 FIR bandpass filter with an antisymmetric impulse response $h[n]$, i.e., $h[n] = -h[4-n]$, $0 \leq n \leq 4$, satisfying the following magnitude response values: $|H(e^{j0.3\pi})| = 0.3$ and $|H(e^{j0.6\pi})| = 0.8$.
(b) Determine the exact expression for the frequency response of the filter designed, and plot its magnitude and phase responses using MATLAB.

7.17 (a) Design a length-4 FIR highpass filter with an antisymmetric impulse response $h[n]$, i.e., $h[n] = -h[3-n]$, $0 \leq n \leq 3$, satisfying the following magnitude response values: $|H(e^{j0.25\pi})| = 0.2$ and $|H(e^{j0.8\pi})| = 0.8$.
(b) Determine the exact expression for the frequency response of the filter designed, and plot its magnitude and phase responses using MATLAB.

7.18 An FIR filter of length 5 is defined by a symmetric impulse response; i.e., $h[n] = h[4-n]$, $0 \leq n \leq 4$. Let the input to this filter be a sum of three cosine sequences of angular frequencies: 0.3 rad/samples, 0.5 rad/samples, and 0.8 rad/samples, respectively. Determine the impulse response coefficients so that the filter blocks only the midfrequency component of the input.

7.19 The frequency response $H(e^{j\omega})$ of a length-4 FIR filter with a real impulse response has the following specific values: $H(e^{j0}) = 13$, $H(e^{j3\pi/4}) = -3 - j4$, and $H(e^{j\pi}) = -3$. Determine $H(z)$.

7.20 The frequency response $H(e^{j\omega})$ of a length-4 FIR filter with a real and antisymmetric impulse response has the following specific values: $H(e^{j\pi}) = 20$ and $H(e^{j3\pi/4}) = -5 - j5$. Determine $H(z)$.

7.21 Consider the two LTI causal digital filters with impulse responses given by

$$h_A[n] = 0.5\delta[n] - \delta[n-1] + 0.5\delta[n-2],$$
$$h_B[n] = 0.5\delta[n] + \delta[n-1] + 0.5\delta[n-2].$$

(a) Sketch the magnitude responses of the two filters and compare their characteristics.
(b) Let $h_A[n]$ be the impulse response of a causal digital filter with a frequency response $H_A(e^{j\omega})$. Define another digital filter whose impulse response $h_C[n]$ is given by

$$h_C[n] = (-1)^n h_A[n], \qquad \text{for all } n.$$

What is the relation between the frequency response $H_C(e^{j\omega})$ of this new filter and the frequency response $H_A(e^{j\omega})$ of the parent filter?

7.22 We have shown that a real-coefficient FIR transfer function $H(z)$ with a symmetric impulse response has a linear-phase response. As a result, the all-pole IIR transfer function $G(z) = 1/H(z)$ will also have a linear-phase response. What are the practical difficulties in implementing $G(z)$? Justify your answer mathematically.

7.23 Prove Eqs. (7.36) and (7.37) of the impulse response coefficients of a causal minimum-phase transfer function.

7.24 Check the stability of each of the following causal IIR transfer functions. If they are not stable, find a stable transfer function with an identical magnitude function. Are there any other transfer functions having the same magnitude response as those shown below?

(a) $H_1(z) = \frac{z^3+3z^2+2z+7}{(2z+3)(z^2+0.5z+0.8)}$, (b) $H_2(z) = \frac{4z^3-2z^2+5z-6}{(1.5z^2+3z-5)(z^2-0.3z+0.7)}$.

7.25 The *notch filter* is used to suppress a particular sinusoidal component of frequency ω_o of an input signal $x[n]$ and has a transfer function with zeros at $z = e^{\pm j\omega_o}$. For each filter given below, (i) determine the notch frequency ω_o, (ii) show the form of the corresponding sinusoidal sequence to be suppressed, and (iii) verify by computing the output $y[n]$ by convolution that in the steady state, $y[n] = 0$ when the sinusoidal sequence is applied at the input of the filter.

(a) $H_1(z) = 1 - z^{-1} + z^{-2}$, (b) $H_2(z) = 1 - 0.8z^{-1} + z^{-2}$, (c) $H_3(z) = 1 - 1.6z^{-1} + z^{-2}$.

7.26 Let $G_L(z)$ and $G_H(z)$ represent ideal lowpass and highpass filters with magnitude responses as sketched in Figure P7.2(a). Determine the transfer functions $H_k(z) = Y_k(z)/X(z)$ of the discrete-time system of Figure P7.2(b), $k = 0, 1, 2, 3$, and sketch their magnitude responses.

7.27 Let $H_{LP}(z)$ denote the transfer function of an ideal real coefficient lowpass filter with a cutoff frequency of ω_p. Sketch the magnitude response of $H_{LP}(-z)$, and show that it is a highpass filter. Determine the relation between the cutoff frequency of this highpass filter in terms of ω_p and its impulse response in terms of the impulse response $h_{LP}[n]$ of the parent lowpass filter.

7.28 Let $H_{LP}(z)$ denote the transfer function of an ideal real coefficient lowpass filter having a cutoff frequency of ω_p, with $\omega_p < \pi/2$. Consider the complex coefficient transfer function $H_{LP}(e^{j\omega_o}z)$, where $\omega_p < \omega_o < \pi - \omega_p$. Sketch its magnitude response for $-\pi \le \omega \le \pi$. What type of filter does it represent? Now consider the transfer function $G(z) = H_{LP}(e^{j\omega_o}z) + H_{LP}(e^{-j\omega_o}z)$. Sketch its magnitude response for $-\pi \le \omega \le \pi$. Show that $G(z)$ is a real-coefficient bandpass filter with a passband centered at ω_o. Determine the width of its passband in terms of ω_p and its impulse response $g[n]$ in terms of the impulse response $h_{LP}[n]$ of the parent lowpass filter.

7.29 Let $H_{LP}(z)$ denote the transfer function of an ideal real coefficient lowpass filter with a cutoff frequency of ω_p with $0 < \omega_p < \pi/3$. Show that the transfer function $F(z) = H_{LP}(e^{j\omega_o}z) + H_{LP}(e^{-j\omega_o}z) + H_{LP}(z)$, where $\omega_o = \pi - \omega_p$, is a real-coefficient bandstop filter with the stopband centered at $\omega_o/2$. Determine the width of its stopband in terms of ω_p and its impulse response $f[n]$ in terms of the impulse response $h_{LP}[n]$ of the parent lowpass filter.

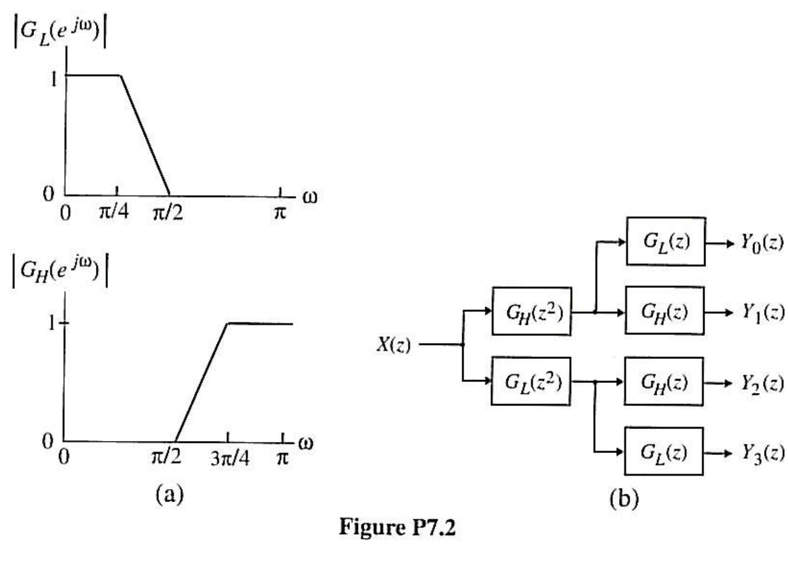

Figure P7.2

7.30 Show that the structure shown in Figure P7.3 implements the highpass filter of Problem 7.27.

7.31 Show that the structure shown in Figure P7.4 implements the bandpass filter of Problem 7.28.

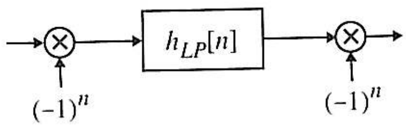

Figure P7.3

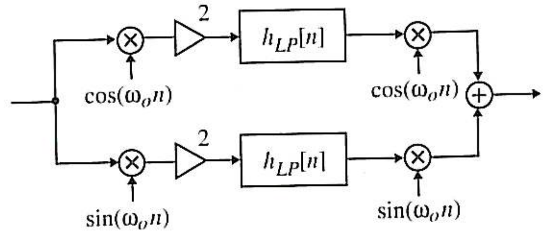

Figure P7.4

7.32 Let $H(z)$ be an ideal real-coefficient lowpass filter with a cutoff at ω_c, where $\omega_c = \pi/M$. Figure P7.5 shows a single-input, M-output filter structure, called an M-band analysis filter bank, where $H_k(z) = H(ze^{-j2\pi k/M})$, $k = 0, 1, \ldots, M-1$. Sketch the magnitude response of each filter, and describe the operation of the filter bank.

7.33 Consider a cascade of M sections of the first-order FIR lowpass filter of Eq. (7.64). Show that its 3-dB cutoff frequency is given by Eq. (7.66).

7.34 Consider a cascade of M sections of the first-order FIR highpass filter of Eq. (7.67). Develop the expression for its 3-dB cutoff frequency.

7.35 Verify that the value of α given by Eq. (7.73b) ensures that the transfer function $H_{LP}(z)$ of Eq. (7.71) is stable.

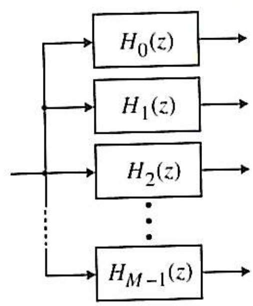

Figure P7.5

7.36 Show by trigonometric manipulation that Eq. (7.73a) can be alternately expressed as

$$\tan\left(\frac{\omega_c}{2}\right) = \frac{1-\alpha}{1+\alpha}. \tag{7.158}$$

Next show that the transfer function $H_{LP}(z)$ of Eq. (7.71) is stable for a value of α given by

$$\alpha = \frac{1-\tan(\omega_c/2)}{1+\tan(\omega_c/2)}. \tag{7.159}$$

7.37 Design a first-order lowpass IIR digital filter for each of the following normalized 3-dB cutoff frequencies: (a) 0.6 rad/samples, (b) 0.45π.

7.38 Show that the 3-dB cutoff frequency ω_c of the first-order highpass IIR digital filter of Eq. (7.74) is given by Eq. (7.73a).

7.39 Design a first-order highpass IIR digital filter for each of the following normalized 3-dB cutoff frequencies: (a) 0.6 rad/samples, (b) 0.55π.

7.40 The following first-order IIR transfer function has been proposed for clutter removal in MTI radars [Urk58]:

$$H(z) = \frac{1 - z^{-1}}{1 - kz^{-1}}.$$

Determine the magnitude response of the above transfer function and show that it has a highpass response. Scale the transfer function so that it has a 0-dB gain at $\omega = \pi$. Sketch the magnitude responses for $k = 0.95, 0.9$, and -0.5, respectively.

7.41 Show that the center frequency ω_o and the 3-dB bandwidth B_w of the second-order IIR bandpass filter of Eq. (7.75) are given by Eqs. (7.76) and (7.78), respectively.

7.42 Design a second-order bandpass IIR digital filter for each of the following specifications: (a) $\omega_o = 0.55\pi$, $B_w = 0.25\pi$, (b) $\omega_o = 0.3\pi$, $B_w = 0.3\pi$.

7.43 Show that the notch frequency ω_o and the 3-dB notch bandwidth B_ω of the second-order IIR bandstop filter of Eq. (7.80) are given by Eqs. (7.76) and (7.78), respectively.

7.44 Design a second-order bandstop IIR digital filter for each of the following specifications: (a) $\omega_o = 0.35\pi$, $B_w = 0.2\pi$, (b) $\omega_o = 0.6\pi$, $B_w = 0.15\pi$.

7.45 Consider a cascade of K identical first-order lowpass digital filters with a transfer function given by Eq. (7.71). Show that the coefficient α of the first-order section is related to the 3-dB cutoff frequency ω_c of the cascade according to Eq. (7.84), with the parameter C given by Eq. (7.85).

7.46 Consider a cascade of K identical first-order highpass digital filters with a transfer function given by Eq. (7.74). Express the coefficient α of the first-order section in terms of the 3-dB cutoff frequency ω_c of the cascade.

7.47 The filter structures shown in Figure P7.6, where $\mathcal{A}_1(z)$ is a stable first-order allpass filter, can be used as low-frequency shelving filters in digital audio equalization [Zöl97] (see Section 15.5.2). Determine the transfer function of each structure.

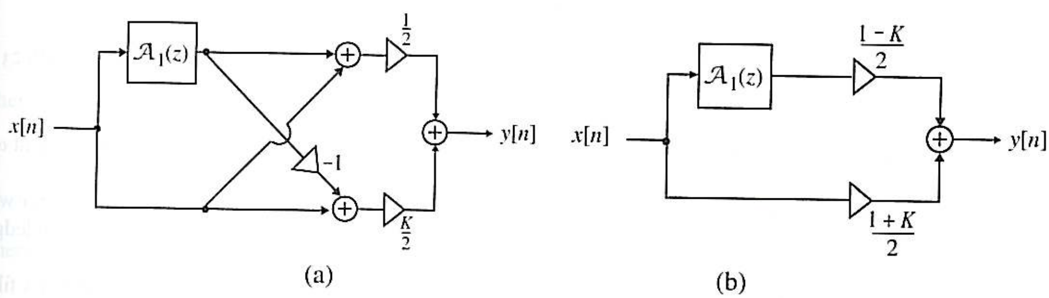

Figure P7.6

7.48 The filter structure shown in Figure P7.7(a), where $\mathcal{A}_1(z)$ is a stable first-order allpass filter, can be used as a low-frequency shelving filter in digital audio equalization [Zöl97] (see Section 15.5.2). Likewise, the filter structure shown in Figure P7.7(b), where $\mathcal{A}_1(z)$ is a stable first-order allpass filter, can be used as a high-frequency shelving filter in digital audio equalization [Zöl97]. Determine the transfer function of each structure.

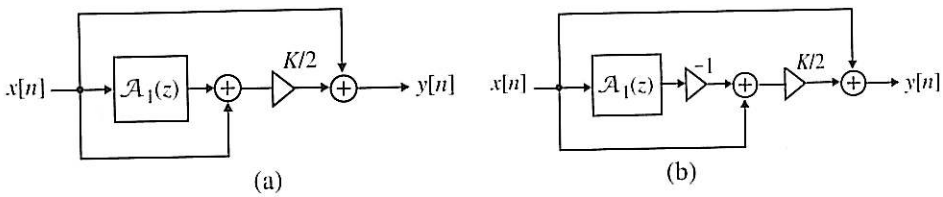

Figure P7.7

7.49 If $H(z)$ is a bandpass filter with passband edges at ω_{p1} and ω_{p2}, and stopband edges at ω_{s1} and ω_{s2}, with $\omega_{s1} < \omega_{p1} < \omega_{p2} < \omega_{s2}$, what type of filter is $H(-z)$? Determine the locations of the bandedges of $F(-z)$ in terms of the bandedges of $H(z)$.

7.50 Using the method of Problem ??, develop the transfer function $G_{HP}(z)$ of a first-order IIR highpass filter from the transfer function $H_{LP}(z)$ of the first-order IIR lowpass filter given by Eq. (7.71). Is it the same as that of the highpass transfer function of Eq. (7.74)? If not, determine the location of its 3-dB cutoff frequency as a function of the parameter α.

7.51 Let $H(z)$ be an ideal lowpass filter with a cutoff frequency at $\pi/3$. Sketch the magnitude responses of the following systems: (a) $H(z^3)$, (b) $H(z)H(z^3)$, (c) $H(-z)H(z^3)$, and (d) $H(z)H(-z^3)$.

7.52 Show that the amplitude response $\breve{H}(\omega)$ of Type 1 and Type 3 linear-phase FIR transfer functions is a periodic function of ω with a period 2π and the amplitude response $\breve{H}(\omega)$ of Type 2 and Type 4 linear-phase FIR transfer functions is a periodic function of ω with a period 4π.

7.53 A length-13 Type 1 real-coefficient FIR filter has the following zeros: $z_1 = 0.8$, $z_2 = -j$, $z_3 = 2 - j2$, $z_4 = -0.5 + j0.3$. (a) Determine the locations of the remaining zeros. (b) What is the transfer function $H_1(z)$ of the filter?

7.54 A length-12 Type 2 real-coefficient FIR filter has the following zeros: $z_1 = 3.1$, $z_2 = -2 + j4$, $z_3 = 0.8 + j0.4$. (a) Determine the locations of the remaining zeros. (b) What is the transfer function $H_2(z)$ of the filter?

7.55 A length-13 Type 3 real-coefficient FIR filter has the following zeros: $z_1 = 0.1 - j0.599$, $z_2 = -0.3 + j0.4$, $z_3 = 2$. (a) Determine the locations of the remaining zeros. (b) What is the transfer function $H_3(z)$ of the filter?

7.56 A length-12 Type 4 real-coefficient FIR filter has the following zeros: $z_1 = 2.2 + j3.4$, $z_2 = 0.6 + j0.9$, $z_3 = -0.5$. (a) Determine the locations of the remaining zeros. (b) What is the transfer function $H_4(z)$ of the filter?

7.57 Let $H(z)$ be a lowpass filter with unity passband magnitude, a passband edge at ω_p, and a stopband edge at ω_s, as shown in Figure P7.8.

(a) Sketch the magnitude response of the digital filter $G_1(z) = H(z^M)F_1(z)$, where $F_1(z)$ is a lowpass filter with unity passband magnitude, a passband edge at ω_p/M, and a stopband edge at $(2\pi - \omega_s)/M$. What are the bandedges of $G_1(z)$?

(b) Sketch the magnitude response of the digital filter $G_2(z) = H(z^M)F_2(z)$, where $F_2(z)$ is a bandpass filter with unity passband magnitude, and with passband edges at $(2\pi - \omega_p)/M$ and $(2\pi + \omega_p)/M$ and stopband edges at $(2\pi - \omega_s)/M$ and $(2\pi + \omega_s)/M$, respectively. What are the bandedges of $G_2(z)$?

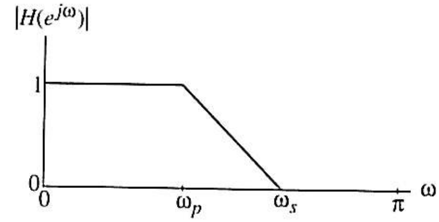

Figure P7.8

7.58 Show analytically that an FIR filter with a constant group delay must have either a symmetric or an antisymmetric impulse response.

7.59 Let the first five impulse response samples of a causal linear-phase FIR filter be given by $h[0] = a,\ h[1] = -b,\ h[2] = -c,\ h[3] = d$, and $h[4] = e$. Determine the remaining impulse response samples of the transfer function of lowest order for each type of linear-phase filter.

7.60 The first five samples of the impulse response of an FIR filter $H(z)$ are given by $h[0] = 1, h[1] = -3, h[2] = -4, h[3] = 6$, and $h[4] = 8$. Determine the remaining impulse response samples of $H(z)$ of lowest order for each type of linear-phase filter. Using `zplane`, determine the zero locations for $H(z)$ for each type of linear-phase filter. Does $H(z)$ have a zero at $z = 1$ and/or $z = -1$? Do the zeros on the unit circle appear in complex conjugate pairs? Do the zeros not on the unit circle appear in mirror-image symmetry? Justify your answers.

7.61 Let $H_1(z)$, $H_2(z)$, $H_3(z)$, and $H_4(z)$ be, respectively, Type 1, Type 2, Type 3, and Type 4 linear-phase FIR filters. Are the following filters composed of a cascade of the above filters' linear phase? If they are, what are their types?

(a) $G_a(z) = H_1(z)H_1(z)$, (b) $G_b(z) = H_1(z)H_2(z)$, (c) $G_c(z) = H_1(z)H_3(z)$,
(d) $G_d(z) = H_1(z)H_4(z)$, (e) $G_e(z) = H_2(z)H_2(z)$, (f) $G_f(z) = H_3(z)H_3(z)$,
(g) $G_g(z) = H_4(z)H_4(z)$, (h) $G_h(z) = H_2(z)H_3(z)$, (i) $G_i(z) = H_3(z)H_4(z)$.

7.62 Consider a linear-phase FIR transfer function given by $H(z) = F_1(z)F_2(z)$. Determine the factor $F_2(z)$ of lowest order for each of the following choices for $F_1(z)$:

(a) $F_1(z) = 2.1 - 3.5z^{-1} + 4.2z^{-2}$, (b) $F_1(z) = 1.4 + 5.2z^{-1} - 2.2z^{-2} + 3.3z^{-3}$.

7.63 Consider a causal FIR transfer function given by

$$H(z) = K\left(1 + h[1]z^{-1} + h[2]z^{-2} + \cdots + h[N]z^{-N}\right) = K\prod_{i=1}^{N}(1 - \lambda_i z^{-1}), \qquad (7.160)$$

where K is a constant. The *root moment* of $H(z)$ is defined by [Fot2001]

$$S_m = \sum_{i=0}^{N}\lambda_i^m, \qquad 1 \le m \le N, \qquad (7.161)$$

where m is the degree of the moment. Prove the *Newton Identities* given by

$$S_m + h[1]S_{m-1} + h[2]S_{m-2} + \cdots + mh[m] = 0, \qquad 1 \le m \le N. \qquad (7.162)$$

Note: The above identities can be iteratively solved for all N root moments.

7.64 Assume that the causal FIR transfer function $H(z)$ of Eq. (7.160) has M_α roots $\{\alpha_i\}$ inside the unit circle, and M_β roots $\{\beta_i\}$ outside the unit circle, where $M_\alpha + M_\beta = N$. We can thus rewrite $H(z)$ as

$$H(z) = K\prod_{i=1}^{M_\alpha}(1-\alpha_i z^{-1})\prod_{i=1}^{M_\beta}(1-\beta_i z^{-1}) = KG_\alpha(z)G_\beta(z), \qquad (7.163)$$

where $G_\alpha(z) = \prod_{i=1}^{M_\alpha}(1-\alpha_i z^{-1})$ is the minimum-phase factor and $G_\beta(z) = \prod_{i=1}^{M_\beta}(1-\beta_i z^{-1})$ is the maximum-phase factor of $H(z)$.

(a) Show that the root moments of a $H(z)$ with real coefficients are real.

(b) Show that the root moments of a minimum-phase $H(z)$ decrease exponentially with increasing m.

(c) If $\breve{H}(\omega)$ and $\theta(\omega)$ denote, respectively, the amplitude and phase responses of $H(z)$, show then [Fot2001]

$$\ln \breve{H}(\omega) = \ln(K_1) - \sum_{m=1}^{\infty}\frac{S_m^\alpha - S_{-m}^\beta}{m}\cos(m\omega), \qquad (7.164a)$$

$$\theta(\omega) = -\omega M_\beta + \sum_{m=1}^{\infty}\frac{S_m^\alpha - S_{-m}^\beta}{m}\sin(m\omega), \qquad (7.164b)$$

where K_1 is an appropriate real constant, $\{S_m^\alpha\}$ are the root moments of the minimum-phase factor $G_\alpha(z)$, and $\{S_{-m}^\beta\}$ are the inverse root moments of the maximum-phase factor $G_\beta(z)$.

(d) Show that for the transfer function $H(z)$ to have linear phase, it must have zeros located outside the unit circle, and determine their number in relation to the number of zeros inside the unit circle.

7.65 (a) Show that the phase delay $\tau_p(\omega)$ of the first-order allpass transfer function

$$A_1(z) = \frac{d_1 + z^{-1}}{1 + d_1 z^{-1}},$$

is given by $\tau_p(\omega) \cong (1-d_1)/(1+d_1) = \delta$ [Ste96].

(b) Design a first-order allpass filter with a phase delay of $\delta = 0.5$ sample and operating at a sampling rate of 20 kHz. Determine the error in samples at 1 kHz in the phase delay from its design value of 0.5 sample.

7.66 Consider the second-order allpass transfer function

$$A_2(z) = \frac{d_2 + d_1 z^{-1} + z^{-2}}{1 + d_1 z^{-1} + d_2 z^{-2}}.$$

If δ denotes the desired low-frequency approximate value of the phase delay $\tau_p(\omega) = -\theta(\omega)/\omega$, show that [Fet72]

$$d_1 = 2\left(\frac{2-\delta}{1+\delta}\right), \qquad d_2 = \frac{(2-\delta)(1-\delta)}{(2+\delta)(1+\delta)}.$$

7.67 Let $G(z)$ be a causal stable nonminimum-phase transfer function, and let $H(z)$ denote another causal stable transfer function that is minimum-phase with $\left|G(e^{j\omega})\right| = \left|H(e^{j\omega})\right|$. Show that $G(z) = H(z)A(z)$, where $A(z)$ is a stable causal allpass transfer function.

7.68 A typical transmission channel is characterized by a causal transfer function

$$H(z) = \frac{(3z-2.1)(z^2+2.5z+5)}{(z-0.65)(z+0.48)}.$$

In order to correct for the magnitude distortion introduced by the channel on a signal passing through it, we wish to connect a causal stable digital filter characterized by a transfer function $G(z)$ at the receiving end. Determine $G(z)$.

7.69 Let $H(z)$ be a causal stable minimum-phase transfer function, and let $G(z)$ denote another causal stable transfer function that is nonminimum-phase with $\left|G(e^{j\omega})\right| = \left|H(e^{j\omega})\right|$. If $h[n]$ and $g[n]$ denote their respective impulse responses, show that

(a) $|g[0]| \leq |h[0]|$,
(b) $\sum_{\ell=0}^{n} |g[\ell]|^2 \leq \sum_{\ell=0}^{n} |h[\ell]|^2$.

7.70 Is the transfer function

$$H(z) = \frac{(2z+3)(4z-1)}{(z+0.4)(z-0.6)}$$

minimum-phase? If it is not minimum-phase, then construct a minimum-phase transfer function $G(z)$ such that $\left|G(e^{j\omega})\right| = \left|H(e^{j\omega})\right|$. Determine their corresponding unit sample responses, $g[n]$ and $h[n]$, for $n = 0, 1, 2, 3, 4$. For what values of m is $\sum_{n=0}^{m} |g[n]|^2$ bigger than $\sum_{n=0}^{m} |h[n]|^2$?

7.71 The following bandstop FIR transfer functions $H_{BS}(z)$ have also been proposed for the recovery of vertical details in the structure of Figure 7.30 employed for the separation of the luminance and the chrominance components [Aca83], [Pri80], [Ros75]:

(a) $H_{BS}(z) = \frac{1}{4}(1+z^{-2})^2$,
(b) $H_{BS}(z) = \frac{1}{16}(1+z^{-2})^2(-1+6z^{-2}-z^{-4})$,
(c) $H_{BS}(z) = \frac{1}{32}(1+z^{-2})^2(-3+14z^{-2}-3z^{-4})$.

Develop their delay-complementary transfer functions $H_{BP}(z)$.

7.72 Let $A_0(z)$ and $A_1(z)$ be two causal stable allpass transfer functions. Define two causal stable IIR transfer functions as follows:

$$H_0(z) = A_0(z) + A_1(z), \qquad H_1(z) = A_0(z) - A_1(z).$$

Show that the numerators of $H_0(z)$ and $H_1(z)$ are, respectively, a symmetric and an antisymmetric polynomial.

7.73 Show that the two transfer functions of Eqs. (7.97a) and (7.97b) are a power-complementary pair.

7.74 Show that the two transfer functions of Eqs. (7.97a) and (7.97b) are each a BR function.

7.75 Consider the transfer function $H(z)$ given by

$$H(z) = \frac{1}{M}\sum_{k=0}^{M-1} A_k(z),$$

where $A_k(z)$ are stable real-coefficient allpass functions. Show that $H(z)$ is a BR function.

7.76 Show that the bandpass transfer function $H_{BP}(z)$ of Eq. (7.77) and the bandstop transfer function $H_{BS}(z)$ of Eq. (7.80) form a doubly-complementary pair.

7.77 Show that the value of the gain function $\mathcal{G}(\omega)$ of a power-symmetric transfer function defined by Eq. (7.101) at $\omega = \pi/2$ is given by $10\log_{10} K - 3$ dB.

7.78 Consider the real-coefficient stable IIR transfer function $H(z) = A_0(z^2) + z^{-1}A_1(z^2)$, where $A_0(z)$ and $A_1(z)$ are stable allpass transfer functions. Show that $H(z)$ is a power-symmetric transfer function.

7.79 Show that

$$H(z) = \frac{-0.1 + 0.5z^{-1} + 0.05z^{-2} + 0.05z^{-3} + 0.5z^{-4} - 0.1z^{-5}}{1 + 0.1z^{-2} - 0.2z^{-4}}$$

is a power-symmetric IIR transfer function.

7.80 Show that the following causal FIR transfer functions satisfy the power-symmetric condition:
(a) $H_a(z) = 1 - 2z^{-1} + 4.5z^{-2} + 6z^{-3} + z^{-4} + 0.5z^{-5}$,
(b) $H_b(z) = 1 + \frac{1}{2}z^{-1} + \frac{15}{4}z^{-2} - z^{-4} + 2z^{-5}$.

7.81 Let $H(z) = a(1 + bz^{-1})$, where a and b are constants. Then $H(z)H(z^{-1})$ is of the form $cz + d + cz^{-1}$. Determine the condition on c and d so that $H(z)$ is a power-symmetric FIR transfer function with $K = 1$. Show that $a = 1/2$ and $b = 1$ satisfy the power-symmetric condition. Determine two other possible sets of values for a and b to ensure the power-symmetric condition. Using MATLAB, show that $H(z)$ and $G(z) = -z^{-1}H(-z^{-1})$ are power-complementary for the above values of the constants a and b.

7.82 Let $H(z) = a(1 + bz^{-1})(1 + d_1z^{-1} + d_2z^{-2})$, where a, b, d_1, and d_2 are constants. Then $H(z)H(z^{-1})$ is of the form $(cz + d + cz^{-1})[d_2z^2 + d_1(1 + d_2)z + (1 + d_1^2 + d_2^2) + d_1(1 + d_2)z^{-1} + d_2z^{-2}]$. Determine the condition on c and d in terms of d_1 and d_2 so that $H(z)$ is a power-symmetric FIR transfer function with $K = 1$. For $d_1 = d_2 = 1$, evaluate the constraint on c and d, and using it, determine one realizable set of values for a and b. Using MATLAB, show that $H(z)$ and $G(z) = -z^{-3}H(-z^{-1})$ are power-complementary for these values of the constants a and b.

7.83 A set of M digital filters $\{G_i(z)\}$, $i = 0, 1, \ldots, M-1$, is defined to be *magnitude-complementary* of each other if the sum of their magnitude responses is equal to a constant [Reg87c]; that is,

$$\sum_{i=0}^{M-1} |G_i(e^{j\omega})| = \beta, \qquad \text{for all } \omega, \tag{7.165}$$

where β is a positive nonzero constant. Consider two real-coefficient doubly-complementary transfer functions $H_0(z)$ and $H_1(z)$ that are related according to Eqs. (7.97a) and (7.97b). Define $G_0(z) = H_0^2(z)$ and $G_1(z) = -H_1^2(z)$. Show that $G_0(z)$ and $G_1(z)$ are a pair of magnitude-complementary transfer functions.

7.84 Show analytically that the following causal FIR transfer functions are BR functions:
(a) $H_1(z) = \frac{1}{4}\left(1 + 3z^{-1}\right)$, (b) $H_2(z) = \frac{1}{2.2}\left(1 - 1.2z^{-1}\right)$,
(c) $H_3(z) = \frac{(1+\alpha z^{-1})(1-\beta z^{-1})}{(1+\alpha)(1+\beta)}$, $\alpha > 0$, $\beta > 0$,
(d) $H_4(z) = \frac{1}{2.34}(1 - 0.3z^{-1})(1 + 0.2z^{-1})(1 - 0.5z^{-1})$.

7.85 Show analytically that the following causal IIR transfer functions are BR functions:
(a) $H_1(z) = \dfrac{2.6 + 2.6z^{-1}}{4.2 + z^{-1}}$, (b) $H_2(z) = \dfrac{1.6 - 1.6z^{-1}}{4.2 + z^{-1}}$, (c) $H_3(z) = \dfrac{0.2(1 - z^{-2})}{1 + 0.4z^{-1} + 0.8z^{-2}}$,
(d) $H_4(z) = \dfrac{4.5 + 2z^{-1} + 4.5z^{-2}}{5 + 2z^{-1} + 4z^{-2}}$.

7.86 If $A_1(z)$ and $A_2(z)$ are two LBR functions, show that $A_1(1/A_2(z))$ is also an LBR function.

7.87 Let $G(z)$ be an LBR function of order N. Define

$$F(z) = z\left(\frac{G(z) + \alpha}{1 + \alpha G(z)}\right),$$

where $|\alpha| < 1$. Show that $F(z)$ is also LBR. What is the order of $F(z)$? Develop a realization of $G(z)$ in terms of $F(z)$.

7.88 If $G(z)$ and $A(z)$ are, respectively, a BR function and and an LBR function, show that $G(1/A(z))$ is a BR function.

7.89 Show analytically that each of the following pairs of transfer functions are doubly-complementary:

(a) $H(z) = \dfrac{2.6(1+z^{-1})}{4.2+z^{-1}}$, $G(z) = \dfrac{1.6(1-z^{-1})}{4.2+z^{-1}}$,

(b) $H(z) = \dfrac{0.1(1-z^{-2})}{1+0.4z^{-1}+0.8z^{-2}}$, $G(z) = \dfrac{0.9+0.4z^{-1}+0.9z^{-2}}{1+0.4z^{-1}+0.8z^{-2}}$.

7.90 Determine analytically the power-complementary transfer function of each of the following BR transfer functions:

(a) $H_a(z) = \dfrac{2(2+z^{-1}+2z^{-2})}{5+2z^{-1}+3z^{-2}}$, (b) $H_b(z) = \dfrac{3+7.5z^{-1}+7.5z^{-2}+3z^{-3}}{8+8z^{-1}+4z^{-2}+z^{-3}}$.

7.91 Verify the relations between the transfer parameters and the chain parameters of a two-pair given in Eqs. (7.128a) and (7.128b).

7.92 A two-pair is said to be *reciprocal* if $t_{12} = t_{21}$ [Mit73b]. Show that for a reciprocal two-pair, $AD - BC = 1$.

7.93 Consider the Γ-cascade of Figure P7.9(a), where the two two-pairs are described by the transfer matrices

$$\boldsymbol{\tau}_1 = \begin{bmatrix} k_1 & (1-k_1^2)z^{-1} \\ 1 & -k_1 z^{-1} \end{bmatrix}, \qquad \boldsymbol{\tau}_2 = \begin{bmatrix} k_2 & (1-k_2^2)z^{-1} \\ 1 & -k_2 z^{-1} \end{bmatrix}.$$

Determine the transfer matrix of the cascade.

7.94 Consider the τ-cascade of Figure P7.9(b), where the two two-pairs are described by the chain matrices

$$\boldsymbol{\Gamma}_1 = \begin{bmatrix} 1 & k_1 z^{-1} \\ k_1 & z^{-1} \end{bmatrix}, \qquad \boldsymbol{\Gamma}_2 = \begin{bmatrix} 1 & k_2 z^{-1} \\ k_2 & z^{-1} \end{bmatrix}.$$

Determine the chain matrix of the cascade.

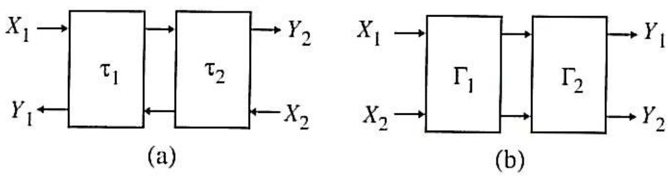

Figure P7.9

7.95 Determine the transfer parameters and the chain parameters of the digital two-pairs of Figure P7.10.

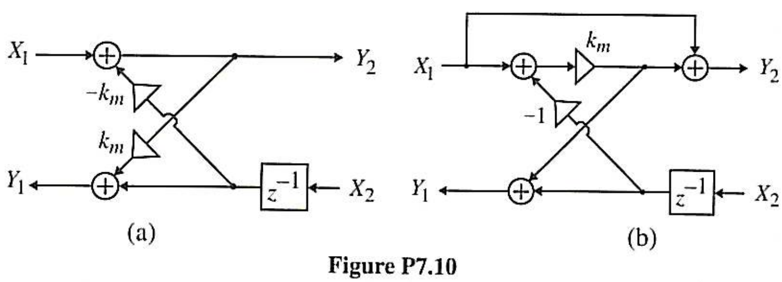

Figure P7.10

7.96 A transfer function $H(z)$ is realized in the form of Figure 7.37, where the constraining transfer function is given by $G(z)$. If the relation between $H(z)$ and $G(z)$ is of the form

$$G(z) = \frac{H(z) - k_m}{z^{-1}[1 - k_m H(z)]},$$

determine the transfer matrix and the chain matrix parameters of the two-pair of Figure 7.37.

7.97 A transfer function $H(z)$ is realized in the form of Figure 7.37, where the constraining transfer function is given by $G(z)$. The relation between $H(z)$ and $G(z)$ is of the form

$$H(z) = \frac{\alpha + z^{-1}G(z)}{1 + \alpha z^{-1}G(z)},$$

with α real and $|\alpha| < 1$.

(a) Determine the chain parameters of the two-pair of Figure 7.37.
(b) If $|G(z)| < 1$ for $|z| = 1$, show that $|H(z)|$ cannot have a maximum value greater than unity.

7.98 Determine chain parameters of the cascade of three lattice two-pairs of Figure P7.11. Using these chain parameters, determine the expression for the transfer function $A_3(z)$.

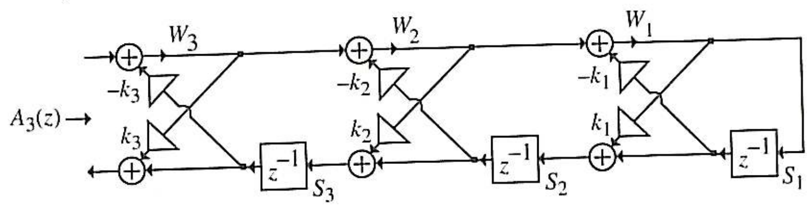

Figure P7.11

7.99 Derive the inequality of Eq. (7.141).

7.100 Determine by inspection which one of the following second-order polynomials has both roots inside the unit circle:

(a) $D_a(z) = 4 + 3z^{-1} + 2z^{-2}$, (b) $D_b(z) = 2 + z^{-1} + z^{-2}$,
(c) $D_c(z) = 3 + 4z^{-1} - 4z^{-2}$, (d) $D_d(z) = 3 - 0.5z^{-1} - z^{-2}$.

7.101 Test analytically the BIBO stability of the following causal IIR transfer functions:

(a) $H_a(z) = \dfrac{2z^2 + 3.75z + 10}{z^3 + 0.75z^2 + 0.5z + 0.25}$, (b) $H_b(z) = \dfrac{0.1z^2 - 0.57z - 1.78}{3z^3 + 2z^2 - 2z - 1}$,

(c) $H_c(z) = \dfrac{8.7z^3 - 15.53z^2 + 33.1z + 12.5}{3z^4 + 2z^3 + 2z^2 + 1.5z - 0.5}$, (d) $H_d(z) = \dfrac{1}{5 + 4z^{-1} + 3z^{-2} + 2z^{-3} + z^{-4}}$,

(e) $H_e(z) = \dfrac{1}{10 + 7z^{-1} + 5z^{-2} + 3z^{-3} + 2z^{-4} + z^{-5}}$.

7.102 Determine analytically whether all roots of the following polynomials are inside the unit circle:

(a) $D_a(z) = 10 + 8z^{-1} + 6z^{-2} + 4z^{-3} + 2z^{-4} + z^{-5}$,
(b) $D_b(z) = 4z^5 + 3.5z^4 + 3z^3 + 2.5z^2 + 2z - 1$.

7.12 MATLAB Exercises

M 7.1 Write a MATLAB program to simulate the filter designed in Problem 7.15, and verify its filtering operation.

M 7.2 Write a MATLAB program to simulate the filter designed in Problem 7.18, and verify its filtering operation.

M 7.3 The following third-order IIR transfer function has been proposed for clutter rejection in MTI radar [Whi58]:

$$H(z) = \frac{z^{-1}(1 - z^{-1})^2}{(1 - 0.4z^{-1})(1 - 0.88z^{-1} + 0.61z^{-2})}.$$

Using MATLAB, determine and plot its gain response, and show that it has a highpass response.

M 7.4 Show that for each case listed below, $H(z)$ and $H(-z)$ are power-complementary.

(a) $H(z) = \dfrac{1 - 2z^{-1} + 3.5z^{-2} + 3.5z^{-3} - 2z^{-4} + z^{-5}}{8 - 2z^{-2} - z^{-4}}$,

(b) $H(z) = \dfrac{1 + 1.5z^{-1} + 5.25z^{-2} + 7.25z^{-3} + 7.25z^{-4} + 5.25z^{-5} + 1.5z^{-6} + z^{-7}}{12 + 13z^{-2} + 4.5z^{-4} + 0.5z^{-6}}$.

To verify the power-complementary property, write a MATLAB program to evaluate $H(z)H(z^{-1}) + H(-z)H(-z^{-1})$, and show that this expression is equal to unity for each of the transfer functions given above.

M 7.5 Plot the magnitude and phase responses of the causal IIR digital transfer function

$$H(z) = \frac{0.2031(1 - z^{-1})(1 - 0.2742z^{-1} + z^{-2})}{(1 + 0.2695z^{-1})(1 + 0.4109z^{-1} + 0.6758z^{-2})}.$$

What type of filter does this transfer function represent? Determine the difference equation representation of the above transfer function.

M 7.6 Plot the magnitude and phase responses of the causal IIR digital transfer function

$$H(z) = \frac{0.2031(1 - z^{-1})(1 - 0.2743z^{-1} + z^{-2})}{(1 + 0.1532z^{-1} + 0.8351z^{-2})(1 + 0.487z^{-1} + 0.84z^{-2})}.$$

What type of filter does this transfer function represent? Determine the difference equation representation of the above transfer function.

M 7.7 Design an FIR lowpass filter with a 3-dB cutoff frequency at 0.45π using a cascade of five first-order lowpass filters of Eq. (7.71). Plot its gain response.

M 7.8 Using the result of Problem 7.46, design an FIR highpass filter with a 3-dB cutoff frequency at 0.4π using a cascade of six first-order highpass filters of Eq. (7.74). Plot its gain response.

M 7.9 Design a first-order IIR lowpass and a first-order IIR highpass filter with a 3-dB cutoff frequency of 0.6π. Using MATLAB, plot their magnitude responses on the same figure. Using MATLAB, show that these filters are both allpass-complementary and power-complementary.

M 7.10 Design a second-order IIR bandpass and a second-order IIR notch filter with a center (notch) frequency $\omega_o = 0.4\pi$ and a 3-dB bandwidth B_w (notch width) of 0.25π. Using MATLAB, plot their magnitude responses on the same figure. Using MATLAB, show that these filters are both allpass-complementary and power-complementary.

M 7.11 Design a stable second-order IIR bandpass filter with a center frequency at 0.6π and a 3-dB bandwidth of 0.2π. Plot its gain response.

M 7.12 Design a stable second-order IIR notch filter with a center frequency at 0.6π and a 3-dB bandwidth of 0.2π. Plot its gain response.

M 7.13 Using MATLAB, show that the transfer function pairs of Exercises M7.11 and M7.12 are both allpass-complementary and power-complementary.

M 7.14 Using MATLAB, show that the transfer function pairs of Problem 7.89 are doubly-complementary.

M 7.15 Develop the pole-zero plots of the transfer functions of Problem 7.90 using the function `zplane` of MATLAB, and show that they are stable. Next, plot the magnitude response of each transfer function using MATLAB, and show that it satisfies the bounded real property.

M 7.16 Using MATLAB, determine the power-complementary transfer function of each of the transfer functions of Problem 7.90.

M 7.17 Develop the pole-zero plots of the transfer functions of Problem 7.101 using the function `zplane` of MATLAB, and then test their stability.

M 7.18 Using Program 7_2, test the stability of the transfer functions of Problem 7.101.

M 7.19 Using Program 7_2, determine whether the roots of the polynomials of Problem 7.102 are inside the unit circle or not.

M 7.20 The FIR digital filter structure of Figure P7.12 is used for aperture correction in television to compensate for high-frequency losses [Dre90]. A cascade of two such circuits is used, with one correcting the vertical aperture and the other correcting the horizontal aperture. In the former case, the delay z^{-1} is a line delay, whereas in the latter case, it is 70 ns for the CCIR standard, and the weighting factor k provides an adjustable amount of correction. Determine the transfer function of this circuit, and plot its magnitude response using MATLAB for two different values of k.

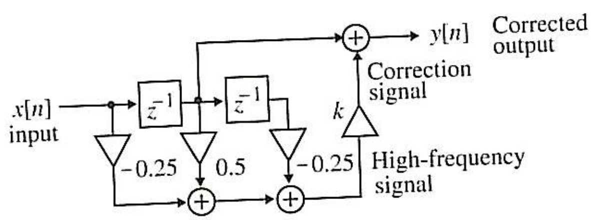

Figure P7.12

M 7.21 An improved aperture correction circuit for digital television is the FIR digital filter structure of Figure P7.13, where the delay z^{-1} is 70 ns for the CCIR standard, and the two weighting factors k_1 and k_2 provide an adjustable amount of correction with $k_1 > 0$ and $k_2 < 0$ [Dre90]. Determine the transfer function of this circuit, and plot its magnitude response using MATLAB for two different values of k_1 and k_2.

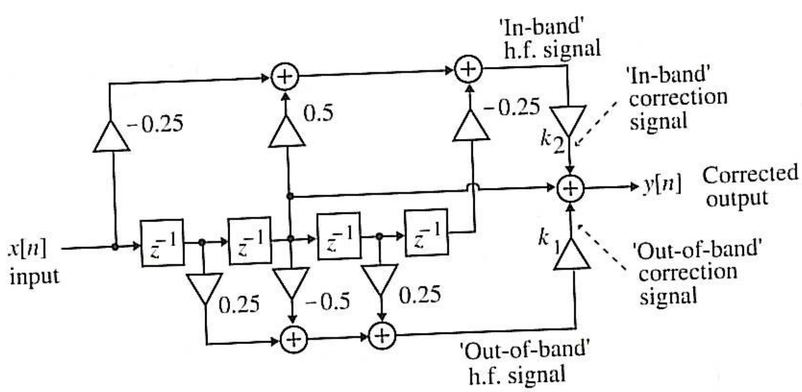

Figure P7.13