EENG 470
Satellite Communications

Lecture # 3 – P2
Orbit control and Launching Methods

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Lecture Content

- **2.4 Describing the Orbit of a Satellite**
- **2.5 Locating the Satellite in the Orbit**

2.4 Describing the Orbit of a Satellite

The quantity θ_0 in Eq. (2.15) serves to orient the ellipse with respect to the orbital plane axes x_0 and y_0. Now that we know that the orbit is an ellipse, we can always choose x_0 and y_0 so that θ_0 is zero. We will assume that this has been done for the rest of this discussion. This now gives the equation of the orbit as

$$r_0 = \frac{p}{1 + e\cos\phi_0} \tag{2.17}$$

The path of the satellite in the orbital plane is shown in Figure 2.6. The lengths a and b of the semimajor and semiminor axes are given by

$$a = p/(1 - e^2) \tag{2.18}$$

$$b = a(1 - e^2)^{1/2} \tag{2.19}$$

The point in the orbit where the satellite is closest to the earth is called the perigee and the point where the satellite is farthest from the earth is called the apogee. The perigee and apogee are **always** exactly opposite each other. To make θ_0 equal to zero, we have

Semi-Axis Lengths of the Orbit

$$a = \frac{p}{1 - e^2}$$ where $$p = \frac{h^2}{\mu}$$

See eq. (2.18) and (2.16)

and ***h*** is the magnitude of the angular momentum

$$b = a\left(1 - e^2\right)^{1/2}$$ where $$e = \frac{h^2 C}{\mu}$$

See eqn. (2.19)

and ***e*** is the eccentricity of the orbit

- If a = semi-major axis,
  b = semi-minor axis, and
  e = eccentricity of the orbit ellipse,

  then

$$e = \frac{a - b}{a + b}$$

NOTE: For a circular orbit, $\boldsymbol{a} = \boldsymbol{b}$ and $\boldsymbol{e} = 0$

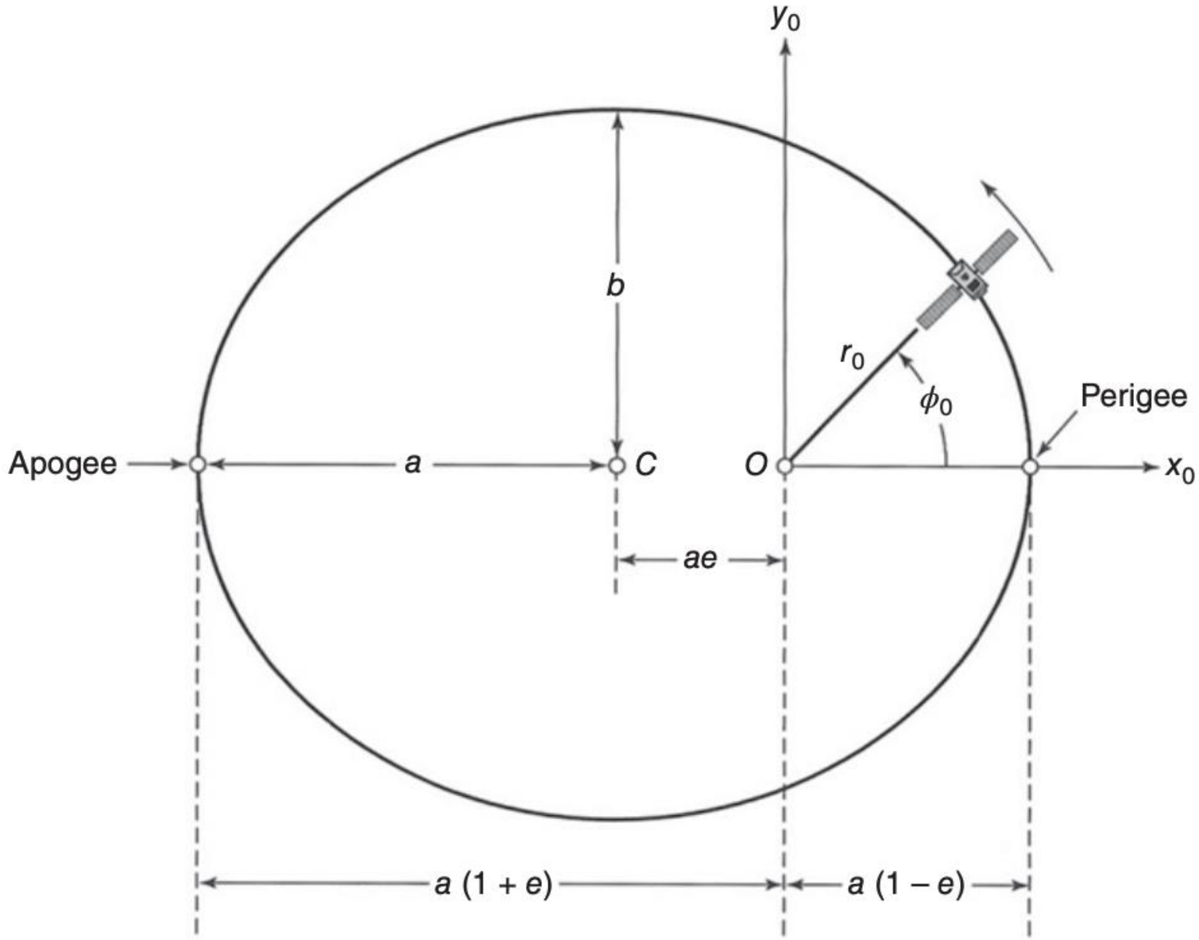


Figure 2.6 The orbit as it appears in the orbital plane. The point O is the center of the earth and the point C is the center of the ellipse. The two centers do not coincide unless the eccentricity, e, of the ellipse is zero (i.e., the ellipse becomes a circle and $a = b$). The dimensions of a and b are the semimajor and semiminor axes of the orbital ellipse, respectively.

chosen the x_0 axis so that both the apogee and the perigee lie along it and the x_0 axis is therefore the major axis of the ellipse.

The differential area swept out by the vector r_0 from the origin to the satellite in time dt is given by

$$dA = 0.5r_0^2 \left(\frac{d\phi_0}{dt} \right) dt = 0.5hdt \tag{2.20}$$

Remembering that h is the magnitude of the orbital angular momentum of the satellite, the radius vector of the satellite can be seen to sweep out equal areas in equal times. This is Kepler's second law of planetary motion. By equating the area of the ellipse (πab) to the area swept out in one orbital revolution, we can derive an expression for the orbital period T as

$$\boxed{T^2 = (4\pi^2 a^3)/\mu} \tag{2.21}$$

This equation is the mathematical expression of Kepler's third law of planetary motion: the square of the period of revolution is proportional to the cube of the semimajor axis. (Note that this is the square of Eq. (2.6) and that in Eq. (2.6) the orbit was assumed to be circular such that semimajor axis a = semiminor axis b = circular orbit radius from the center of the earth r.) Kepler's third law extends the result from Eq. (2.6), which was derived for a circular orbit, to the more general case of an elliptical orbit. Equation (2.21) is extremely important in satellite communications systems. This equation determines the period of the orbit of any satellite, and it is used in every GPS

$$T^2 = (4\pi^2 a^3)/\mu$$

$$T = (2\pi r^{3/2})/(\mu^{1/2})$$

receiver in the calculation of the positions of GPS satellites. Equation (2.21) is also used to find the orbital radius of a GEO satellite, for which the period T must be made exactly equal to the period of one revolution of the earth for the satellite to remain stationary over a point on the equator.

An important point to remember is that the period of revolution, T, is referenced to inertial space, that is, to the galactic background. The orbital period is the time the orbiting body takes to return to the same reference point in space with respect to the galactic background. Nearly always, the primary body will also be rotating and so the period of revolution of the satellite may be different from that perceived by an observer who is standing still on the surface of the primary body. This is most obvious with a GEO satellite (see Table 2.1). The orbital period of a GEO satellite is exactly equal to the period of rotation of the earth, 23 hours 56 minutes 4.1 seconds, but, to an observer on the ground, the satellite appears to have an infinite orbital period: it always stays in the same place in the sky.

geostationary vs. *geosynchronous* orbit.

To be perfectly geostationary, the orbit of a satellite needs to have three features: (a) it must be exactly circular (i.e., have an eccentricity of zero); (b) it must be at the correct altitude (i.e., have the correct period); and (c) it must be in the plane of the equator (i.e., have a zero inclination with respect to the equator). If the inclination of the satellite is not zero and/or if the eccentricity is not zero, but the orbital period is correct, then the satellite will be in a *geosynchronous* orbit. The position of a geosynchronous satellite will appear to oscillate about a mean look angle in the sky with respect to a stationary observer on the earth's surface.

Locating the Satellite in the Orbit

Consider now the problem of locating the satellite in its orbit. The equation of the orbit may be rewritten by combining Eqs. (2.15) and (2.18) to obtain

$$\boxed{r_0 = \frac{a(1 - e^2)}{1 + e \cos \phi_0}} \tag{2.22}$$

The angle ϕ_0 (see Figure 2.6) is measured from the x_0 axis and is called the *true anomaly*. [*Anomaly* was a measure used by astronomers to mean a planet's angular distance from its perihelion (closest approach to the sun), measured as if viewed from the sun. The term was adopted in celestial mechanics for all orbiting bodies.] Since we defined the positive x_0 axis so that it passes through the perigee, ϕ_0 measures the angle from the perigee to the instantaneous position of the satellite. The rectangular coordinates of the satellite are given by

$$x_0 = r_0 \cos \phi_0 \tag{2.23}$$

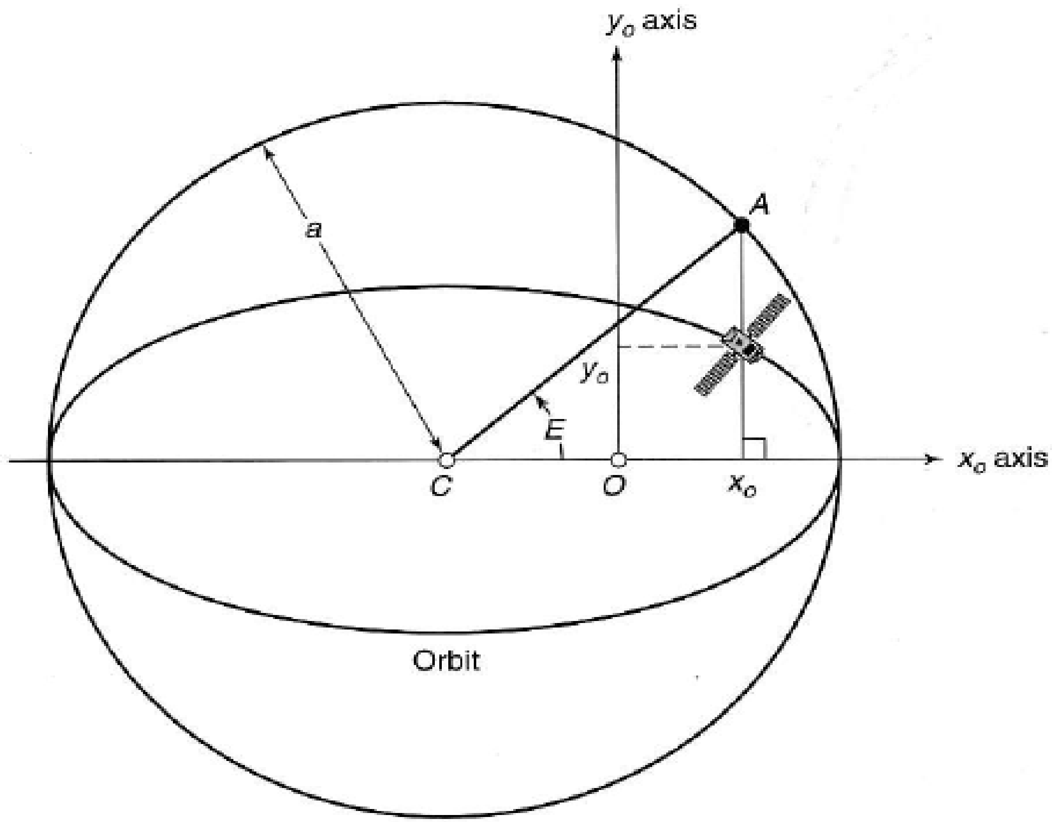
$$y_0 = r_0 \sin \phi_0 \tag{2.24}$$

As noted earlier, the orbital period T is the time for the satellite to complete a revolution in inertial space, traveling a total of 2π radians. The average angular velocity η is thus

$$\boxed{\eta = (2\pi)/T = (\mu^{1/2})/(a^{3/2})} \tag{2.25}$$

If the orbit is an ellipse, the instantaneous angular velocity will vary with the position of the satellite around the orbit. If we enclose the elliptical orbit with a *circumscribed circle* of radius a (see Figure 2.7), then an object going around the circumscribed circle with a constant angular velocity η would complete one revolution in exactly the same period T as the satellite requires to complete one (elliptical) orbital revolution.

Consider the geometry of the circumscribed circle as shown in Figure 2.7. Locate the point (indicated as A) where a vertical line drawn through the position of the satellite intersects the circumscribed circle. A line from the center of the ellipse (C) to this point (A) makes an angle E with the x_0 axis; E is called the *eccentric anomaly* of the satellite.



Circumscribed Circle

FIGURE 2.7 The circumscribed circle and the eccentric anomaly E. Point O is the center of the earth and point C is both the center of the orbital ellipse and the center of the circumscribed circle. The satellite location in the orbital plane coordinate system is specified by (x_0, y_0). A vertical line through the satellite intersects the circumscribed circle at point A. The eccentric anomaly E is the angle from the x_0 axis to the line joining C and A.

It is related to the radius r_0 by

$$r_0 = a(1 - e\cos E) \tag{2.26}$$

Thus

$$a - r_0 = ae\cos E \tag{2.27}$$

We can also develop an expression that relates eccentric anomaly E to the average angular velocity η, which yields

$$\eta\, dt = (1 - e\cos E)dE \tag{2.28}$$

Let t_p be the *time of perigee*. This is simultaneously the time of closest approach to the earth; the time when the satellite is crossing the x_0 axis; and the time when E is zero. If we integrate both sides of Eq. (2.28), we obtain

$$\eta(t - t_p) = E - e\sin E \tag{2.29}$$

The left side of Eq. (2.29) is called the *mean anomaly*, M. Thus

$$M = \eta(t - t_p) = E - e\sin E \tag{2.30}$$

The mean anomaly M is the arc length (in radians) that the satellite would have traversed since the perigee passage if it were moving on the circumscribed circle at the mean angular velocity η.

If we know the time of perigee, t_p, the eccentricity, e, and the length of the semi-major axis, a, we now have the necessary equations to determine the coordinates (r_0, ϕ_0)

locating the satellite at the point (x_0, y_0, z_0)

and (x_0, y_0) of the satellite in the orbital plane. The process is as follows

1. Calculate η using Eq. (2.25). $\eta = (2\pi)/T = (\mu^{1/2})/(a^{3/2})$
2. Calculate M using Eq. (2.30). $M = \eta(t - t_p) = E - e \sin E$
3. Solve Eq. (2.30) for E.
4. Find r_0 from E using Eq. (2.27). $a - r_0 = ae \cos E$
5. Solve Eq. (2.22) for ϕ_0. $r_0 = \dfrac{a(1 - e^2)}{1 + e \cos \phi_0}$
6. Use Eqs. (2.23) and (2.24) to calculate x_0 and y_0.

$$x_0 = r_0 \cos \phi_0$$
$$y_0 = r_0 \sin \phi_0$$

Now we must locate the orbital plane with respect to the earth.

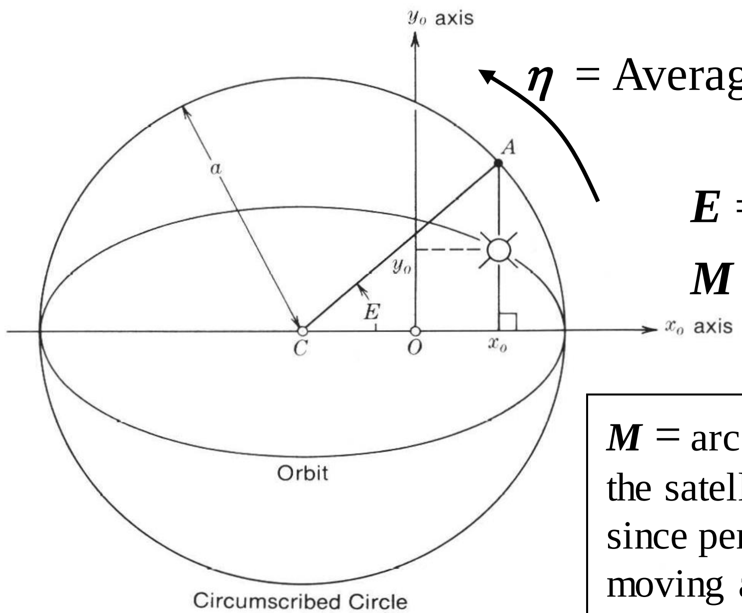
LOCATING THE SATELLITE IN ORBIT SUMMARY

LOCATING THE SATELLITE IN ORBIT: 1

- Need to develop a procedure that will allow the average angular velocity to be used
- If the orbit is not circular, the procedure is to use a ***Circumscribed Circle***
- A circumscribed circle is a circle that has a radius equal to the semi-major axis length of the ellipse and also has the same center

LOCATING THE SATELLITE IN ORBIT: 2

Fig. 2.7 in the text



η = Average angular velocity

$\boldsymbol{E}$ = Eccentric Anomaly

$\boldsymbol{M}$ = Mean Anomaly

$\boldsymbol{M}$ = arc length (in radians) that the satellite would have traversed since perigee passage if it were moving around the circumscribed circle with a mean angular velocity η

ORBIT DETERMINATION 3:
Procedure:

Given the time of perigee t_p, the eccentricity e and the length of the semimajor axis a:

- $\boldsymbol{\eta}$ Average Angular Velocity (eqn. 2.25)
- **M** Mean Anomaly (eqn. 2.30)
- **E** Eccentric Anomaly (solve eqn. 2.30)
- $\mathbf{r_o}$ Radius from orbit center (eqn. 2.27)
- $\boldsymbol{\varphi_o}$ True Anomaly (solve eq. 2.22)
- $\mathbf{x_0}$ and $\mathbf{y_0}$ (using eqn. 2.23 and 2.24)