for $1 \leq \ell \leq M$. The allpass delay equalizer design can be formulated as a minimax optimization problem in which we minimize the peak absolute value of the error

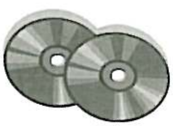
$$\mathcal{E}(\omega) = \tau_{\text{overall}}(\omega) - \tau_o, \tag{9.54}$$

in the passband of the filter. The adjustable parameters in the optimization procedure are the desired delay $\tau_o$ and the coefficients $d_{2,\ell}, d_{1,\ell}$ of the allpass transfer function [Cha80].

The M-file `iirgrpdelay` can be used to design the allpass delay equalizer which is availble with several versions. We illustrate its use in Example 9.17.

**EXAMPLE 9.17 Delay Equalizer Design Using MATLAB**

We design an allpass section of eighth order to equalize group delay in the passband of a fourth-order elliptic lowpass filter with a passband edge at $0.3\pi$, passband ripple of 1 dB and a minimum stopband attenuation of 30 dB. To this end, we make use of Program 9_4. The group delays of the lowpass filter and the overall cascade are shown in Figure 7.8. The numerator and the denominator coefficients of the allpass section are given in `num` and `den`. It can be shown using the statement `poly2rc(den)` that the designed allpass is a stable transfer function as all eight reflection coefficients are of magnitude less than 1.



Program 9_4.m

## 9.8 Summary

The digital filter design problem is concerned with the development of a suitable transfer function meeting the frequency response specifications, which, in this chapter, is restricted to magnitude (or, equivalently, gain) response specifications. These specifications are usually given in terms of the desired passband edge and stopband edge frequencies and the allowable deviations from the desired passband and stopband magnitude (gain) levels. This chapter considered the design of causal, stable infinite impulse response (IIR) digital filters.

IIR filter design is usually carried out by transforming a prototype analog transfer function by means of a suitable mapping of the complex frequency variable $s$ into the complex variable $z$. The widely used bilinear transform method, discussed in this chapter, is based on this approach.

The chapter then discusses some of the algorithms for the design of IIR digital filters that are available in the *Signal Processing Toolbox* of MATLAB as functions. In particular, it includes the design of IIR digital filters with Butterworth, Chebyshev, and elliptic magnitude responses.

Finally, the chapter reviews the basic idea behind the design of IIR digital filters using computer-aided iterative techniques and outlines a specific application of this approach to the design of group delay equalizers.

## 9.9 Problems

**9.1** Determine the peak ripple values $\delta_p$ and $\delta_s$ for each of the following sets of peak passband ripple $\alpha_p$ and minimum stopband attenuation $\alpha_s$:
(a) $\alpha_p = 0.21$ dB, $\alpha_s = 53$ dB, (b) $\alpha_p = 0.17$ dB, $\alpha_s = 78$ dB.

**9.2** Determine the peak passband ripple $\alpha_p$ and minimum stopband attenuation $\alpha_s$ in dB for each of the following sets of peak ripple values $\delta_p$ and $\delta_s$:
(a) $\delta_p = 0.02$, $\delta_s = 0.03$, (b) $\delta_p = 0.055$, $\delta_s = 0.033$.

**9.3** Let $H(z)$ be the transfer function of a lowpass digital filter with a passband edge at $\omega_p$, stopband edge at $\omega_s$, passband ripple of $\delta_p$, and stopband ripple of $\delta_s$, as indicated in Figure 9.1. Consider a cascade of two identical filters with a transfer function $H(z)$. What are the passband and stopband ripples of the cascade at $\omega_p$ and $\omega_s$, respectively? Generalize the results for a cascade of $M$ identical sections.

**9.4** Let $H_{LP}(z)$ denote the transfer function of a real-coefficient lowpass filter with a passband edge at $\omega_p$, stopband edge at $\omega_s$, passband ripple of $\delta_p$, and stopband ripple of $\delta_s$, as indicated in Figure 9.1. Sketch the magnitude response of the highpass transfer function $H_{LP}(-z)$ for $-\pi \le \omega < \pi$, and determine its passband and stopband edges in terms of $\omega_p$ and $\omega_s$.

**9.5** Consider the transfer function $G(z) = H_{LP}(e^{j\omega_o}z)$, where $H_{LP}(z)$ is the lowpass transfer function of Problem 9.4. Sketch its magnitude response for $-\pi \le \omega \le \pi$, and determine its passband and stopband edge frequencies in terms of $\omega_p$, $\omega_s$, and $\omega_o$.

**9.6** The impulse invariance method is another approach to the design of a causal IIR digital filter $G(z)$ based on the transformation of a prototype causal analog transfer function $H_a(s)$. If $h_a(t)$ is the impulse response of $H_a(s)$, in the impulse invariance method, we require that the unit sample response $g[n]$ of $G(z)$ be given by the sampled version of $h_a(t)$ sampled at uniform intervals of $T$ seconds; that is,

$$g[n] = h_a(nT), \qquad n = 0, 1, 2, \ldots.$$

(a) Show that $G(z)$ and $H_a(s)$ are related through

$$G(z) = \mathcal{Z}\{g[n]\} = \mathcal{Z}\{h_a(nT)\}$$
$$= \frac{1}{T}\sum_{k=-\infty}^{\infty} H_a\left(s + j\frac{2\pi k}{T}\right)\bigg|_{s=(1/T)\ln z}. \tag{9.55}$$

(b) Show that the transformation

$$s = \frac{1}{T}\ln z, \tag{9.56}$$

has the desirable properties enumerated in Section 9.1.3.

(c) Develop the condition under which the frequency response $G(e^{j\omega})$ of $G(z)$ will be a scaled replica of the frequency response $H_a(j\Omega)$ of $H_a(s)$.

(d) Show that the normalized digital angular frequency $\omega$ is related to the analog angular frequency $\Omega$ as

$$\omega = \Omega T. \tag{9.57}$$

**9.7** Show that the digital transfer function $G(z)$ obtained from an arbitrary rational analog transfer function $H_a(s)$ with simple poles via the impulse invariance method is given by

$$G(z) = \sum_{\substack{\text{all poles of}\\ H_a(s)}} \text{Residues}\left[\frac{H_a(s)}{1 - e^{sT}z^{-1}}\right]. \tag{9.58}$$

**9.8** Using Eq. (9.58), develop the expression for the causal digital transfer function $G(z)$ obtained from the causal analog transfer function $H(s) = A/(s+\alpha)$ via the impulse invariance method.

**9.9** Determine the digital transfer functions obtained by transforming the following causal analog transfer functions using the impulse invariance method. Assume $T = 0.3$ sec.

(a) $H_a(s) = \dfrac{4(3s+7)}{(s+2)(s^2+4s+5)}$, (b) $H_b(s) = \dfrac{8s^2+37s+56}{(s^2+2s+10)(s+4)}$, (c) $H_c(s) = \dfrac{s^3+s^2+6s+14}{(s^2+2s+5)(s^2+s+4)}$.

**9.10** The following causal IIR digital transfer functions were designed using the impulse invariance method with $T = 0.2$ sec. Determine their respective parent causal analog transfer functions.

$$\text{(a)}\ G_a(z) = \frac{3z}{z - e^{-1.5}} + \frac{4z}{z - e^{-1.8}}, \quad \text{(b)}\ G_b(z) = \frac{z\,e^{-1.2}\sin(1.5)}{z^2 - 2z\,e^{-1.2}\cos(1.5) + e^{-2.4}}.$$

**9.11** The following causal IIR digital transfer functions were designed using the bilinear transformation method with $T = 0.5$. Determine their respective parent causal analog transfer functions.

$$\text{(a)}\ G_a(z) = \frac{4(z^2 + 3z + 4)}{10z^2 + 4z + 6}, \quad \text{(b)}\ G_b(z) = \frac{54z^3 + 62z^2 + 26z + 18}{(3z + 1)(12z^2 - 4z + 8)}.$$

**9.12** An IIR digital lowpass filter is to be designed by transforming an analog lowpass filter with a passband edge frequency $F_p$ at 0.45 kHz using the impulse invariance method with $T = 0.3$ ms. What is the normalized passband edge angular frequency $\omega_p$ of the digital filter if there is no aliasing? What would be the normalized passband edge angular frequency $\omega_p$ of the digital filter if it is designed using the bilinear transformation with $T = 0.3$ ms?

**9.13** An IIR lowpass digital filter has a normalized passband edge frequency $\omega = 0.56\pi$. What is the passband edge frequency in Hz of the prototype analog lowpass filter if the digital filter has been designed using the impulse invariance method with $T = 0.2$ ms? What is the passband edge frequency in Hz of the prototype analog lowpass filter if the digital filter has been designed using the bilinear transformation method with $T = 0.2$ ms?

**9.14** Design an IIR lowpass digital filter $G(z)$ with a maximally flat magnitude response and meeting the specifications given by Eqs. (9.34a) and (9.34b) using the impulse invariance method. How does this filter compare with that designed via the bilinear transformation method in Section 9.2?

**9.15** An LTI continuous-time system described by a linear constant coefficient differential equation is often solved numerically by developing an equivalent linear constant coefficient difference equation by replacing the derivative operators in the differential equation by their approximate difference equation representation. A commonly used difference equation representation of the first derivative at time $t = nT$ is given by

$$\left.\frac{d\,y(t)}{dt}\right|_{t=nT} \cong \frac{1}{T}(y[n] - y[n-1]),$$

where $T$ is the sampling period and $y[n] = y(nT)$. The corresponding mapping from the $s$-domain to the $z$-domain is obtained by replacing $s$ with the *backward difference operator* $\frac{1}{T}(1 - z^{-1})$. Investigate the above mapping and its properties. Does a stable $H_a(s)$ result in a stable $H(z)$? How useful is this mapping for digital filter design?

**9.16** This problem illustrates how aliasing can be suitably exploited in order to realize interesting frequency response characteristics. An ideal causal analog lowpass filter with an impulse response $h_a(t)$ has a frequency response given by

$$H_a(j\Omega) = \begin{cases} 1, & |\Omega| < \Omega_c, \\ 0, & \text{otherwise.} \end{cases}$$

Let $H_1(e^{j\omega})$ and $H_2(e^{j\omega})$ be the frequency responses of digital filters obtained by sampling $h_a(t)$ at $t = nT$, where $T = 3\pi/2\Omega_c$ and $\pi/\Omega_c$, respectively. Assume the transfer functions are later normalized so that $H_1(e^{j0}) = H_2(e^{j0}) = 1$.

(a) Sketch the frequency responses $G_1(e^{j\omega})$ and $G_2(e^{j\omega})$ of the two digital filter structures shown in Figure P9.1.

(b) What type of filters are $G_1(z)$ and $G_2(z)$ (lowpass, highpass, etc.)?

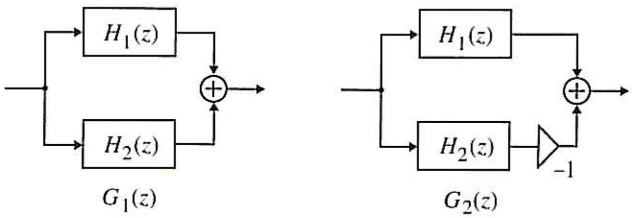



**Figure P9.1**

**9.17** Let $H_a(s)$ be a real-coefficient causal and stable analog transfer function with a magnitude response bounded above by unity. Show that the digital transfer function $G(z)$ obtained by a bilinear transformation of $H_a(s)$ is a BR function.

**9.18** We have shown in Section 8.7.2 that the transfer function $G(z)$ of a second-order IIR notch filter as given in Eq. (9.32) can be expressed in the form $G(z) = \frac{1}{2}[1 + A_2(z)]$, where $A_2(z)$ is a second-order allpass transfer function given by Eq. (8.66). Consider a notch filter with a notch frequency at $\omega = \pi/2$. Show that a notch filter with multiple notch frequencies is obtained if $z^{-1}$ is replaced with $z^{-N}$ [Reg88]. What are the locations of the new notch frequencies?

**9.19** A notch filter with $N$ notch frequencies can be realized by replacing the allpass filter $A_2(z)$ in Problem 9.18 with a cascade of $N$ second-order allpass filters [Jos99]. In this problem, we consider the design of a notch filter with two notch frequencies $\omega_1, \omega_2$, and corresponding 3-dB notch bandwidths $B_1, B_2$. We thus replace $A_2(z)$ with a fourth-order allpass transfer function $A_4(z)$,

$$A_4(z) = \left(\frac{\alpha_1 - \beta_1(1+\alpha_1)z^{-1} + z^{-2}}{1 - \beta_1(1+\alpha_1)z^{-1} + \alpha_1 z^{-2}}\right)\left(\frac{\alpha_2 - \beta_2(1+\alpha_2)z^{-1} + z^{-2}}{1 - \beta_2(1+\alpha_2)z^{-1} + \alpha_2 z^{-2}}\right),$$

obtained by cascading two second-order allpass filters. The constants $\alpha_1$ and $\alpha_2$ are chosen as

$$\alpha_i = \frac{1 - \tan(B_i/2)}{1 + \tan(B_i/2)}, \quad i = 1, 2.$$

The transfer function of the modified structure is now given by $H(z) = \frac{1}{2}[1 + A_4(z)] = N(z)/D(z)$.

(a) Show that $N(z)$ is a mirror-image polynomial of the form $a(1 + b_1 z^{-1} + b_2 z^{-2} + b_1 z^{-3} + z^{-4})$, and express the constants $b_1$ and $b_2$ in terms of the coefficients of $A_4(z)$.

(b) Show that $a = (1 + \alpha_1\alpha_2)/2$.

(c) By setting $N(e^{j\omega_i}) = 0$, $i = 1, 2$, solve for the constants $b_1$ and $b_2$ in terms of $\omega_1$ and $\omega_2$. From the equations in Parts (a) and (b), determine the expressions for the coefficients $\beta_1$ and $\beta_2$.

(d) Using the design equations derived above, design a double notch filter with the following specifications: $\omega_1 = 0.2\pi$, $\omega_2 = 0.6\pi$, $B_1 = 0.2\pi$, and $B_2 = 0.25\pi$. Using MATLAB, plot the magnitude response of the designed notch filter.

**9.20** Let $H_{LP}(z)$ be an IIR lowpass transfer function with a zero (pole) at $z = z_k$. Let $H_D(\hat{z})$ denote the lowpass transfer function obtained by applying the lowpass-to-lowpass transformation given in Table 9.1, which moves the zero (pole) at $z = z_k$ of $H_{LP}(z)$ to a new location at $\hat{z} = \hat{z}_k$. Express $\hat{z}_k$ in terms of $z_k$. If $H_{LP}(z)$ has a zero at $z = -1$, show that $H_D(\hat{z})$ also has a zero at $z = -1$.

**9.21** Let $H_{LP}(z)$ be an IIR lowpass transfer function with a zero (pole) at $z = z_k$. Let $H_D(\hat{z})$ denote the bandpass transfer function obtained by applying the lowpass-to-bandpass transformation given in Table 9.1, which moves the zero (pole) at $z = z_k$ of $H_{LP}(z)$ to a new location at $\hat{z} = \hat{z}_k$. Express $\hat{z}_k$ in terms of $z_k$. If $H_{LP}(z)$ has a zero at $z = -1$, show that $H_D(\hat{z})$ also has a zero at $z = \pm 1$.

**9.22** A second-order lowpass IIR digital filter with a 3-dB cutoff frequency at $\omega_c = 0.55\pi$ has a transfer function

$$G_{LP}(z) = \frac{0.3404(1 + z^{-1})^2}{1 + 0.1842z^{-1} + 0.1776z^{-2}}. \tag{9.59}$$

Design a second-order lowpass filter $H_{LP}(z)$ with a 3-dB cutoff frequency at $\hat{\omega}_c = 0.42\pi$ by transforming the above lowpass transfer function using a lowpass-to-lowpass spectral transformation. Using MATLAB, plot the gain responses of the two lowpass filters on the same figure.

**9.23** Design a second-order highpass filter $H_{HP}(z)$ with a 3-dB cutoff frequency at $\hat{\omega}_c = 0.47\pi$ by transforming the lowpass transfer function of Eq. (9.59) using a lowpass-to-highpass spectral transformation. Using MATLAB, plot the gain responses of the highpass and the lowpass filters on the same figure.

**9.24** A second-order lowpass Type 2 Chebyshev IIR digital filter $G_{LP}(z)$ with a minimum attenuation of 20-dB at $\omega_c = 0.56\pi$ has a transfer function

$$G_{LP}(z) = \frac{0.1944(1 + 0.9802z^{-1} + z^{-2})}{1 - 0.7016z^{-1} + 0.281z^{-2}}. \tag{9.60}$$

Design a fourth-order bandpass filter $H_{BP}(z)$ with a center frequency at $\hat{\omega}_c = 0.41\pi$ by transforming the above lowpass transfer function using a lowpass-to-bandpass spectral transformation. Using MATLAB, plot the gain responses of the lowpass and the bandpass filters on the same figure.

**9.25** A third-order elliptic highpass filter with a passband edge at $\omega_p = 0.52\pi$ has a transfer function

$$G_{HP}(z) = \frac{0.2397(1 - 1.5858z^{-1} + 1.5858z^{-2} - z^{-3})}{1 + 0.3272z^{-1} + 0.7459z^{-2} + 0.179z^{-3}}. \tag{9.61}$$

Design a highpass filter $H_{HP}(z)$ with a passband edge at $\omega_p = 0.45\pi$ by transforming the above highpass transfer function using the lowpass-to-lowpass spectral transformation. Using MATLAB, plot the gain responses of the two highpass filters on the same figure.

**9.26** Design a second-order bandpass filter with a center frequency at $\omega_o = 0.5\pi$ by transforming the bandpass transfer function of Eq. (7.79) using the lowpass-to-lowpass spectral transformation. Using MATLAB, plot the gain responses of the two bandpass filters on the same figure.

**9.27** Design a second-order notch filter operating at a sampling rate of 400 Hz with a notch frequency at 80 Hz and a 3-dB bandwidth of 5 Hz. By applying the lowpass-to-lowpass spectral transformation to this filter, design a notch filter with a notch frequency at 50 Hz. Using MATLAB, plot the gain responses of the two notch filters on the same figure.

**9.28** Design a lowpass filter with a cutoff at $\omega_p = 0.45\pi$ by transforming the highpass transfer function of Eq. (9.61) using the lowpass-to-highpass spectral transformation. Using MATLAB, plot the gain responses of the highpass and the lowpass filters on the same figure.

**9.29** A maximally flat group delay IIR allpass filter can be designed to approximate a fractional delay $z^{-D}$:

$$z^{-D} \cong \frac{d_N + d_{N-1}z^{-1} + \cdots + d_1 z^{-(N-1)} + z^{-N}}{1 + d_1 z^{-1} + d_2 z^{-2} + \cdots + d_{N-1} z^{-(N-1)} + d_N z^{-N}}.$$

By expressing the desired positive delay as $D = N + \delta$, where $N$ is a positive integer and $\delta$ a fractional number, it can be shown that the coefficient $\{d_k\}$ of the allpass filter is given by [Fet71]

$$d_k = (-1)^k C_k^N \prod_{n=0}^{N} \frac{D - N + n}{D - N + k + n},$$

where $C_k^N = N!/k!(N-k)!$ is a binomial coefficient. Design an allpass fractional delay filter of order 11 with a delay of 90/13 samples. Plot using MATLAB, the group delay response of the designed filter along with that of the ideal fractional delay filter. Comment on your results.

**9.30** The desired frequency response of an ideal *integrator* is given by

$$H_{\text{int}}(e^{j\omega}) = \frac{1}{j\omega}. \tag{9.62}$$

Determine the transfer function $H_R(z)$ of an IIR integrator derived via the rectangular numerical integration method given by Eq. (2.141) and the transfer function $H_T(z)$ of an IIR integrator derived via the trapezoidal numerical integration method given by Eq. (2.119). Using MATLAB, plot the magnitude responses of $H_{\text{int}}(z)$, $H_R(z)$, and $H_T(z)$ for $T = 1$. Comment on your results.

**9.31** An improved IIR digital integrator can be obtained by interpolating the rectangular and the trapezoidal integrators according to [Ala93]

$$H_N(z) = \tfrac{3}{4}H_R(z) + \tfrac{1}{4}H_T(z).$$

Using MATLAB, plot the magnitude responses of $H_N(z)$, $H_R(z)$, and $H_T(z)$ for $T = 1$. Comment on your results.

**9.32** Develop an IIR digital differentiator by inverting the IIR digital integrator of Problem 9.31 [Ala93]. Is this a stable transfer function? If not, develop a stable equivalent. Using MATLAB, plot the magnitude responses of the ideal differentiator and the digital differentiator designed here. Comment on your results.

## 9.10 MATLAB Exercises

**M 9.1** Design a digital Butterworth lowpass filter operating at a sampling rate of 100 kHz with a 0.4-dB cutoff frequency at 10 kHz and a minimum stopband attenuation of 50 dB at 30 kHz using the bilinear transformation method. Determine the order of the analog filter prototype using the formula given in Eq. (4.35), and then design the analog prototype filter using the M-file `buttap` of MATLAB. Transform the analog filter transfer function to the desired digital transfer function using the M-file `bilinear`. Plot the gain and phase responses using MATLAB. Show all steps used in the design.

**M 9.2** Modify Program 9_3 to design a digital Butterworth lowpass filter using the bilinear transformation method. The input data required by the modified program should be the desired passband and stopband edges and maximum passband deviation and the minimum stopband attenuation in dB. Using the modified program, design the digital Butterworth lowpass filter of Exercise M9.1.

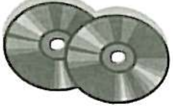
Program 9_3.m

**M 9.3** Design a digital filter by an impulse invariant transformation of a fifth-order analog Bessel transfer function for the following values of sampling frequencies: (a) $F_T = 1$ Hz and (b) $F_T = 2$ Hz. Plot the gain and the group delay responses of both designs using MATLAB, and compare these responses with that of the original Bessel transfer function. Comment on your results.

**M 9.4** Using the M-file `impinvar`, design the digital Butterworth lowpass filter of Exercise M9.1. Use the analog prototype filter order determined using the formula given in Eq. (4.35).

**M 9.5** Design a digital Type 1 Chebyshev lowpass filter operating at a sampling rate of 100 kHz with a passband edge frequency at 10 kHz, a passband ripple of 0.4 dB, and a minimum stopband attenuation of 50 dB at 30 kHz using the impulse invariance method and the bilinear transformation method. Determine the order of the analog filter prototype using the formula given in Eq. (4.43), and then design the analog prototype filter using the M-file `cheb1ap` of MATLAB. Transform the analog filter transfer function to the desired digital transfer function using the M-file `bilinear`. Plot the gain and phase responses of both designs using MATLAB. Compare the performances of the two filters. Show all steps used in the design.

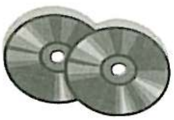
**M 9.6** Modify Program 9_2 to design a digital Type 1 Chebyshev lowpass filter using the bilinear transformation method. The input data required by the modified program should be the desired passband and stopband edges and maximum passband deviation and the minimum stopband attenuation in dB. Using the modified program, design the digital Type 1 Chebyshev lowpass filter of Exercise M9.5.



Program 9_2.m

**M 9.7** Using the M-file `impinvar`, write a MATLAB program to design a digital Type 1 Chebyshev lowpass filter using the impulse invariance method. The input data required by the modified program should be the desired passband and stopband edges and maximum passband deviation and the minimum stopband attenuation in dB. Using your program, design the digital Type 1 Chebyshev lowpass filter of Exercise M9.5.

**M 9.8** Design a digital elliptic lowpass filter operating at a sampling rate of 100 kHz with a passband edge frequency at 10 kHz, a stopband edge frequency at 30 kHz, passband ripple of 0.4 dB, and a stopband ripple of 50 dB using the impulse invariance method and the bilinear transformation method. Determine the order of the analog filter prototype using the formula given in Eq. (4.54), and then design the analog prototype filter using the M-file `ellipap` of MATLAB. Transform the analog filter transfer function to the desired digital transfer function using the M-file `bilinear`. Plot the gain and phase responses of both designs using MATLAB. Compare the performances of the two filters. Show all steps used in the design.



Program 9_3.m

**M 9.9** Modify Program 9_3 to design a digital elliptic lowpass filter using the bilinear transformation method. The input data required by the modified program should be the desired passband and stopband edges and the maximum passband deviation and the minimum stopband attenuation in dB. Using the modified program, design the digital elliptic lowpass filter of Exercise M9.8.

**M 9.10** Using the bilinear transformation method, design a digital Butterworth highpass filter operating at a sampling rate of 1.5 MHz with the following specifications: passband edge at 600 kHz, stopband edge at 210 kHz, peak passband ripple of 0.4 dB, and minimum stopband attenuation of 45 dB. (a) What are the specifications of the analog highpass filter? (b) What are the specifications of the analog prototype lowpass filter? (c) Show all pertinent transfer functions. Plot the gain responses of the prototype analog lowpass filter, analog highpass filter, and desired digital highpass filter. Show all steps.

**M 9.11** Using the bilinear transformation method, design a digital Type 1 Chebyshev bandpass filter operating at a sampling rate of 9 kHz with the following specifications: passband edges at 1.2 kHz and 2.2 kHz, stopband edges at 650 Hz and 3 kHz, peak passband ripple of 0.8 dB, and minimum stopband attenuation of 31 dB. (a) What are the specifications of the analog bandpass filter? (b) What are the specifications of the analog prototype lowpass filter? (c) Show all pertinent transfer functions. Plot the gain responses of the prototype analog lowpass filter, the analog bandpass filter, and desired digital bandpass filter. Show all steps.

**M 9.12** Using the bilinear transformation method, design a digital elliptic bandstop filter operating at a sampling rate of 8 kHz with the following specifications: passband edges at 0.9 kHz and 2.1 kHz, stopband edges at 0.6 kHz and 3 kHz, peak passband ripple of 1.5 dB, and minimum stopband attenuation of 30 dB. (a) What are the specifications of the analog bandstop filter? (b) What are the specifications of the analog prototype lowpass filter? (c) Show all pertinent transfer functions. Plot the gain responses of the prototype analog lowpass filter, the analog bandstop filter, and desired digital bandstop filter. Show all steps.

**M 9.13** Using the M-file `iirgrpdelay`, design an allpass section to equalize group delay in the passband of the Type 1 Chebyshev IIR highpass filter of Example 9.15.

**M 9.14** Using the M-file `iirgrpdelay`, design an allpass section to equalize group delay in the passband of the Butterworth IIR bandpass filter of Example 9.16.