EEG372 (Communication systems I)

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Assignment (4)

Question (1)

Classify the following signals as energy signals or power signals. Find the normalized energy or normalized power of each.

(a) $x(t) = A \cos 2\pi f_0 t \qquad \text{for } -\infty < t < \infty$

(b) $x(t) = \begin{cases} A \cos 2\pi f_0 t & \text{for } -T_0/2 \le t \le T_0/2, \text{ where } T_0 = 1/f_0 \\ 0 & \text{elsewhere} \end{cases}$

(c) $x(t) = \begin{cases} A \exp(-at) & \text{for } t > 0, a > 0 \\ 0 & \text{elsewhere} \end{cases}$

(d) $x(t) = \cos t + 5 \cos 2t \qquad \text{for } -\infty < t < \infty$

Question (2)

Determine which, if any, of the following functions have the properties of autocorrelation functions. Justify your determination. [*Note:* $\mathcal{F}\{R(\tau)\}$ must be a nonnegative function. Why?]

(a) $x(\tau) = \begin{cases} 1 & \text{for } -1 \le \tau \le 1 \\ 0 & \text{otherwise} \end{cases}$

(b) $x(\tau) = \delta(\tau) + \sin 2\pi f_0 \tau$

(c) $x(\tau) = \exp(|\tau|)$

(d) $x(\tau) = 1 - |\tau| \qquad \text{for } -1 \le \tau \le 1, 0 \text{ elsewhere}$

Question (3)

The power spectral density of a random process $X(t)$ is shown in Figure P1.12. It consists of a delta function at $f = 0$ and a triangular component.

(a) Determine and sketch the autocorrelation function $R_X(\tau)$ of $X(t)$.

(b) What is the DC power contained in $X(t)$?

(c) What is the AC power contained in $X(t)$?

(d) What sampling rates will give uncorrelated samples of $X(t)$? Are the samples statistically independent?

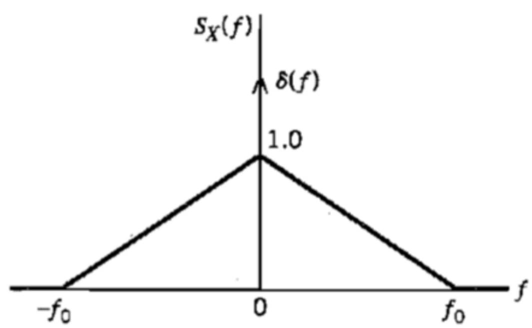
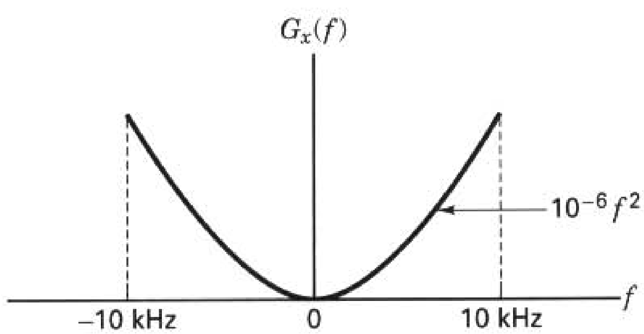


FIGURE P1.12

Question (4)

The two-sided power spectral density, $G_x(f) = 10^{-6} f^2$, of a waveform $x(t)$ is shown in



(a) Find the normalized average power in $x(t)$ over the frequency band from 0 to 10 kHz.

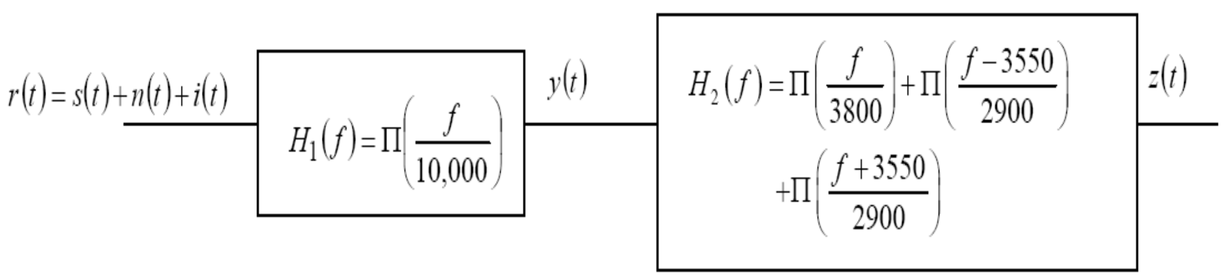
(b) Find the normalized average power contained in the frequency band from 5 to 6 kHz.

Question (5)

(1) A signal $s(t)$ having power spectral density $\Phi_s(f) = 5\Pi\left(\frac{f}{10{,}000}\right)$ is passed through a channel in which additive white Gaussian noise $n(t)$ and interference $i(t)$ are present, resulting in a received signal of $r(t) = s(t) + i(t)$. You may assume that the signal, noise and interference are all independent and that the noise has power spectral density $\Phi_n(f) = 0.1$, and the interference has power spectral density

$$\Phi_i(f) = 100\Lambda\left(\frac{f-2000}{100}\right) + 100\Lambda\left(\frac{f+2000}{100}\right).$$

The received signal is then filtered twice, first to bandlimit the signal and then to excise the interference, according to the following block diagram:



(a) Find the Signal to Interference and Noise Ratio (SINR) of $r(t)$

(b) Find the SINR of $y(t)$.

(c) Find the SINR of $z(t)$.

Question (5)

(1) A signal $s(t)$ having power spectral density $\Phi_s(f) = 5\Pi\left(\frac{f}{10{,}000}\right)$ is passed through a channel in which additive white Gaussian noise $n(t)$ and interference $i(t)$ are present, resulting in a received signal of $r(t) = s(t) + i(t)$. You may assume that the signal, noise and interference are all independent and that the noise has power spectral density $\Phi_n(f) = 0.1$, and the interference has power spectral density

$$\Phi_i(f) = 100\Delta\left(\frac{f-2000}{100}\right) + 100\Delta\left(\frac{f+2000}{100}\right).$$

The received signal is then filtered twice, first to bandlimit the signal and then to excise the interference, according to the following block diagram: