EEENG 470
Satellite Communications

Lecture # 3 - P3
Orbit control and Launching Methods

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Lecture Content

- **2.6 Locating the Satellite With Respect to the Earth**
- **2.7 Orbital Elements**
- **2.8 Look Angle Determination (Next Lecture)**

2.6 Locating the Satellite With Respect to the Earth

The process for locating satellite in rectangular coordinate system of orbital plane:

- Locate satellite at point (x0, y0, z0) with respect to center of Earth.
- Need to locate satellite from observation point not at center of Earth.
- Development of transformations to locate satellite from point on rotating surface of Earth.
- Begin with geocentric equatorial coordinate system (Fig 2.8)
- Rotational axis of Earth is zi axis, through geographic North Pole.
- xi axis from center of Earth toward fixed location in space called first point of Aries.
- Coordinate system translates as Earth moves in orbit around sun but does not rotate with Earth.
- xi direction remains constant regardless of Earth's position around sun, in direction of first point of Aries.
- (xi, yi) plane contains Earth's equator, known as equatorial plane.

• Angular distance measured eastward in equatorial plane from xi axis is right ascension (RA), symbolized as RA.

•Points where orbit intersects equatorial plane are nodes:

- @Ascending node: Satellite moves upward through equatorial plane.
- @Descending node: Satellite moves downward through equatorial plane.

Figure 2.9:

•Right ascension of ascending node is Ω.

•Inclination (i): Angle between orbital plane and equatorial plane.

•ω (omega): Argument of perigee west. This is the angle measured along orbit from ascending node to perigee.

•Variables Ω and i together locate orbital plane with respect to equatorial plane.

•To locate orbital coordinate system with respect to equatorial coordinate system, ω is needed.

Locating the Satellite with Respect to the Earth

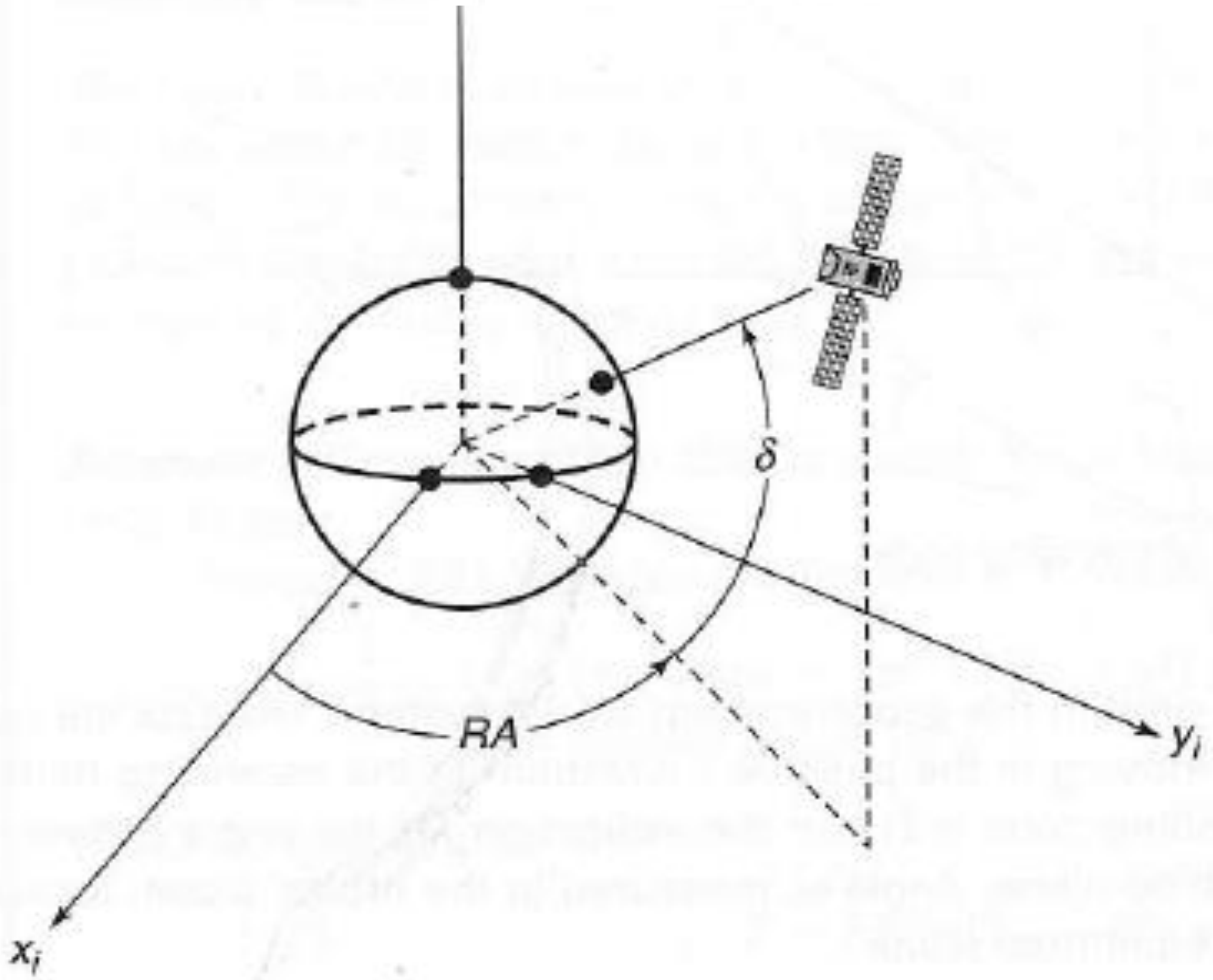


FIGURE 2.8 The geocentric equatorial system. This geocentric system differs from that shown in Figure 2.1 only in that the x_i axis points to the first point of Aries. The first point of Aries is the direction of a line from the center of the earth through the center of the sun at the vernal equinox (about March 21 in the Northern Hemisphere), the instant when the subsolar point crosses the equator from south to north. In the above system, an object may be located by its right ascension *RA* and its declination δ.

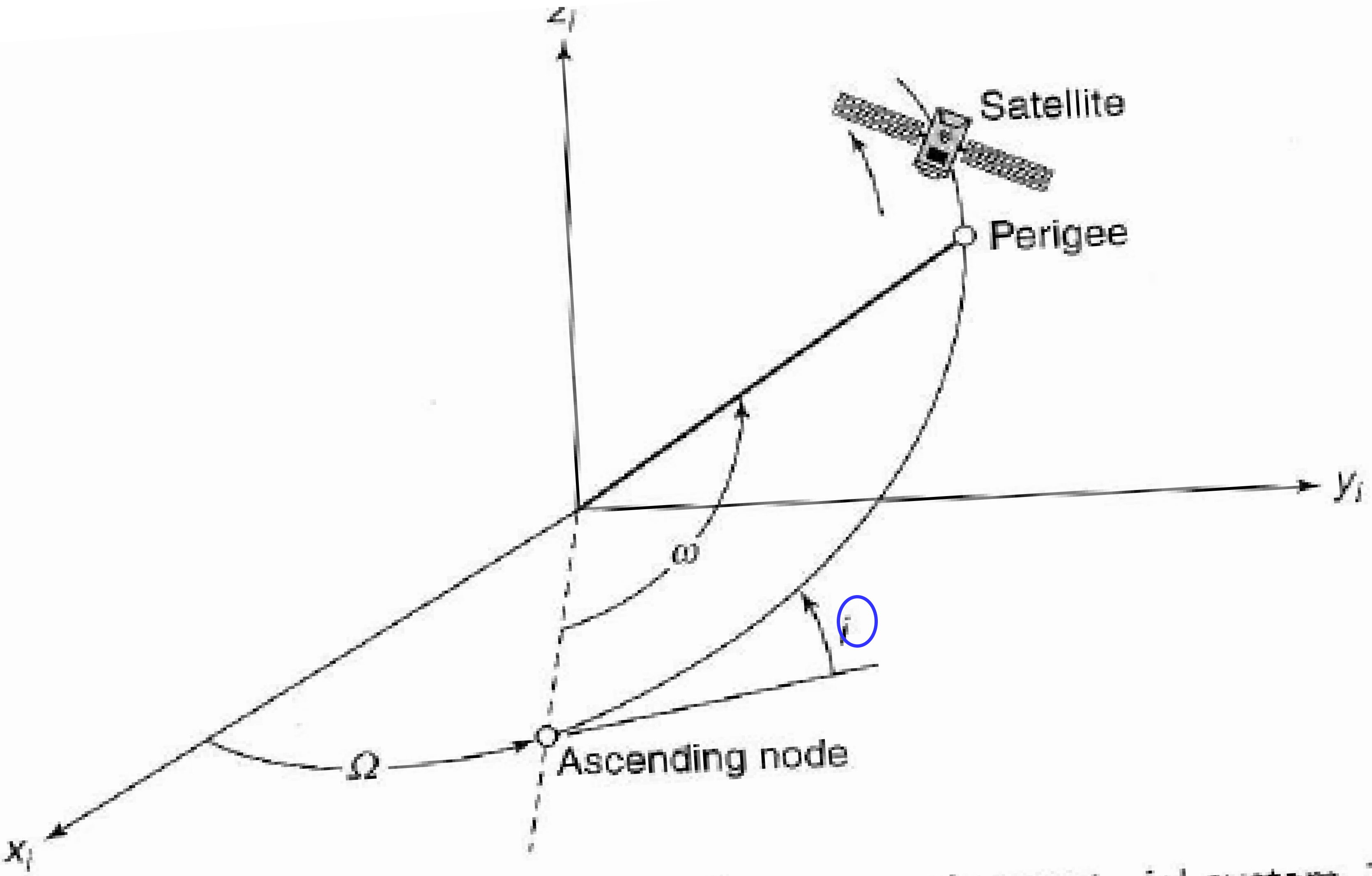


FIGURE 2.9 Locating the orbit in the geocentric equatorial system. The satellite penetrates the equatorial plane (while moving in the positive z direction) at the ascending node. The right ascension of the ascending node is Ω and the inclination i is the angle between the equatorial plane and the orbital plane. Angle ω, measured in the orbital plane, locates the perigee with respect to the equatorial plane.

2.7 Orbital Elements

Orbital elements are necessary to specify the absolute coordinates of a satellite at a particular time. Six quantities constitute the orbital elements, as determined by the satellite's equation of motion:

1. Eccentricity (e)
2. Semimajor axis (a)
3. Time of perigee (tp)
4. Right ascension of ascending node (Ω)
5. Inclination (i)
6. Argument of perigee (ω)

More than six quantities can describe a unique orbital path, with some flexibility in their selection. Commonly used orbital elements in satellite communications are eccentricity, semimajor axis, time of perigee, right ascension of ascending node, inclination, and argument of perigee. Mean anomaly (M) at a given time can sometimes substitute for time of perigee (tp).

Example 2.3 Geostationary Satellite Orbit (GEO) Radius

Question: The earth rotates once per sidereal day (23 hours 56 minutes 4.09 seconds). Use Eq. (2.21) to show that the radius of the GEO is 42 164.17 km as given in Table 2.1.

Answer

Equation (2.21) enables us to find the period of a satellite's orbit given the radius of the orbit. Namely

$$T^2 = (4\pi^2 a^3)/\mu \text{ seconds}$$

Rearranging the equation, the orbital radius a is given by

$$a^3 = T^2\mu/(4\pi^2)$$

For one sidereal day, $T = 86\,164.09$ seconds. Hence

$$a^3 = (86164.1)^2 \times 3.986004418 \times 10^5/(4\pi^2) = 7.496020251 \times 10^{13} \text{ km}^3$$

Thus $a = 42\,164.17$ km

This is the orbital radius for a geostationary satellite, as given in Table 2.1.

Example 2.4 Low Earth Orbit

Question: A SpaceX mission to the ISS is an example of a LEO satellite mission. Before rendezvousing with the ISS on this mission, SpaceX inserted the *Dragon* capsule into an initial circular orbit 250 km above the earth's surface, where there are still a finite number of molecules from the atmosphere. The mean earth's radius, r_e, is approximately 6378.14 km. Using these numbers, calculate the period of the Dragon capsule of SpaceX in its 250 km orbit. Find also the linear velocity of the Dragon capsule along this orbit.

Answer

The radius from the center of the earth of the 250 km altitude Dragon orbit is $(r_e + h)$, where h is the orbital altitude, and this = 6378.14 + 250.0 = 6628.14 km.

From Eq. 2.21, the period of the orbit is T where

$$T^2 = (4\pi^2 a^3)/\mu = 4\pi^2 \times (6628.14)^3/3.986004418 \times 10^5 \text{ s}^2$$
$$= 2.88401145 \times 10^7 \text{ s}^2$$

Hence the period of the orbit is

$$T = 5370.30 \text{ seconds} = 89 \text{ minutes } 30.3 \text{ seconds}$$

This orbit period is about as small as possible. At a lower altitude, friction with the earth's atmosphere will quickly slow the Dragon capsule down and it will return to earth. Thus, all spacecraft in stable earth orbit tend to have orbital periods exceeding 89 minutes 30 seconds.

The circumference of the orbit is $2\pi a = 41\,645.83$ km.

Hence the velocity of the Dragon in orbit is

$$2\pi a/T = 41645.83/5370.30 = 7.755 \text{ km/s}$$

Alternatively, you could use Eq. (2.5): $v = (\mu/r)^{1/2}$.

The term $\mu = 3.986\,004\,418 \times 10^5$ km^3/s^2 and the term $r = (6378.14 + 250.0)$ km, yielding $v = 7.755$ km/s.

Note: If μ and r had been quoted in units of m^3/s^2 and m, respectively, the answer would have been in meters/second. Be sure to keep the units the same during a calculation procedure. A velocity of about 7.8 km/s is a typical velocity for a LEO satellite. As the altitude of a satellite increases, its velocity becomes smaller.

Example 2.5 Elliptical Orbit

Question: A satellite is in an elliptical orbit with a perigee of 1000 km and an apogee of 4000 km. Using a mean earth radius of 6378.14 km, find the period of the orbit in hours, minutes, and seconds, and the eccentricity of the orbit.

Answer

The major axis of the elliptical orbit is a straight line between the apogee and perigee, as seen in Figure 2.6. Hence, for a semimajor axis length a, earth radius r_e, perigee height h_p, and apogee height h_a,

$$2a = 2r_e + h_p + h_a = 2 \times 6378.14 + 1000.0 + 4000.0 = 17\,756.28 \text{ km}$$

Thus the semimajor axis of the orbit has a length $a = 8878.14$ km. Using this value of a in Eq. (2.21) gives an orbital period T seconds where

$$\begin{aligned} T^2 &= (4\pi^2 a^3)/\mu = 4\pi^2 \times (8878.07)^3/3.986004418 \times 10^5 \text{ s}^2 \\ &= 6.930872802 \times 10^7 \text{ s}^2 \end{aligned}$$

which gives

$$\begin{aligned} T &= 8\,325.186\,4 \text{ seconds} = 138 \text{ minutes } 45.19 \text{ seconds} \\ &= 2 \text{ hours } 18 \text{ minutes } 45.19 \text{ seconds} \end{aligned}$$

The eccentricity of the orbit is given by e, which can be found from Eq. 2.27 by considering the instant at which the satellite is at perigee. Referring to Figure 2.7, when the satellite is at perigee, the eccentric anomaly $E = 0$ and $r_0 = r_e + h_p$. From Eq. (2.27), at perigee

$$r_0 = a(1 - e\cos E) \text{ and } \cos E = 1$$

Hence

$$r_e + h_p = a(1 - e)$$

which gives

$$e = 1 - (r_e + h_p)/a = 1 - 7378.14/8878.14 = 0.169.$$

- Book Lecture notes

ORBIT DETERMINATION 2:

- ***Orbital Constants*** allow you to determine coordinates ***(r_o, φ_o)*** and ***(x_o, y_o)*** in the orbital plane

- Now need to locate the orbital plane with respect to the earth

- More specifically: need to locate the orbital location with respect to a **point on the surface of the earth**

LOCATING THE SATELLITE WITH RESPECT TO THE EARTH

- The orbital constants define the orbit of the satellite with respect to the **CENTER** of the earth

- To know where to look for the satellite in space, we must relate the orbital plane and time of perigee to the earth's axis

NOTE: Need a ***Time Reference*** to locate the satellite. The time reference most often used is the ***Time of Perigee,*** t_p

Time reference:

- $\mathbf{t_p}$ Time of Perigee = Time of closest approach to the earth, at the same time, time the satellite is crossing the x_0 axis, according to the reference used.

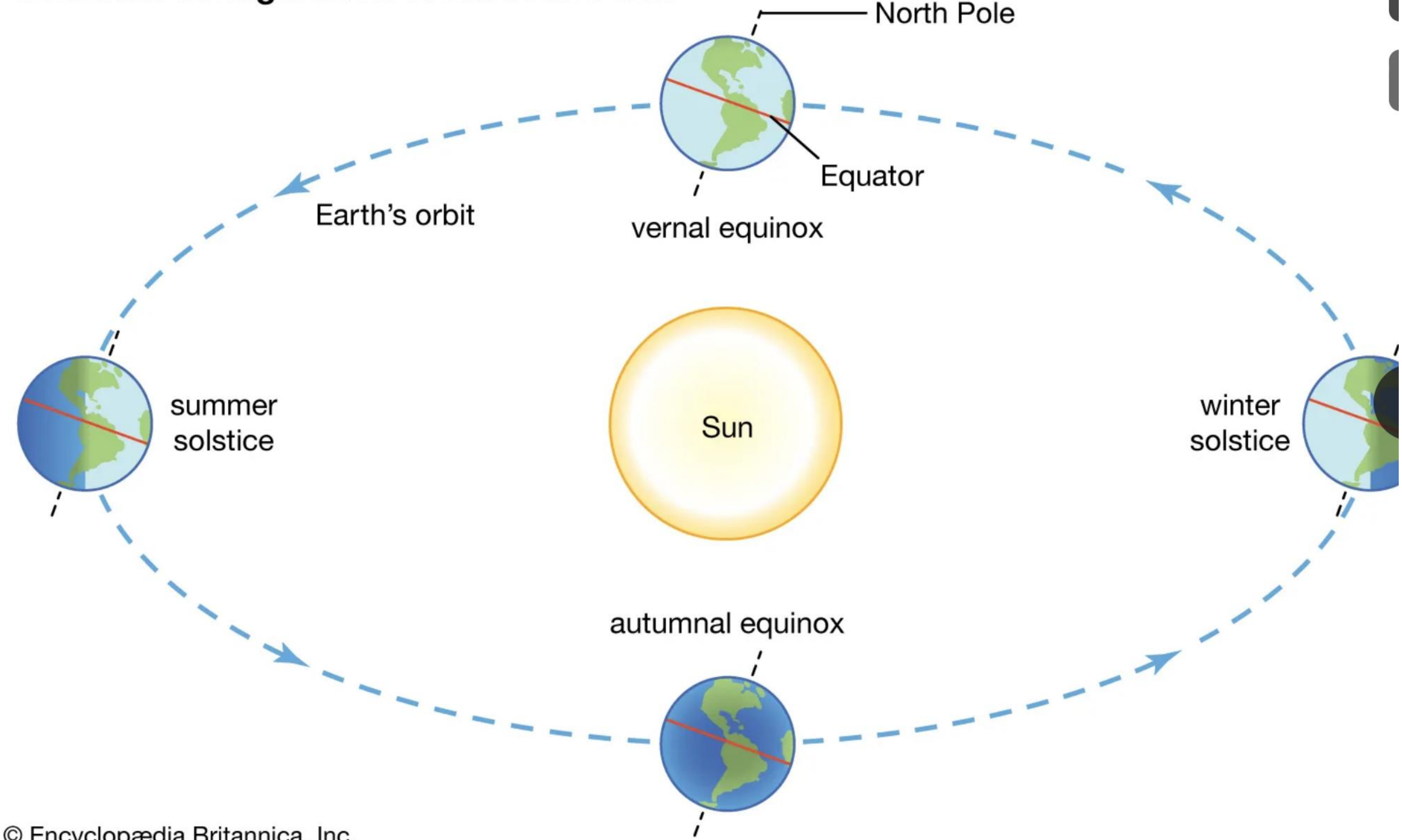
- t- $\mathbf{t_p}$ = time elapsed since satellite last passed the perigee.

GEOCENTRIC EQUATORIAL COORDINATES - 1

- $\mathbf{z_i}$ axis — Earth's rotational axis (N-S poles with N as positive z)
- $\mathbf{x_i}$ axis — In equatorial plane towards **FIRST POINT OF ARIES**
- $\mathbf{y_i}$ axis — Orthogonal to $\mathbf{z_i}$ and $\mathbf{x_i}$

NOTE: The ***First Point of Aries*** is a line from the center of the earth through the center of the sun at the vernal equinox (spring) in the northern hemisphere

Seasonal configuration of Earth and Sun

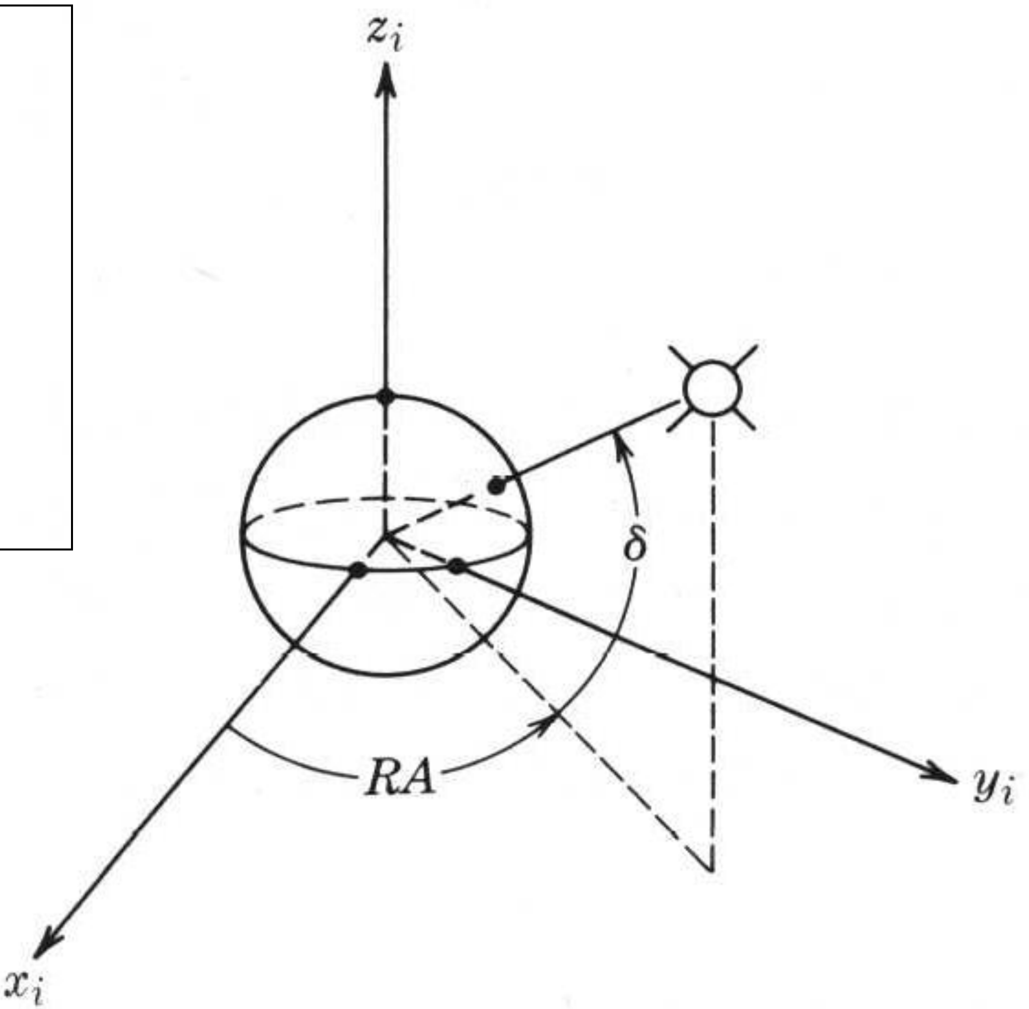


GEOCENTRIC EQUATORIAL COORDINATES - 2

RA = Right Ascension (in the x_i,y_i plane)

δ = Declination (the angle from the x_i,y_i plane to the satellite radius)

NOTE: Direction to ***First Point of Aries*** does **NOT** rotate with earth's motion around; the direction only ***translates***



To ***First Point of Aries***

LOCATING THE SATELLITE - 1

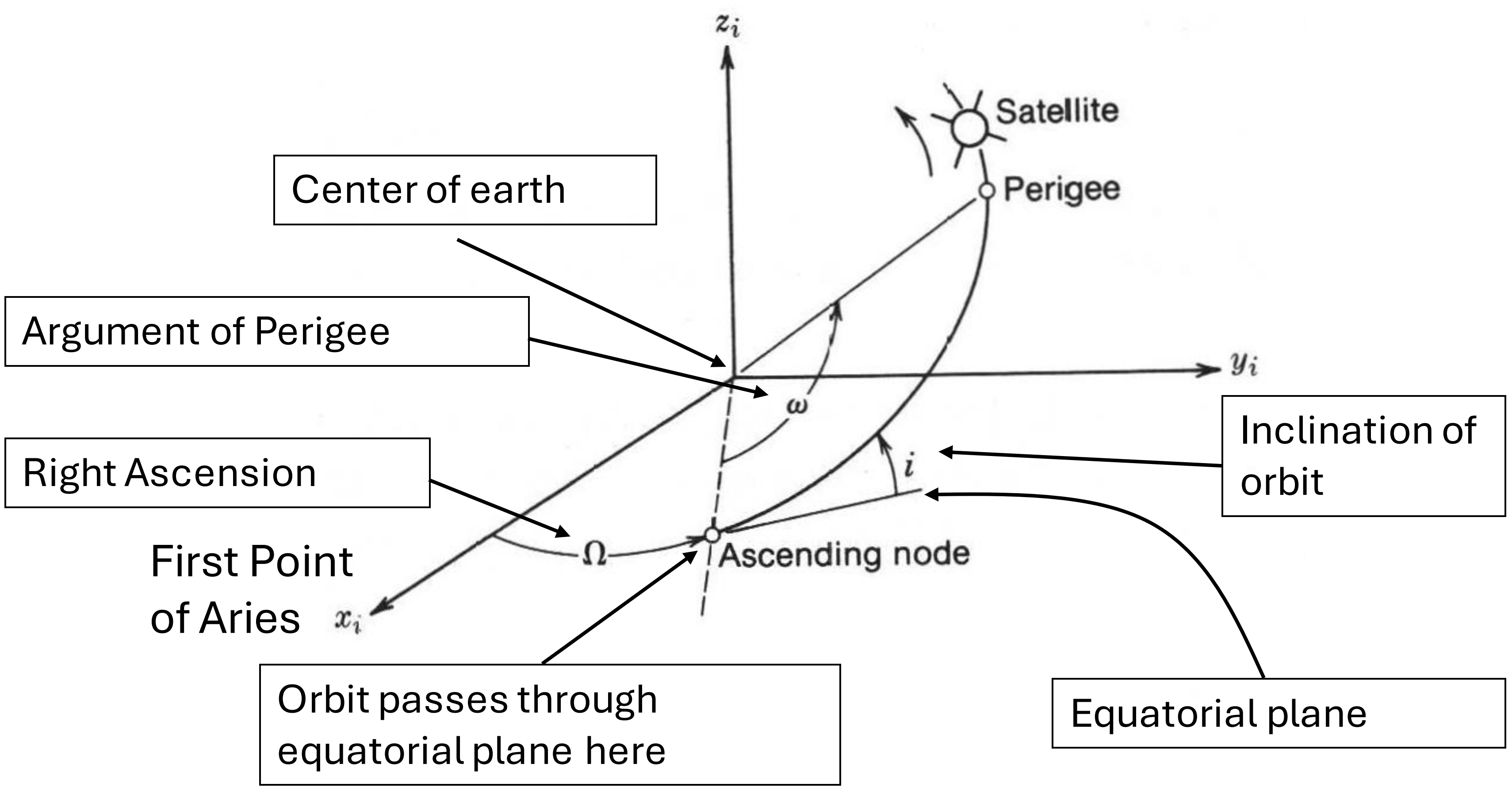
- Find the ***Ascending Node***
  - Point where the satellite crosses the equatorial plane from South to North
  - Define Ω and **i**
  - Define ω

Inclination

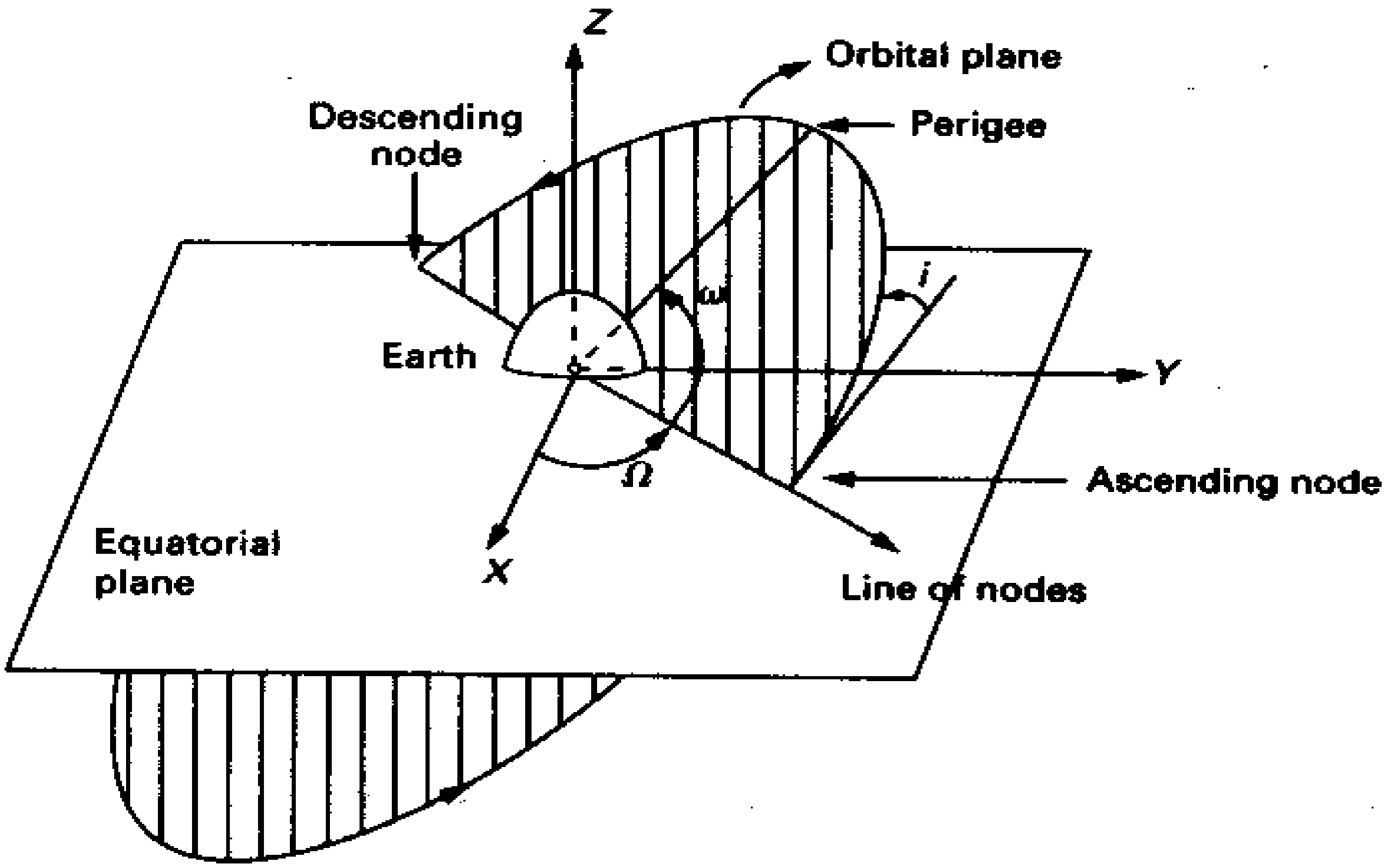
Right Ascension of the Ascending Node

(= ***RA*** from Fig. 2.6 in text)

DEFINING PARAMETERS



DEFINING PARAMETERS 2



(Source: M.Richaria, Satellite Communication Systems, Fig.2.9)