Question 1.

Explain what the terms centrifugal and centripetal mean with regard to a satellite in orbit around the earth.

A satellite is in a circular orbit around the earth. The altitude of the satellite's orbit above the surface of the earth is 1,400 km. (i) What are the centripetal and centrifugal accelerations acting on the satellite in its orbit? Give your answer in m/s^2. (ii) What is the velocity of the satellite in this orbit? Give your answer in km/s. (iii) What is the orbital period of the satellite in this orbit? Give your answer in hours, minutes, and seconds. Note: assume the average radius of the earth is 6,378.137 km and Kepler's constant has the value 3.986004418×10^5 km^3/s^2

Solution to question 1:

In the case of a satellite orbiting the earth, the centrifugal force on the satellite is a force on the satellite that is directly away from the center of gravity of the earth (F_{OUT} in Fig. 2.1) and the centripetal force is one directly towards the center of gravity of the earth (F in Fig. 2.1). The centrifugal force on a satellite will therefore try to fling the satellite away from the earth while the centripetal force will try to bring the satellite down towards the earth.

(i) From equation (2.1) centripetal acceleration $a = \mu/r^2$, where μ is Kepler's constant. The value of r = 6,378.137 + 1,400 = 7,778.137 km, thus $a = 3.986004418 \times 10^5 / (7{,}778.137)^2 = 0.0065885$ km/s^2 = 6.5885007 m/s^2. From equation (2.3), the centrifugal acceleration is given by $a = v^2/r$, where v = the velocity of the satellite in a circular orbit From equation (2.5) $v = (\mu/r)^{1/2} = (3.986004418 \times 10^5 / 7{,}778.137)^{1/2} = 7.1586494$ km/s and so a = 0.0065885007 km/s^2 = 6.5885007 m/s^2. **NOTE:** since the satellite was in stable orbit, the centrifugal acceleration must be equal to the centripetal acceleration, which we have found to be true here (but we needed only to calculate one of them).

(ii) We have already found out the velocity of the satellite in orbit in part (i) (using equation (2.5)) to be 7.1586494 km/s

(iii) From equation (2.6), the orbital period $T = (2\pi r^{3/2})/(\mu^{1/2}) = (2\pi 7{,}778.137^{3/2})/(3.986004418 \times 10^5)^{1/2}$ = (4,310,158.598)/(631.3481146) = 6,826.912916 s = 1 hour 53 minutes 46.92 seconds

Question 2

A satellite is in a 322 km high circular orbit. Determine:

a. The orbital angular velocity in radians per second;

b. The orbital period in minutes; and

c. The orbital velocity in meters per second.

Note: assume the average radius of the earth is 6,378.137 km and Kepler's constant has the value 3.986004418×10^5 km^3/s^2.

Solution to question 2:

It is actually easier to answer the three parts of this question backwards, beginning with the orbital velocity, then calculating the period, and hence the orbital angular velocity. First we will find the total radius of the orbit r = 322 + 6,378.137 km = 6700.137 km

(c) From eqn. (2.5), the orbital velocity $v = (\mu/r)^{1/2} = (3.986004418 \times 10^5 / 6700.137)^{1/2} =$ 7.713066 km/s = 7,713.066 m/s.

(b) From eqn. (2.6), $T = (2\pi r^{3/2})/(\mu^{1/2}) = (2\pi 6,700.137^{3/2})/(3.986004418 \times 10^5)^{1/2} =$ (3,445,921.604)/(631.3481146) = 5,458.037372 seconds = 90.9672895 minutes = 90.97 minutes.

(a) The orbital period from above is 5,458.037372 seconds. One revolution of the earth covers 360° or 2π radians. Hence 2π radians are covered in 5,458.037372 seconds, giving the orbital angular velocity as $2\pi/5{,}458.037372$ radians/s = 0.0011512 radians/s. An alternative calculation procedure would calculate the distance traveled in one orbit ($2\pi r = 2\pi 6700.137 = 42{,}098.20236$ km). This distance is equivalent to 2π radians and so 1 km is equivalent to $2\pi/42{,}098.20236$ radians = 0.0001493 radians. From above, the orbital velocity was 7.713066 km/s = $7.713066 \times 0.0001493$ radians/s = 0.0011512 radians/s.

Question 4

What are Kepler's three laws of planetary motion? Give the mathematical formulation of Kepler's third law of planetary motion. What do the terms perigee and apogee mean when used to describe the orbit of a satellite orbiting the earth?

A satellite in an elliptical orbit around the earth has an apogee of 39,152 km and a perigee of 500 km. What is the orbital period of this satellite? Give your answer in hours. Note: assume the average radius of the earth is 6,378.137 km and Kepler's constant has the value 3.986004418×10^5 km^3/s^2.

Solution to question 4

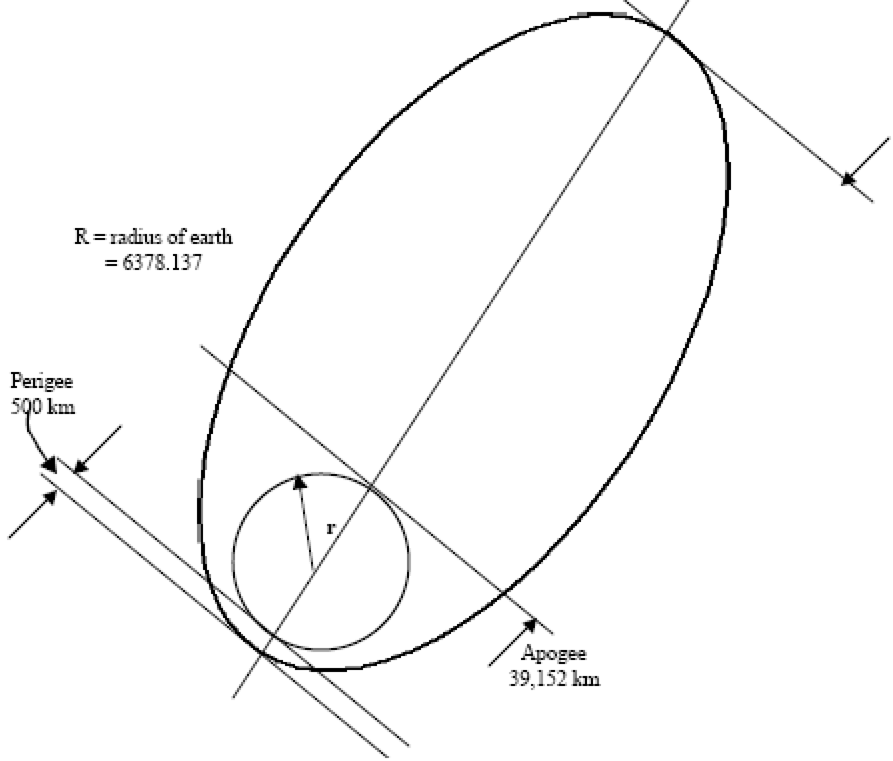
Kepler's three laws of planetary motion are (see page 22)

1. The orbit of any smaller body about a larger body is always an ellipse, with the center of mass of the larger body as one of the two foci.
2. The orbit of the smaller body sweeps out equal areas in time (see Fig. 2.5).
3. The square of the period of revolution of the smaller body about the larger body equals a constant multiplied by the third power of the semimajor axis of the orbital ellipse.

The mathematical formulation of the third law is $T^2 = (4\pi^2 a^3)/\mu$, where T is the orbital period, a is the semimajor axis of the orbital ellipse, and μ is Kepler's constant.

The perigee of a satellite is the closest distance in the orbit to the earth; the apogee of a satellite is the furthest distance in the orbit from the earth.

For the last part, draw a diagram to illustrate the geometry.



The semimajor axis of the ellipse = (39,152 + (2 × 6378.137) + 500)/2 = 26,204.137 km

The orbital period is

$$T^2 = (4\pi^2 a^3)/\mu = (4\pi^2(26{,}204.137)^3)/\ 3.986004418 \times 10^5 = 1{,}782{,}097{,}845.0$$

Therefore, T = 42,214.90075 seconds = 11 hours 43 minutes 34.9 seconds

Question 6

What is the difference, or are the differences, between a *geosynchronous* satellite and a *geostationary* satellite orbit? What is the period of a geostationary satellite? What is the name given to this orbital period? What is the velocity of a geostationary satellite in its orbit? Give your answer in km/s.

A particular shuttle mission released a TDRSS satellite into a circular low orbit, with an orbital height of 270 km. The shuttle orbit was inclined to the earth's equator by approximately 28°. The TDRSS satellite needed to be placed into a geostationary transfer orbit (GTO) once released from the shuttle cargo bay, with the apogee of the GTO at geostationary altitude and the perigee at the height of the shuttle's orbit. (i) What was the eccentricity of the GTO? (ii) What was the period of the GTO? (iii) What was the difference in velocity of the satellite in GTO between when it was at apogee and when it was at perigee? Note: assume the average radius of the earth is 6,378.137 km and Kepler's constant has the value 3.986004418×10^5 km^3/s^2.

Solution to question 6

A *geostationary* satellite orbit is one that has zero inclination to the equatorial plane, is perfectly circular (eccentricity is zero), and is at the correct orbital height to remain apparently stationary in orbit as viewed from the surface of the earth. A *geosynchronous* satellite orbit has most of the attributes of a geostationary orbit, but is either not exactly circular, not in the equatorial plane, or not at exactly the correct orbital height.

From Table 2.1, the orbital period of a geostationary satellite is 23 hours, 56 minutes, and 4.1 seconds.

The orbital period of a geostationary satellite is called a sidereal day.

From Table 2.1, the velocity of a geostationary satellite is 3.0747 km/s.

(i) The Geostationary Transfer Orbit (GTO) will have an apogee of 35,786.03 km (the geostationary altitude) and a perigee of 270 km (the release altitude of the TDRSS).

The semimajor axis $a = (2r_e + h_p + h_a)/2 = (2 \times 6378.137 + 270 + 35{,}786.03)/2 =$ 24,406.152 km

From equation (2.27) and example 2.1.3, $r_o = r_e + h_p$ and the eccentric anomaly $E = 0$ when the satellite is at perigee. From equation (2.27) $r_o = a(1 - e\cos E)$, with $\cos E = 1$. Therefore, $r_e + h_p = a(1 - e)$ and, rearranging the equation, $e = 1 - (r_e + h_p)/a = 1 -$ $(6378.137 + 270)/24{,}406.152 = 0.727604$. The eccentricity of the GTO is 0.728.

(ii) The orbital period $T = ((4\pi^2 a^3)/\mu)^{1/2} = ((4\pi^2 \times 24{,}406.152^3)/\ 3.986004418 \times 10^5)^{1/2} =$ 37,945.47102 seconds = 10 hours 32 minutes 25.47 seconds.

Exercises

2.1 Explain what the terms centrifugal and centripetal mean with regard to a satellite in orbit around the earth.

A satellite is in a circular orbit around the earth. The altitude of the satellite's orbit above the surface of the earth is 1500 km.

a. What are the centripetal and centrifugal accelerations acting on the satellite in its orbit? Give your answer in m/s.

b. What is the velocity of the satellite in this orbit? Give your answer in km/s.

c. What is the orbital period of the satellite in this orbit? Give your answer in hours, minutes, and seconds.

Note: Assume the average radius of the earth is 6378.137 km and Kepler's constant has the value $3.986\,004\,418 \times 10^5\ \text{km}^3/\text{s}^2$.

2.2 A satellite is in a circular orbit at an altitude of 350 km. Determine

a. The orbital angular velocity in radians per second.

b. The orbital period in minutes.

c. The orbital velocity in meters per second.

Note: Assume the average radius of the earth is 6378.137 km and Kepler's constant has the value $3.986\,004\,418 \times 10^5\ \text{km}^3/\text{s}^2$.

2.3 A LEO satellite is in a circular equatorial orbit with an altitude of 1000 km. What is the orbital period in hours, minutes, and seconds to the nearest 1/100 second?

2.4 An observer is located on the equator at longitude 0°. How long is the LEO satellite described in Question 3 visible to this observer, assuming that the observer can see down to the horizon at zero degrees elevation? Give your answer in minutes and seconds to the nearest second.

Hint. This is a problem in geometry. Calculate the angle at the center of the earth that defines visibility. Then find the relative angular velocity of the satellite, assuming a prograde orbit (satellite travels in same direction as earth's rotation). Visibility time is central angle times angular velocity.

2.5 A LEO satellite has an apogee altitude of 5000 km and a perigee altitude of 800 km.

What is the eccentricity of the orbit?

2.6 What are Kepler's three laws of planetary motion? Give the mathematical formulation of Kepler's third law of planetary motion. What do the terms perigee and apogee mean when used to describe the orbit of a satellite orbiting the earth?

A satellite in an elliptical orbit around the earth has an apogee of 39 152 km and a perigee of 500 km. What is the orbital period of this satellite? Give your answer in hours, minutes, and seconds. Note: Assume the average radius of the earth is 6378.137 km and Kepler's constant has the value $3.986\,004\,418 \times 10^5\ \text{km}^3/\text{s}^2$.

2.7 An observation satellite is to be placed into a circular equatorial orbit so that it moves in the same direction as the earth's rotation. Using a synthetic aperture radar system, the satellite stores data on weather related parameters as it flies overhead. These data will be downloaded to a controlling earth station after each trip around the world.

The orbit is designed so that the satellite is directly above the controlling earth station, which is located on the equator, once every four hours. The controlling earth station's antenna is unable to operate below an elevation angle of 10° to the horizontal in any direction. Taking the earth's rotational period to be exactly 23 hours 56 minutes 4.09 seconds, find the following quantities:

a. The satellite's angular velocity in radians per second.
b. The orbital period in hours, minutes, and seconds.
c. The orbital radius in kilometers.
d. The orbital height in kilometers.
e. The satellite's linear velocity in meters per second.
f. The time interval in minutes and seconds for which the controlling earth station can communicate with the satellite on each pass.

2.11 A GEO satellite is located at longitude 109° west. The satellite broadcasts television programming to the continental United States.

a. Calculate the look angles for an earth station located near Blacksburg, Virginia, latitude 37.22°N, longitude 80.42°W.

b. Calculate the look angles for an earth station located near Billings, Montana, latitude 46.00°N, longitude 109.0°W.

c. Calculate the look angles for an earth station located near Los Angles, California, longitude 118.0°W, latitude 34.00°N.

2.12 A GEO satellite is located at longitude 343° (17° west), over the Atlantic Ocean. Communication is established through this satellite between two earth stations. One earth station is near Washington, D.C., at latitude 38.9°N, longitude 77.2°W. The other station is near Cape Town South Africa, at latitude 34.0°S, longitude 19.0°E.

a. Calculate the look angles for each earth station.

Don't forget that the earth station in Africa looks north, and that it has a longitude in degrees east. The Washington, D.C. station has a longitude in degrees west and looks south. The numerical values of the earth station longitudes must be added when finding the separation of the stations in longitude.

b. Calculate the delay, in milliseconds, for a signal to travel from one earth station to the other via the GEO satellite.

2.13 A link is established through a GEO satellite at longitude 30°W between an earth station near Rio de Janeiro, Brazil, latitude 22.91°S, longitude 43.17°W, and an earth station near Santiago, Chile, latitude 33.45°S, longitude 70.67°W. Calculate the look angles for each earth station.