8.13 Summary

This chapter considered the realization of a causal digital transfer function. Such realizations, called structures, are usually represented in block diagram forms that are formed by interconnecting adders, multipliers, and unit delays. A digital filter structure represented in block diagram form can be analyzed to develop its input–output relationship either in the time domain or in the transform domain. Often, for analysis purposes, it is convenient to represent the digital filter structure as a signal flow-graph.

Several basic FIR and IIR digital filter structures are then reviewed. These structures, in most cases, can be developed from the transfer function description of the digital filter essentially by inspection.

The digital allpass filter is a versatile building block and has a number of attractive digital signal processing applications. Since the numerator and the denominator polynomials of a digital transfer function exhibit mirror-image symmetry, an Mth-order digital allpass filter can be realized with only M distinct multipliers. Two different approaches to the minimum-multiplier realization of a digital allpass transfer function are described. One approach is based on a realization in the form of a cascade of first- and second-order allpass filters. The second approach results in a cascaded lattice realization. The final realizations in both cases remain allpass independent of the actual values of the multiplier coefficients and are thus less sensitive to multiplier coefficient quantization. An elegant application of the first- and second-order minimum multiplier allpass structures, considered here, is in the implementation of some simple transfer functions with parametrically tunable properties.

The Gray-Markel method, to realize any arbitrary transfer function using the cascaded lattice form of allpass structure, is outlined. The realization of a large class of arbitrary Nth-order transfer functions using a parallel connection of two allpass filters is described. The final structure is shown to require only N multipliers. In Section 12.9.2, we demonstrate the low passband sensitivity property of these structures to small changes in the multiplier coefficients.

The cascaded lattice realization of an FIR transfer function is considered. The realization of a digital sine-cosine generator is then described, and various oscillator structures are systematically developed. The chapter concludes with a comparison of the computational complexities of FIR and IIR digital filter structures.

The digital filter realization methods outlined in this chapter assume the existence of the corresponding transfer function. Chapter 9 considers the development of such transfer functions meeting the given frequency response specifications. The analysis of the finite wordlength effects on the performance of the digital filter structures is treated in Chapter 12.

8.14 Problems

8.1 The digital filter structure of Figure P8.1 has a delay-free loop and is therefore unrealizable. Determine a realizable equivalent structure with identical input–output relations and without any delay-free loop. (*Hint:* Express the output variables $y[n]$ and $w[n]$ in terms of the input variables $x[n]$ and $u[n]$ only, and develop the corresponding block diagram representation from these input–output relations.)

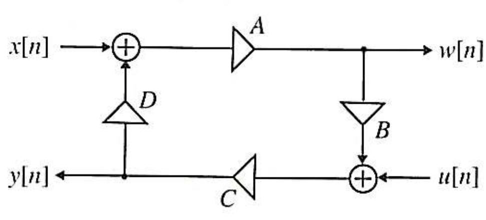


Figure P8.1

8.2 Determine by inspection whether or not the digital filter structures in Figure P8.2 have delay-free loops. Identify these loops if they exist. Develop equivalent structures without delay-free loops.

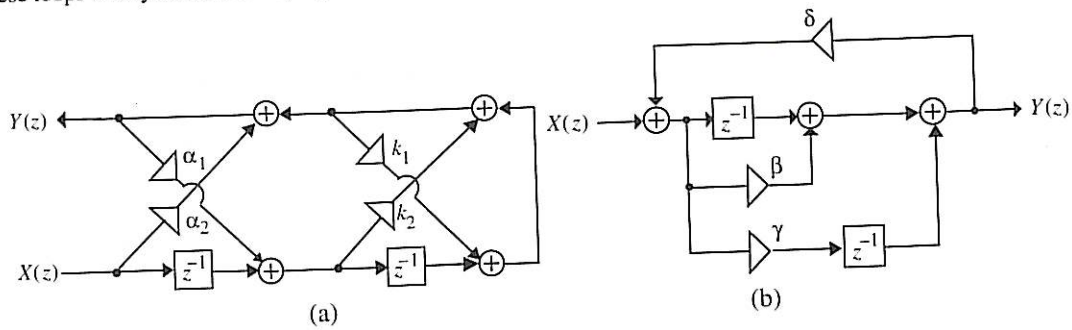


Figure P8.2

8.3 Figure P8.3 shows a typical closed-loop discrete-time feedback control system in which $G(z)$ is the plant and $C(z)$ is the compensator. If $G(z) = \frac{2}{1+3z^{-1}}$ and $C(z) = K$, determine the range of values of K for which the feedback structure is stable.

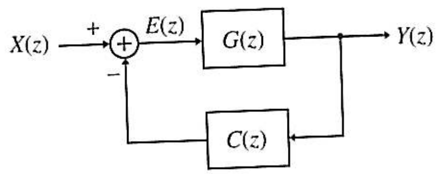


Figure P8.3

8.4 Figure P8.3 shows a typical closed-loop discrete-time feedback control system in which $G(z)$ is the plant and $C(z)$ is the compensator. If

$$G(z) = \frac{0.2(1+0.4z^{-1})}{1+0.9z^{-1}+0.6z^{-2}},$$

determine the transfer function $C(z)$ of the compensator so that the overall closed-loop transfer function of the feedback system is

$$H(z) = \frac{0.2z^{-1}+0.28z^{-2}+0.18z^{-3}+0.04z^{-4}}{1+0.5z^{-1}+0.74z^{-2}+0.18z^{-3}+0.28z^{-4}}.$$

Check the stability of $G(z)$, $C(z)$, and $H(z)$ using the M-file `zplane`.

8.5 Analyze the digital filter structure of Figure P8.4, and determine its transfer function $H(z) = Y(z)/X(z)$. (a) Is this a canonic structure? (b) What should be the value of the multiplier coefficient K so that $H(z)$ has a unity gain at $\omega = 0$? (c) What should be the value of the multiplier coefficient K so that $H(z)$ has a unity gain at $\omega = \pi$? (d) Is there a difference between these two values of K? If not, why not?

8.6 Analyze the digital filter structure of Figure P8.5, where all multiplier coefficients are real, and determine the transfer function $H(z) = Y(z)/X(z)$. What are the range of values of the multiplier coefficients for which the filter is BIBO stable?

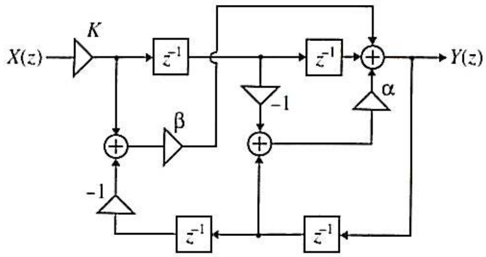


Figure P8.4

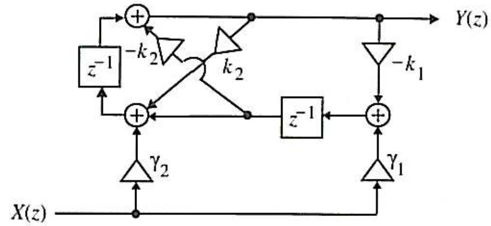


Figure P8.5

8.7 Determine the transfer function $H(z) = Y(z)/X(z)$ of the digital filter structure of Figure P8.6.

8.8 Determine the transfer function $H(z) = Y(z)/X(z)$ of the digital filter structure of Figure P8.7.

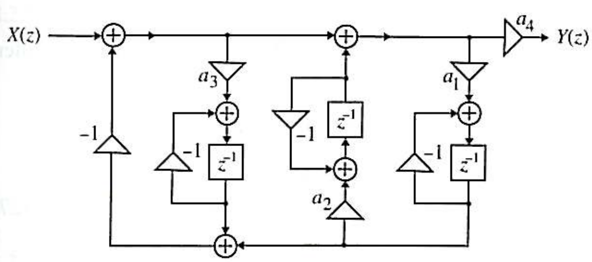


Figure P8.6

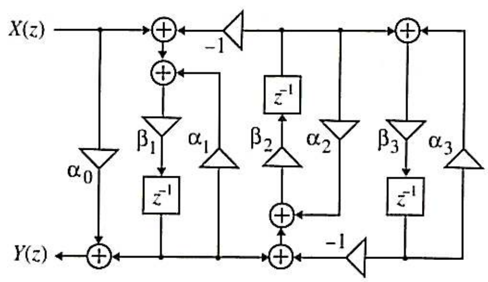


Figure P8.7

8.9 Determine the transfer function of the digital filter structure of Figure P8.9 [Kin72].

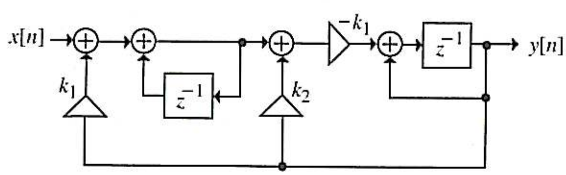


Figure P8.9

8.10 Realize the FIR transfer function

$$H(z) = (1 - 0.6z^{-1})^6 = 1 - 3.6z^{-1} + 5.4z^{-2} - 4.32z^{-3} + 1.944z^{-4} - 0.4666z^{-5} + 0.0467z^{-6}$$

in the following forms: (a) two different direct forms, (b) cascade of six first-order sections, (c) cascade of three second-order sections,(d) cascade of two third-order sections, and (e) cascade of two second-order sections and two first-order sections.

Compare the computational complexity of each of the above realizations.

8.11 Consider a length-10 FIR transfer function given by

$$H(z) = h[0] + h[1]z^{-1} + h[2]z^{-2} + h[3]z^{-3} + h[4]z^{-4}$$
$$+ h[5]z^{-5} + h[6]z^{-6} + h[7]z^{-7} + h[8]z^{-8} + h[9]z^{-9}.$$

(a) Develop a four-branch polyphase realization of $H(z)$ in the form of Figure 8.7(a), and determine the expressions for the polyphase transfer functions $E_0(z)$, $E_1(z)$, $E_2(z)$, and $E_3(z)$.

(b) From this realization, develop a canonic four-branch polyphase realization.

8.12 (a) Develop a three-branch polyphase realization of $H(z)$ of Problem 8.11 in the form of Figure 8.7(b), and determine the expressions for the polyphase transfer functions $E_0(z)$, $E_1(z)$, and $E_2(z)$.

(b) From this realization, develop a canonic three-branch polyphase realization of $H(z)$.

8.13 (a) Develop a two-branch polyphase realization of $H(z)$ of Problem 8.11 in the form of Figure 8.7(c), and determine the expressions for the polyphase transfer functions $E_0(z)$ and $E_1(z)$.

(b) From this realization, develop a canonic two-branch polyphase realization of $H(z)$.

8.14 Develop a minimum-multiplier realization of a length-7 Type 3 FIR transfer function.

8.15 Develop a minimum-multiplier realization of a length-8 Type 4 FIR transfer function.

8.16 Let $H(z)$ be a Type 1 linear-phase FIR filter of order N, with $G(z)$ denoting its delay-complementary filter. Develop a realization of both filters using only N delays and $(N+2)/2$ multipliers.

8.17 Show that a Type 1 linear-phase FIR transfer function $H(z)$ of length $2M+1$ can be expressed as

$$H(z) = z^{-M}\left[h[M] + \sum_{n=1}^{M} h[M-n]\left(z^{n} + z^{-n}\right)\right]. \tag{8.127}$$

By using the relation

$$z^{\ell} + z^{-\ell} = 2T_{\ell}\left(\frac{z+z^{-1}}{2}\right),$$

where $T_{\ell}(x)$ is the ℓth-order Chebyshev polynomial[10] in x, express $H(z)$ in the form

$$H(z) = z^{-M}\sum_{n=0}^{M} a[n]\left(\frac{z+z^{-1}}{2}\right)^{n}. \tag{8.128}$$

Determine the relation between $a[n]$ and $h[n]$. Develop a realization of $H(z)$ based on Eq. (8.128) in the form of Figure P8.10, where $F_1(z^{-1})$ and $F_2(z^{-1})$ are causal structures. Determine the form of $F_1(z^{-1})$ and $F_2(z^{-1})$. The structure of Figure P8.9 is called the *Taylor structure* for linear-phase FIR filters [Sch72].

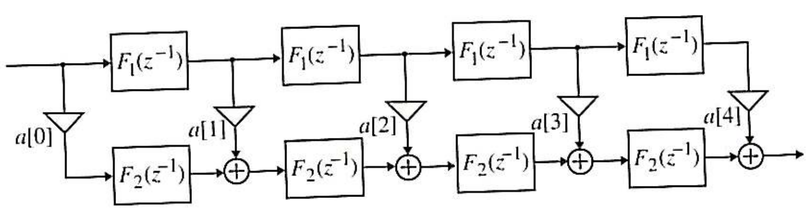


Figure P8.10 The Taylor structure shown for $M = 4$.

[10] For a definition of the Chebyshev polynomial and a recursive equation for generating such a polynomial, see Section 4.4.3.

8.18 Show that there are 36 distinct cascade realizations of the transfer function $H(z)$ of Eq. (8.27) obtained by different pole-zero pairings and different orderings of the individual sections.

8.19 Consider a real coefficient IIR transfer function $H(z)$ with its numerator and denominator expressed as a product of polynomials,

$$H(z) = \prod_{i=1}^{K} \frac{P_i(z)}{D_i(z)},$$

where $P_i(z)$ and $D_i(z)$ are either first-order or second-order polynomials with real coefficients. Determine the total number of distinct cascade realizations that can be obtained by different pole-zero pairings and different orderings of the individual sections.

8.20 Develop a canonic direct form realization of the transfer function

$$H(z) = \frac{2 - 5z^{-2} + 3z^{-3}}{1 - 4z^{-1} + 3z^{-2} + 6z^{-4}},$$

and then determine its transpose configuration.

8.21 Develop two different cascade canonic realizations of the following causal IIR transfer functions:

(a) $H_1(z) = \dfrac{(0.2z^{-1} + 0.6z^{-2})(3 - 2.4z^{-1})}{(2 - 3.2z^{-1} + 4.2z^{-2})(1 - 0.75z^{-1})}$,

(b) $H_2(z) = \dfrac{(1.7 + 6.2z)(z - 4.5)(3z^2 - 0.5)}{(2z - 0.3)(z + 0.2)(z^2 + 0.5z + 0.1)}$,

(c) $H_3(z) = \dfrac{(6 + 2.8z^{-1})(5.2 - 8.4z^{-1} + 7z^{-2})}{(4 + 12.4z^{-1})(1 - 2.4z^{-1} + 0.76z^{-2})}$.

8.22 Realize the transfer functions of Problem 8.21 in parallel forms I and II.

8.23 Develop a cascade realization of the transfer function of Eq. (8.26) using the factorization given in Eq. (8.29). Compare the computational complexity of this realization with the one shown in Figure 8.19(b).

8.24 Consider the cascade of three causal first-order LTI discrete-time systems shown in Figure P8.11, where

$$H_1(z) = \frac{2 - 0.3z^{-1}}{1 + 0.5z^{-1}}, \quad H_2(z) = \frac{0.4 + z^{-1}}{1 + 0.4z^{-1}}, \quad H_3(z) = \frac{3}{1 + 0.5z^{-1}}.$$

(a) Determine the transfer function of the overall system as a ratio of two polynomials in z^{-1}.
(b) Determine the difference equation characterizing the overall system.
(c) Develop the realization of the overall system with each section realized in direct form II.
(d) Develop a parallel form I realization of the overall system.
(e) Determine the impulse response of the overall system in closed form.

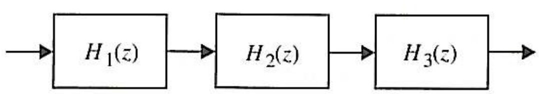


Figure P8.11

8.25 A causal LTI discrete-time system develops an output $y[n] = (0.4)^n \mu[n] - 0.3(0.4)^{n-1}\mu[n-1]$, for an input $x[n] = (0.2)^n \mu[n]$.

(a) Determine the transfer function of the system.
(b) Determine the difference equation characterizing the system.
(c) Develop a canonic direct form realization of the system with no more than three multipliers.
(d) Develop a parallel form I realization of the system.
(e) Determine the impulse response of the system in closed form.
(f) Determine the output $y[n]$ of the system for an input $x[n] = (0.3)^n \mu[n] - 0.4(0.3)^{n-1}\mu[n-1]$.

8.26 The structure shown in Figure P8.12 was developed in the course of a realization of the IIR digital transfer function

$$H(z) = \frac{2z^2 - 3.2z - 18.6}{z^2 - 2.6z - 1.2}.$$

However, by a mistake in the labeling, two of the multiplier coefficients in this structure have incorrect values. Find these two multipliers, and determine their correct values.

8.27 Figure P8.13 shows an incomplete realization of the causal IIR transfer function

$$H(z) = \frac{4z^2 - 5.6z}{z^2 + 0.2z - 0.08}.$$

Determine the values of the multiplier coefficients A and B.

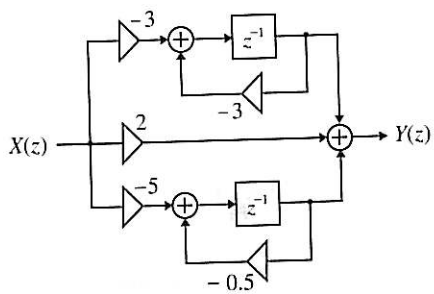


Figure P8.12

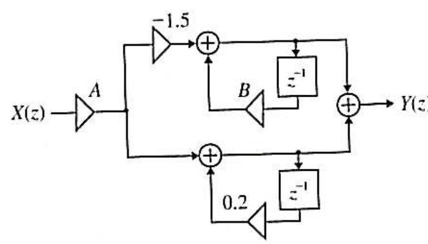


Figure P8.13

8.28 Develop a two-multiplier canonic realization of each of the following second-order transfer functions where the multiplier coefficients are α_1 and $-\alpha_2$, respectively [Hir73]:

$$\text{(a) } H_1(z) = \frac{(1-\alpha_1+\alpha_2)(1+z^{-1})^2}{1-\alpha_1 z^{-1}+\alpha_2 z^{-2}}, \quad \text{(b) } H_2(z) = \frac{(1-\alpha_2)(1-z^{-2})}{1-\alpha_1 z^{-1}+\alpha_2 z^{-2}}.$$

8.29 Develop a four-multiplier canonic realization with multiplier coefficients $\alpha_1, \alpha_2, \alpha_3$ and α_4 for each one of the following second-order transfer functions [Szc75a]:

$$\text{(a) } H_a(z) = \frac{1-\alpha_3 z^{-1}+(\alpha_2-\alpha_4)z^{-2}}{1-(\alpha_1+\alpha_3)z^{-1}-\alpha_4 z^{-2}}, \quad \text{(b) } H_b(z) = \frac{1-\alpha_3 z^{-1}+\alpha_2 z^{-2}}{1-(\alpha_1+\alpha_3)z^{-1}-\alpha_4 z^{-2}},$$

8.30 In this problem, we develop an alternative cascaded lattice realization of an Nth-order IIR transfer function [Mit77b]

$$H_N(z) = \frac{p_0 + p_1 z^{-1} + \cdots + p_{N-1}z^{-N+1} + p_N z^{-N}}{1 + d_1 z^{-1} + \cdots + d_{N-1}z^{-N+1} + d_N z^{-N}}. \qquad (8.129)$$

The first stage of the realization process is shown in Figure P8.14.

(a) Show that if the two-pair chain parameters are chosen as

$$A = 1, \qquad B = d_N z^{-1}, \qquad C = p_0, \qquad D = p_N z^{-1}, \tag{8.130}$$

then $H_{N-1}(z)$ is an $(N-1)$th-order IIR transfer function of the form

$$H_{N-1}(z) = \frac{p'_0 + p'_1 z^{-1} + \cdots + p'_{N-1} z^{-N+1}}{1 + d'_1 z^{-1} + \cdots + d'_{N-1} z^{-N+1}}, \tag{8.131}$$

with coefficients given by

$$p'_k = \frac{p_0 d_{k+1} - p_{k+1}}{p_0 d_N - p_N}, \quad k = 0, 1, \ldots, N-1, \tag{8.132a}$$

$$d'_k = \frac{p_k d_N - p_N d_k}{p_0 d_N - p_N}, \quad k = 1, 2, \ldots, N-1. \tag{8.132b}$$

(b) Develop a lattice realization of the two-pair.

(c) Continuing the above process, we can realize $H_N(z)$ as a cascade connection of N lattice sections constrained by a transfer function $H_0(z) = 1$. What are the total number of multipliers and two-input adders in the final realization of $H_N(z)$?

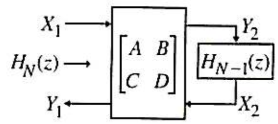


Figure P8.14

8.31 Realize the transfer functions of Problem 8.21 using the cascaded lattice realization method of Problem 8.30.

8.32 Show that the cascaded lattice realization method of Problem 8.30 results in the cascaded lattice structure described in Section 8.6.2 when $H_N(z)$ is an allpass transfer function.

8.33 Develop the structures of Types 1B, $1A_t$, and $1B_t$ first-order allpass transfer functions shown in Figure 8.24(b), (c), and (d), from Eqs. (8.50b), (8.50c), and (8.50d), respectively.

8.34 (a) Develop a cascade realization of the fourth-order allpass transfer function

$$A(z) = \left(\frac{a + z^{-1}}{1 + az^{-1}}\right)\left(\frac{b + z^{-1}}{1 + bz^{-1}}\right)\left(\frac{c + z^{-1}}{1 + cz^{-1}}\right)\left(\frac{d + z^{-1}}{1 + dz^{-1}}\right),$$

with each allpass section realized in Type 1A form. By sharing the delays between adjacent allpass sections, show that the total number of delays in the overall structure can be reduced from 8 to 6 [Mit74a].

(b) Repeat Part (a) with each allpass section realized in Type $1A_t$ form.

8.35 Analyze the digital filter structure of Figure P8.15, and show that it is a first-order allpass filter [Sto94].

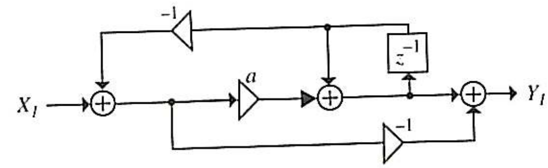


Figure P8.15

8.36 Develop formally the realizations of a second-order Type 2 allpass transfer function of Eq. (8.37), shown in Figure 8.25, using the multiplier extraction approach. Are there other Type 2 allpass structures?

8.37 Show that a cascade of two Type 2D second-order allpass structures can be realized with six delays by sharing the delays between adjacent sections. What is the minimum number of delays needed to implement a cascade of M Type 2D second-order allpass structures?

8.38 Develop formally the realizations of a second-order Type 3 allpass transfer function of Eq. (8.38), shown in Figure 8.26, using the multiplier extraction approach. Are there other Type 3 allpass structures?

8.39 Show that a cascade of two Type 3B second-order allpass structures can be realized with six delays by sharing the delays between adjacent sections. What is the minimum number of delays needed to implement a cascade of M Type 3H second-order allpass structures?

8.40 Develop a three-multiplier realization of the two-pair described by Eq. (8.50d).

8.41 Develop a lattice realization of the two-pair given by Eq. (8.50d). Determine the transfer function of an all-pole second-order cascaded lattice filter realized using this lattice structure and the transfer function of an all-pole second-order cascaded lattice filter realized using the lattice structure of Figure 8.30(a). Evaluate the approximate expressions for the gain of both second-order filters at resonance when the poles are close to the unit circle. Show that the gain of the first all-pole filter is approximately independent of the pole radius, whereas that of the second filter is not [Lar99].

8.42 Realize each of the following IIR transfer functions in the Gray-Markel form, and check the BIBO stability of each transfer function:

$$\text{(a)}\, H_1(z) = \frac{3.9 + 2.3z^{-1} + z^{-2}}{1 + 0.3z^{-1} + 0.5z^{-2}}, \qquad \text{(b)}\, H_2(z) = \frac{2.6 + 0.74z^{-1} + 3z^{-2}}{1 - 0.42z^{-1} + 0.4z^{-2}},$$

$$\text{(c)}\, H_3(z) = \frac{-3 + 5.192z^{-1} - 3.56z^{-2} + 2z^{-3}}{1 - 0.28z^{-1} + 0.056z^{-2} + 0.4z^{-3}}, \qquad \text{(d)}\, H_4(z) = \frac{1.6z^3 + 3.112z^2 + 0.84z + 2}{z^3 + 0.42z^2 + 0.056z - 0.3},$$

$$\text{(e)}\, H_5(z) = \frac{2z(z + 0.3)(z^2 - 0.5)}{(z + 0.5)(z - 0.4)(z^2 - 0.2z + 0.4)}.$$

8.43 Realize each of the IIR transfer functions of Problem 8.21 in the Gray-Markel form, and check their BIBO stability.

8.44 Realize the IIR transfer function

$$H(z) = \frac{0.2545(1 + z^{-1})(1 + 0.6985z^{-1} + z^{-2})}{(1 - 0.2169z^{-1})(1 + 0.0278z^{-1} + 0.7258z^{-2})}$$

in the following forms: (a) direct canonic form, (b) cascade form, (c) Gray-Markel form, and (d) cascaded lattice structure described in Problem 8.30. Compare their hardware requirements.

8.45 In this problem, the realization of an even-order real-coefficient transfer function using complex arithmetic is illustrated [Reg87a]. Let $G(z)$ be an Nth-order-real coefficient transfer function with simple poles in complex-conjugate pairs and with numerator degree less than or equal to that of the denominator.

(a) Show that $G(z)$ can be expressed as a sum of two complex coefficient transfer functions of order $N/2$,

$$G(z) = H(z) + H^*(z^*), \tag{8.133}$$

where the coefficients of $H^*(z^*)$ are complex conjugate of their corresponding coefficients of $H(z)$.

(b) Generalize the above decomposition to the case when $G(z)$ has one or more simple real poles.

(c) Consider a realization of $H(z)$ that has one real input $x[n]$ and a complex output $y[n]$. Show that the transfer function from the input to the real part of the output is simply $G(z)$. [*Hint:* Use a partial-fraction expansion to obtain the decomposition of Eq. (8.133).]

8.46 Develop a realization of a first-order complex coefficient transfer function $H(z)$ given by

$$H(z) = \frac{A + jB}{1 + (\alpha + j\beta)z^{-1}},$$

where A, B, α, and β are real constants. Show the real and imaginary parts of all signal variables separately. Determine the transfer functions from the input to the real and imaginary parts of the output.

8.47 Develop a cascaded lattice realization of an Nth-order complex coefficient allpass transfer function $A_N(z)$.

8.48 Realize the following transfer functions in the form of a parallel connection of two allpass filters:

(a) $H_1(z) = 4\left(\dfrac{1+z^{-1}}{5+3z^{-1}}\right)$, (b) $H_2(z) = \dfrac{5 - z^{-1}}{6 - 4z^{-1}}$,

(c) $$H_3(z) = \frac{0.5414(1 - z^{-1})(1 - 0.0757z^{-1} + z^{-2})}{(1 - 0.1768z^{-1})(1 - 0.004z^{-1} + 0.9061z^{-2})},$$

(d) $$H_4(z) = \frac{0.4547(1 + z^{-1})(1 - 0.2859z^{-1} + z^{-2})}{(1 + 0.0712z^{-1})(1 - 0.0377z^{-1} + 0.8482z^{-2})}.$$

8.49 Consider a causal length-6 FIR filter described by the convolution sum

$$y[n] = \sum_{k=0}^{5} h[k]x[n-k], \qquad n \geq 0,$$

where $y[n]$ and $x[n]$ denote, respectively, the output and the input sequences.

(a) Let the output and input sequences be blocked into length-2 vectors

$$\mathbf{Y}_\ell = \begin{bmatrix} y[2\ell] \\ y[2\ell+1] \end{bmatrix}, \quad \mathbf{X}_\ell = \begin{bmatrix} x[2\ell] \\ x[2\ell+1] \end{bmatrix}.$$

Show that the above FIR filter can be equivalently described by a block convolution sum given by

$$\mathbf{Y}_\ell = \sum_{r=0}^{3} \mathbf{H}_r \mathbf{X}_{\ell-r},$$

where $\mathbf{H}_r$ is a 2×2 matrix composed of the impulse response coefficients. Determine the block convolution matrices $\mathbf{H}_r$. An implementation of the FIR filter based on the above block convolution sum is shown in Figure P8.16, where the block labeled "S/P" is a serial-to-parallel converter and the block marked "P/S" is a parallel-to-serial converter.

(b) Develop the block convolution sum description of the above FIR filter for a block length of 3 for the input and the output.

(c) Develop the block convolution sum description of the above FIR filter for a block length of 4 for the input and the output.

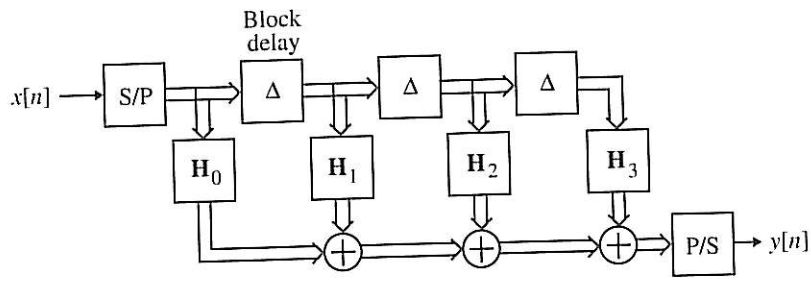


Figure P8.16

8.50 Consider a causal IIR filter described by a difference equation

$$\sum_{k=0}^{4} d_k y[n-k] = \sum_{k=0}^{4} p_k x[n-k], \qquad n \geq 0,$$

where $y[n]$ and $x[n]$ denote, respectively, the output and the input sequences.

(a) Let the output and input sequences be blocked into length-2 vectors

$$\mathbf{Y}_\ell = \begin{bmatrix} y[2\ell] \\ y[2\ell+1] \end{bmatrix}, \mathbf{X}_\ell = \begin{bmatrix} x[2\ell] \\ x[2\ell+1] \end{bmatrix}.$$

Show that the above IIR filter can be equivalently described by a block difference equation given by [Bur72]

$$\sum_{r=0}^{2} \mathbf{D}_r \mathbf{Y}_{\ell-r} = \sum_{r=0}^{2} \mathbf{P}_r \mathbf{X}_{\ell-r},$$

where $\mathbf{D}_r$ and $\mathbf{P}_r$ are 2×2 matrices composed of the difference equation coefficients $\{d_k\}$ and $\{p_k\}$, respectively. Determine the block difference equation matrices $\mathbf{D}_r$ and $\mathbf{P}_r$. An implementation of the IIR filter based on the above block difference equation is shown in Figure P8.17.

(b) Develop the block difference equation description of the above IIR filter for a block length of 3 for the input and the output.

(c) Develop the block difference equation description of the above IIR filter for a block length of 4 for the input and the output.

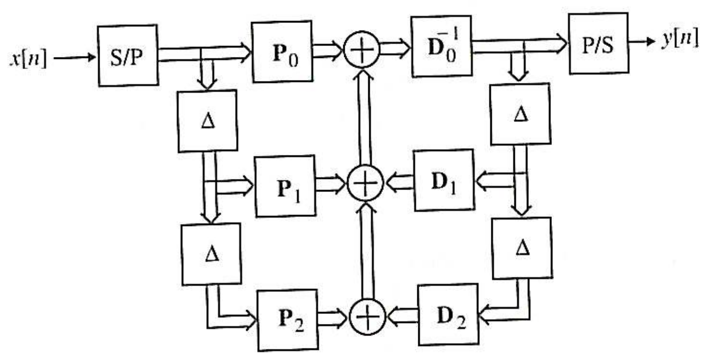


Figure P8.17

8.51 Develop a canonic realization of the block digital filter of Figure P8.17 employing only two block delays.

8.52 Develop the three-multiplier structure of a digital sine-cosine generator obtained by setting $\alpha = \pm\beta \sin \omega_o$ in Eq. (8.125).

8.53 Develop a one-multiplier structure of a digital sine-cosine generator obtained by setting $-\beta \sin \omega_o/\alpha = 1+\cos \omega_o$ in Eq. (8.125).

8.54 Develop a one-multiplier structure of a digital sine-cosine generator obtained by setting $C = -1$ in Eq. (8.124) and then choosing α and β properly. Show the final structure.

8.55 Signals generated by multiple sources or multiple sensors, called *multichannel signals*, are usually transmitted through independent channels in close proximity to each other. As a result, each component of the multichannel signal often gets corrupted by signals from adjacent channels during transmission, resulting in *cross talk*. Separation of the multichannel signal at the receiver is thus of practical interest. A model representing the cross talk between a pair of channels for a two-channel signal is depicted in Figure P8.18(a), and the corresponding discrete-time system for channel separation is as shown in Figure P8.18(b) [Yel96]. Determine two possible sets of conditions for perfect channel separation.

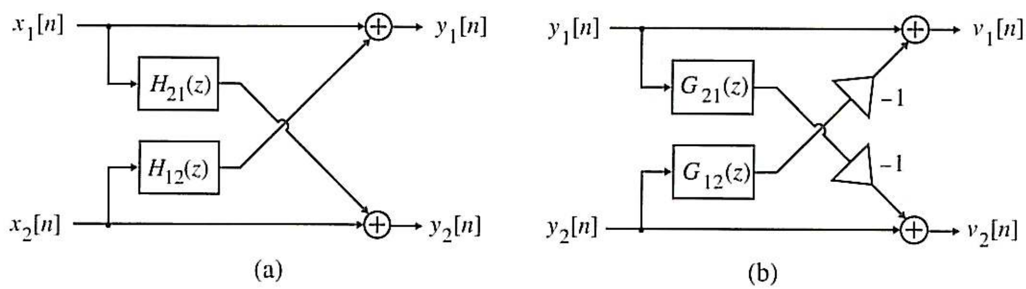


Figure P8.18

8.15 MATLAB Exercises

M 8.1 Using MATLAB, develop a cascade realization of each of the following linear-phase FIR transfer functions:
(a) $H_1(z) = -0.24 + 0.184z^{-1} + 0.4448z^{-2} + 1.296z^{-3} + 0.4448z^{-4} + 0.184z^{-5} - 0.24z^{-6}$,
(b) $H_2(z) = 4 - 13.6z^{-1} - 25.08z^{-2} + 77.2z^{-3} - 25.08z^{-4} - 13.6z^{-5} + 4z^{-6}$,
(c) $H_3(z) = -0.24 + 0.184z^{-1} + 0.4448z^{-2} - 0.4448z^{-4} - 0.184z^{-5} + 0.24z^{-6}$,
(d) $H_4(z) = 4 - 13.6z^{-1} - 25.08z^{-2} + 25.08z^{-4} + 13.6z^{-5} - 4z^{-6}$.

M 8.2 Consider the fourth-order IIR transfer function

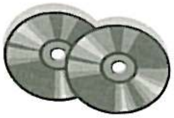
$$G(z) = \frac{0.3901 + 0.6426z^{-1} + 0.8721z^{-2} + 0.6426z^{-3} + 0.3901z^{-4}}{1 + 0.5038z^{-1} + 0.8923z^{-2} + 0.3844z^{-3} + 0.1569z^{-4}}.$$

(a) Using MATLAB, express $G(z)$ in factored form.
(b) Develop two different cascade realizations of $G(z)$.
(c) Develop two different parallel form realizations of $G(z)$.
Realize each second-order section in direct form II.

M 8.3 Consider the fourth-order IIR transfer function given below:

$$H(z) = \frac{0.3549 + 0.2002z^{-1} + 0.7031z^{-2} + 0.2002z^{-3} + 0.3549z^{-4}}{1 + 1.2522z^{-1} + 1.9448z^{-2} + 0.9774z^{-3} + 0.5595z^{-4}}.$$

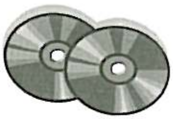
(a) Using MATLAB, express $H(z)$ in factored form.
(b) Develop two different cascade realizations of $H(z)$.
(c) Realize $H(z)$ in parallel forms I and II.
Realize each second-order section in direct form II.



M 8.4 Using Program 8_5, develop a Gray-Markel tapped cascaded lattice realization of the IIR transfer function $G(z)$ of Problem M8.2.

Program 8_5.m

M 8.5 Using Program 8_5, develop a Gray-Markel tapped cascaded lattice realization of the IIR transfer function $H(z)$ of Problem M8.3.



M 8.6 Using Program 8_7, develop a cascaded lattice realization of each of the FIR transfer functions of Problem M8.1.

Program 8_7.m

M 8.7 (a) Realize the following IIR lowpass transfer function $G(z)$ in the form of a parallel allpass structure:

$$G(z) = \frac{0.2801(1 - 0.6006z^{-1} + 1.0338z^{-2} + 1.0338z^{-3} - 0.6006z^{-4} + z^{-5})}{1 - 1.9607z^{-1} + 2.9395z^{-2} - 2.14486z^{-3} + 1.165z^{-4} - 0.1962z^{-5}}.$$

(b) From the allpass decomposition, determine its power-complementary transfer function $H(z)$.
(c) Plot the square of the magnitude responses of the original transfer function $G(z)$ and its power-complementary transfer function $H(z)$ derived in Part (b), and verify that their sum is equal to 1 at all frequencies.

M 8.8 (a) Realize the following IIR highpass transfer function $G(z)$ in the form of a parallel allpass structure:

$$G(z) = \frac{0.2876(1 + 0.1318z^{-1} + 1.1861z^{-2} - 1.1861z^{-3} - 0.1318z^{-4} - z^{-5})}{1 + 1.57274z^{-1} + 2.712z^{-2} + 1.9431z^{-3} + 1.2979z^{-4} + 0.3018z^{-5}}.$$

(b) From the allpass decomposition, determine its power-complementary transfer function $H(z)$.
(c) Plot the square of the magnitude responses of the original transfer function $G(z)$ and its power-complementary transfer function $H(z)$ derived in Part (b), and verify that their sum is equal to 1 at all frequencies.

M 8.9 (a) Realize the following IIR bandpass transfer function $G(z)$ in the form of a parallel allpass structure.

$$G(z) = \frac{0.3082(1 - 1.9622z^{-1} + 2.3876z^{-2} - 1.9622z^{-3} + z^{-4})}{1 - 2.0936z^{-1} + 2.5697z^{-2} - 1.7322z^{-3} + 0.7076z^{-4}}.$$

(b) From the allpass decomposition, determine its power-complementary transfer function $H(z)$.
(c) Plot the square of the magnitude responses of the original transfer function $G(z)$ and its power-complementary transfer function $H(z)$ derived in Part (b), and verify that their sum is equal to 1 at all frequencies.

M 8.10 Using MATLAB, simulate the single-multiplier sine-cosine generator of Problem 8.53 with $\cos\omega_o = 0.8$, and plot the first 50 samples of its two output sequences. Scale the outputs so that they both have a maximum amplitude of ± 1. What is the effect of initial values of the variables $s_i[n]$?

M 8.11 Using MATLAB, simulate the single-multiplier sine-cosine generator of Problem 8.54 with $\cos\omega_o = 0.8$, and plot the first 50 samples of its two output sequences. Scale the outputs so that they both have a maximum amplitude of ± 1. What is the effect of initial values of the variables $s_i[n]$ on the outputs?