EENG 470
Satellite Communications

Lecture # 3 – P1
Orbit control and Launching Methods

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Lecture Content

- **2.2 Achieving a Stable Orbit**
- **2.3 Kepler's Three Laws of Planetary Motion**

Chapter (2)

- Sets out the basics of satellite orbits.
- The factors that influence a satellite once in orbit.
- Calculation of look angles – where to look for a satellite in the sky – is restricted to GEO satellites.

• The following sections explore how earth orbit is achieved, the laws governing objects in orbit, satellite maneuvers, and determining look angles

- Stable orbit around Earth requires being beyond the bulk of Earth's atmosphere, commonly known as space. Different definitions of space exist; US astronauts receive "space wings" at altitudes exceeding 50 miles (≈80 km).

- Some international treaties designate the space frontier at a height of 100 miles (≈160 km), requiring permission to overfly below that altitude.

- Atmospheric drag during re-entry begins around 400,000 ft (≈76 miles, ≈122 km).

- Satellites for missions lasting more than a few months are typically placed in orbits at least 250 miles (≈400 km) above Earth. Even at 250 miles, atmospheric drag is significant.

- The International Space Station (ISS) initially injected into orbit at 397 km decayed to 360 km by the end of 1999, necessitating orbit-raising maneuvers.

- Without onboard thrusters and sufficient orbital maneuvering fuel, the ISS in a low orbit would not last more than a few years.

- Understanding the basic laws of celestial mechanics starts with Newtonian equations describing the motion of a celestial body. Coordinate axes are established to set the orbit of a satellite and determine various forces acting on the Earth satellite.

Newton's laws of motion can be encapsulated into four equations:

$$s = ut + (1/2)at^2 \quad (2.1a)$$

$$v^2 = u^2 + 2at \quad (2.1b)$$

$$v = u + at \quad (2.1c)$$

$$P = ma \quad (2.1d)$$

- Distance traveled at time t (s).
- Initial velocity at $t = 0$ (u).
- Final velocity at time t (v).
- Acceleration of the object (a).
- Force acting on the object (P).
- Mass of the object (m).

Understanding Equation (2.1d):

1. States that the force acting on a body equals the mass of the body multiplied by the resulting acceleration.
2. Alternatively, the resulting acceleration is the ratio of the force acting on the body to the mass of the body.
3. Indicates that, for a given force, lighter masses result in higher accelerations.

Forces Acting on a Satellite in Stable Orbit:

In a stable orbit, two main forces act on a satellite:

Centrifugal force due to the satellite's kinetic energy, attempting to move it into a higher orbit.

Centripetal force due to gravitational attraction from the planet it orbits, attempting to pull it toward the planet.

Conditions for Stable Orbit:

1. If centrifugal and centripetal forces are equal, the satellite remains in a stable orbit.
2. The satellite continually falls toward the planet but compensates by moving forward in its orbit.
3. Maintains the same orbital height, described as being in free fall.

Visual Representation: Figure 2.1 illustrates the two opposing forces on a satellite in a stable orbit.

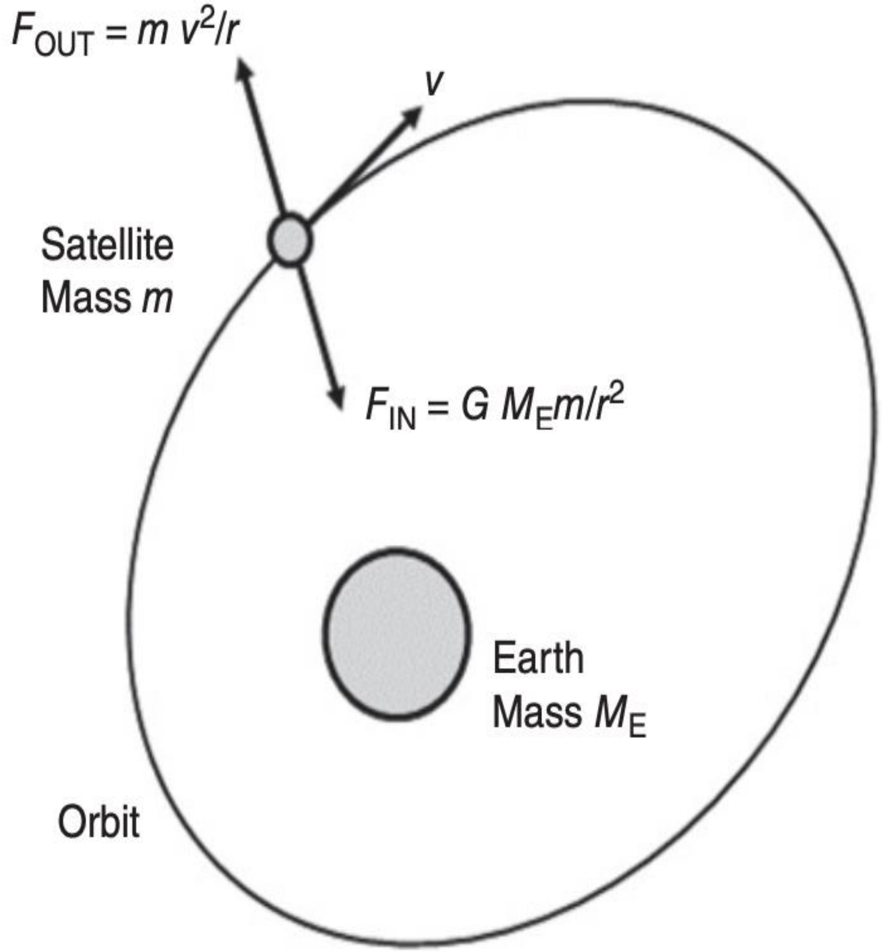

Figure2.1 Forces acting on a satellite in a stable orbit around the earth.

Gravitational force is inversely proportional to the square of the distance between the centers of gravity of the satellite and the planet the satellite is orbiting, in this case the earth. The gravitational force inward (***F_{IN}***, the centripetal force) is directed toward the center of gravity of the earth.

The kinetic energy of the satellite (***F_{OUT}***, the centrifugal force) is directed diametrically opposite the gravitational force. Kinetic energy is proportional to the square of the velocity *v* of the satellite.

When these inward and outward forces are balanced, the satellite moves around the earth in a *free fall* trajectory: the satellite's orbit. For a description of the units, please see the text.

Definition of Force:

- Force (F) is calculated as the product of mass (m) and acceleration (a): $F = m \times a$.
- The unit of force is the Newton (N).

Newton as a Unit of Force:

- One Newton (N) is defined as the force required to accelerate a mass of 1 kg with an acceleration of 1 m/s^2.
- The underlying units of a Newton are $(\mathrm{kg}) \times (\mathrm{m/s^2})$.

Imperial Units Conversion:

- In Imperial Units, one Newton is equivalent to 0.2248 foot-pounds ($\mathrm{ft.\ lb.}$).

Standard Acceleration due to Gravity:

- The standard acceleration due to gravity at Earth's surface is approximately 9.80665×10^{-3} km/s^2.
- This value is often quoted as $981\ \mathrm{cm/s^2}$.

Note on Gravity:

- The provided information suggests that the acceleration due to gravity decreases with altitude or distance from the Earth's surface.

height above the earth's surface. The acceleration, a, due to gravity at a distance r from the center of the earth is (Gordon and Morgan 1993)

$$a = \mu/r^2 \text{ km/s}^2 \tag{2.1e}$$

where the constant μ is the product of the universal gravitational constant G and the mass of the earth M_E.

The product GM_E is called Kepler's constant and has the value $3.986\,004\,418 \times 10^5 \text{ km}^3/\text{s}^2$.

The universal gravitational constant is

$$G = 6.672 \times 10^{-11} \text{ Nm}^2/kg^2 \text{ or } 6.672 \times 10^{-20} \text{ km}^3/\text{kg s}^2$$

in the older units. Since force = mass × acceleration, the centripetal force acting on the satellite, F_{IN}, is given by

$$F_{IN} = m \times (\mu/r^2) \tag{2.2a}$$

$$= m \times \left(GM_E/r^2\right) \tag{2.2b}$$

In a similar fashion, the centrifugal acceleration is given by

$$a = (v^2/r) \tag{2.3}$$

which will give the centrifugal force, F_{OUT}, as

$$F_{OUT} = m \times (v^2/r) \tag{2.4}$$

If the forces on the satellite are balanced, $F_{IN} = F_{OUT}$ and, using Eqs. (2.2a) and (2.4),

$$m \times \mu/r^2 = m \times v^2/r$$

hence the velocity v of a satellite in a circular orbit is given by

$$v = (\mu/r)^{1/2} \tag{2.5}$$

If the orbit is circular, the distance traveled by a satellite in one orbit around a planet is $2\pi r$, where r is the radius of the orbit from the satellite to the center of the planet. Since distance divided by velocity equals time to travel that distance, the period of the satellite's orbit, T, will be

Since distance divided by velocity equals time to travel that distance, the period of the satellite's orbit, T, will be

$$T = (2\pi r)/v = (2\pi r)/[(\mu/r)^{1/2}]$$

giving

$$T = (2\pi r^{3/2})/(\mu^{1/2}) \tag{2.6}$$

Table 2.1 Orbital velocity, height, and period for five satellite systems

Satellite system	Orbital height (km)	Orbital velocity (km/s)	Orbital period (h)	Orbital period (min)	Orbital period (s)
Intelsat (GEO)	35 786.03	3.074 7	23	56	4.08
Other 3 billion (O3B) (MEO)	8 062	5.253 9	4	47	0.01
Globalstar (LEO)	1 414	7.152 2	1	54	5.35
Iridium (LEO)	780	7.462 4	1	40	27.0
SpaceX (VLEO)	345.6	7.699 51	1	31	26.90

Table 2.1 gives the velocity, *v*, and orbital period, *T*, for four satellite systems that occupy typical low earth orbit (LEO), medium earth orbit (MEO), and geostationary earth orbit (GEO) orbits around the earth. In each case, the orbits are circular and the average radius of the earth is taken as 6378.137 km (Gordon and Morgan 1993).

Note the reduction in orbital period as the satellites move from GEO (essentially zero movement as observed from the ground) to very low earth orbit (VLEO). There are two immediate consequences for non-geostationary satellites as far as connections to a fixed earth station on the surface of the earth: (i) there will be gaps in coverage unless a con- stellation of the same satellites are orbiting, usually in the same plane; (ii) the observation time is significantly reduced as the orbital altitude is reduced. What is gained in lower signal delay with altitude is lost with the need for complex fixed earth station antennas and smaller observation time per satellite.

Example 2.1

Question: A satellite is in a 322 km high circular orbit. Determine:

a. The orbital angular velocity in radians per second;

b. The orbital period in minutes; and

c. The orbital velocity in meters per second.

Note: Assume the average radius of the earth is 6378.137 km and Kepler's constant has the value 3.986 004 418 × 105 km3/s2.

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Answer

It is actually easier to answer the three parts of this question backward, beginning with the orbital velocity, then calculating the period, and hence the orbital angular velocity. First we will find the total radius of the orbit $r = 322 + 6378.137$ km $= 6700.137$ km

(c) From Eq. (2.5), the orbital velocity $v = (\mu/r)^{1/2} = (3.986\,004\,418 \times 10^5/6700.137)^{1/2}$ $= 7.713\,066$ km/s $= 7713.066$ m/s.

(b) From Eq. (2.6), $T = (2\pi r^{3/2})/(\mu^{1/2}) = (2\pi 6700.137^{3/2})/(3.986\,004\,418 \times 10^5)^{1/2} =$ $(3\,445\,921.604)/(631.348\,114\,6) = 5\,458.037\,372$ seconds $= 90.967\,2895$ minutes $=$ 90.97 minutes.

(a) The orbital period from above is 5 458.037 372 seconds. One revolution of the earth covers 360° or 2π radians. Hence 2π radians are covered in 5458.037 372 seconds, giving the orbital angular velocity as 2π/5458.037 372 rad/s = 0.001 1512 rad/s. An alternative calculation procedure would calculate the distance traveled in one orbit ($2\pi r = 2\pi 6700.137 = 42\,098.202\,36$ km). This distance is equivalent to 2π radians and so 1 km is equivalent to 2π/42 098.202 36 rad = 0.000 149 3 rad. From above, the orbital velocity was 7.713 066 km/s = 7.713 066 × 0.000 149 3 rad/s = 0.001 1512 rad/s.

A number of coordinate systems and reference planes can be used to describe the orbit of a satellite around a planet. Figure 2.2 illustrates one of these using a Cartesian coordinate system with the earth at the center and the reference planes coinciding with the equator and the polar axis. This is referred to as a geocentric coordinate system.

With the coordinate system set up as in Figure 2.2, and with the satellite mass m located at a vector distance r from the center of the earth, the gravitational force $\boldsymbol{F}$ on the satellite is given by

$$\bar{F} = -\frac{GM_E m\bar{r}}{r^3} \tag{2.7}$$

where M_E is the mass of the earth and $G = 6.672 \times 10^{-11}$ Nm^2/kg^2. But force = mass × acceleration and Eq. (2.7) can be written as

$$\bar{F} = m\frac{d^2\bar{r}}{dt^2} \tag{2.8}$$

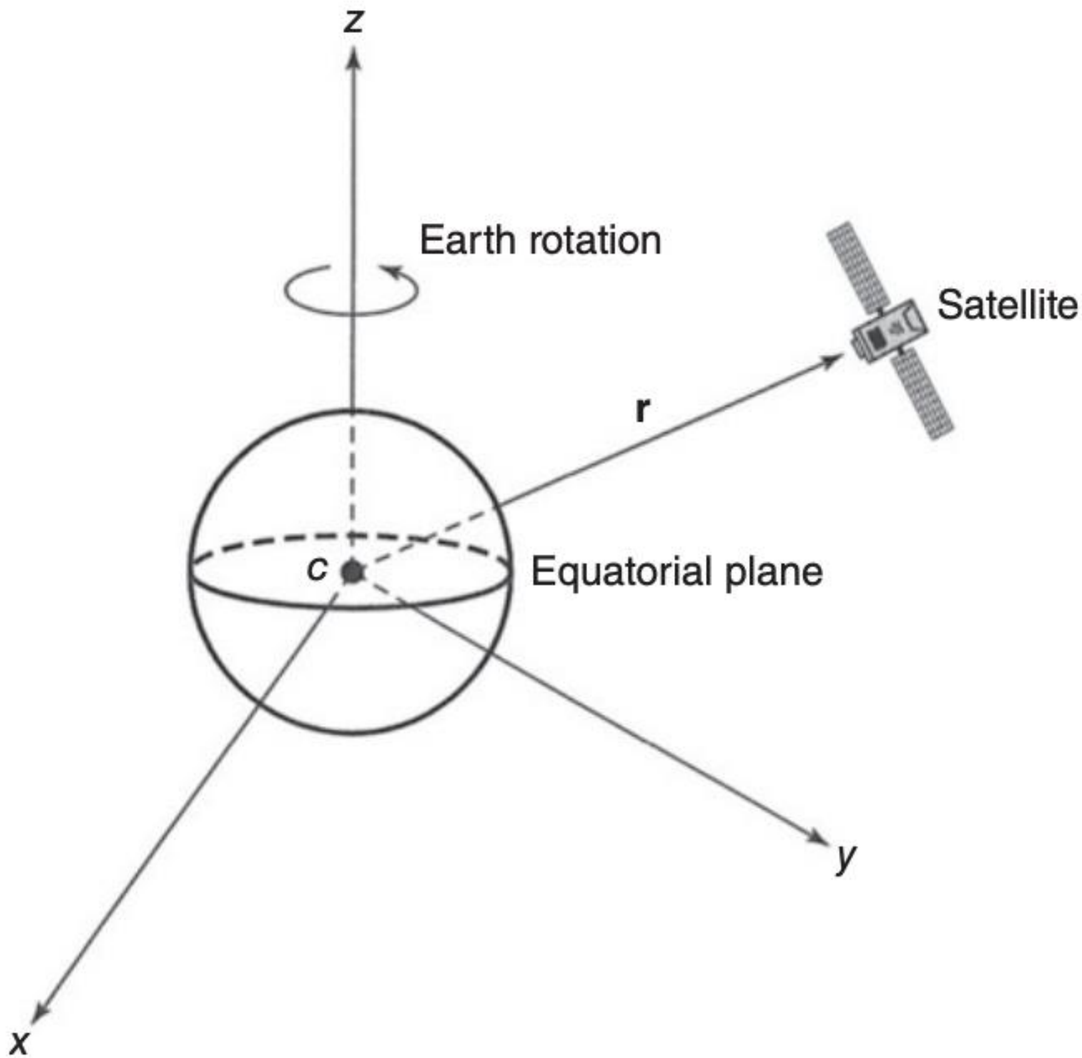

Figure2.2 The initial coordinate system used to describe the relationship between the earth and a satellite.

A Cartesian coordinate system with the geographical axes of the earth as the principal axes is the simplest coordinate system and the origin at the center of the earth.

The rotational axis of the earth is about the *z* axis, which passes through the geographic north pole.

The *x* and *y* axes are mutually orthogonal to the *z* axis and lie in the earth's equatorial plane.

The vector *r* locates the satellite with respect to the center of the earth.

From Eqs. (2.7) and (2.8) we have

$$-\frac{\bar{r}}{r^3}\mu = \frac{d^2\bar{r}}{dt^2} \tag{2.9}$$

which yields

$$\frac{d^2\bar{r}}{dt^2} + \frac{\bar{r}}{r^3}\mu = 0 \tag{2.10}$$

This is a second order linear differential equation and its solution will involve six undetermined constants called the *orbital elements*. The orbit described by these orbital elements can be shown to lie in a plane and to have a constant angular momentum. The solution to Eq. (2.10) is difficult since the second derivative of *r* involves the second derivative of the unit vector *r*. To remove this dependence, a different set of coordinates can be chosen to describe the location of the satellite such that the unit vectors in the three axes are constant. This coordinate system uses the plane of the satellite's orbit as the reference plane. This is shown in Figure 2.3.

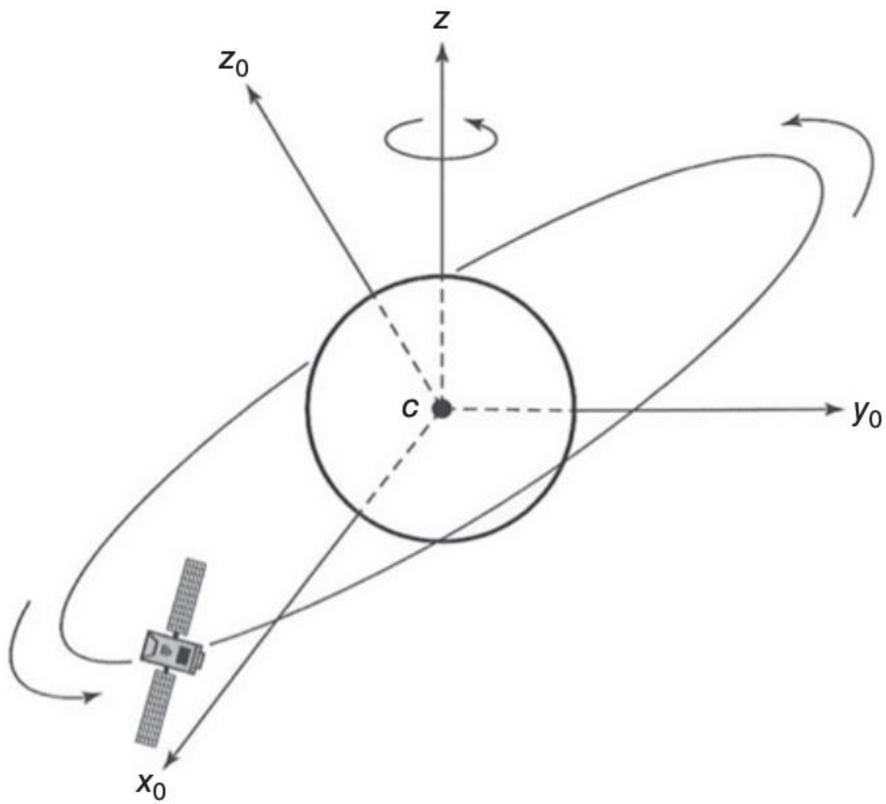

Figure2.3 The orbital plane coordinate system.

In this coordinate system the orbital plane is used as the reference plane.

The orthogonal axes *x*0 and *y*0 lie in the orbital plane.

The third axis, *z*0 is orthogonal to the *x*0 and *y*0 axes to form a right hand coordinate set.

The *z*0 axis is not coincident with the earth's *z* axis through the earth's north pole unless the orbital plane lies exactly in the earth's equatorial plane.

Expressing Eq. (2.10) in terms of the new coordinate axes x_0, y_0, and z_0 gives

$$\hat{x}_0\left(\frac{d^2x_0}{dt^2}\right)+\hat{y}_0\left(\frac{d^2y_0}{dt^2}\right)+\frac{\mu\left(x_0\hat{x}_0+y_0\hat{y}_0\right)}{\left(x_0^2+y_0^2\right)^{3/2}}=0 \tag{2.11}$$

Equation (2.11) is easier to solve if it is expressed in a polar coordinate system rather than a Cartesian coordinate system. The polar coordinate system is shown in Figure 2.4.

With the polar coordinate system shown in Figure 2.4 and using the transformations

$$x_0 = r_0 \cos\phi_0 \tag{2.12a}$$

$$y_0 = r_0 \sin\phi_0 \tag{2.12b}$$

$$\hat{x}_0 = \hat{r}_0 \cos\phi_0 - \hat{\phi}_0 \sin\phi_0 \tag{2.12c}$$

$$\hat{y}_0 = \hat{\phi}_0 \cos\phi_0 + \hat{r}_0 \sin\phi_0 \tag{2.12d}$$

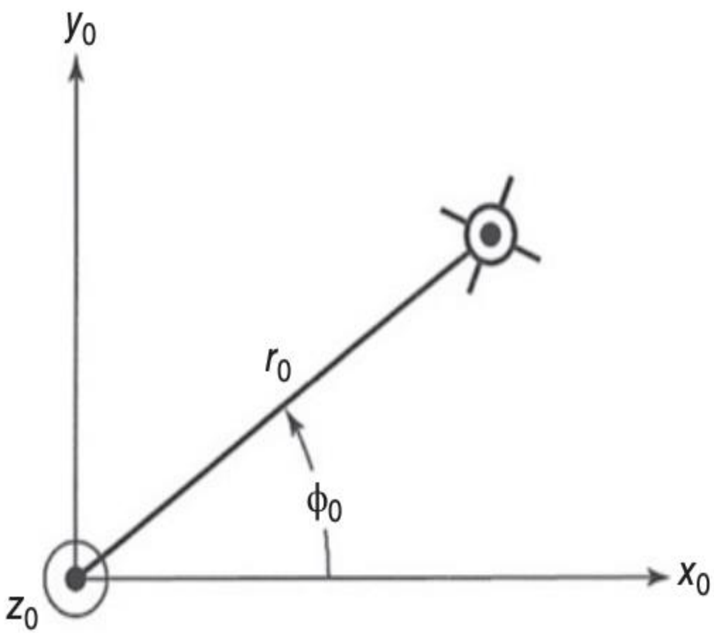

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Figure 2.4 Polar coordinate system in the plane of the satellite's orbit. The axis z_0 is straight out of the paper from the center of the earth, and is normal to the plane of the satellite's orbit. The satellite's position is described in terms of the distance r_0 from the center of the earth and the angle this makes with the x_0 axis, ϕ_0.

and equating the vector components of r_0 and ϕ_0 in turn in Eq. (2.11) yields

$$\frac{d^2 r_0}{dt^2} - r_0 \left(\frac{d\phi_0}{dt} \right) = -\frac{\mu}{r_0^2} \tag{2.13}$$

and

$$r_0 \left(\frac{d^2 \phi_0}{dt^2} \right) + 2 \left(\frac{dr_0}{dt} \right) \left(\frac{d\phi_0}{dt} \right) = 0 \tag{2.14}$$

Using standard mathematical procedures, we can develop an equation for the radius of the satellite's orbit, r_0, namely

$$r_0 = \frac{p}{1 + e\cos(\phi_0 - \theta_0)} \tag{2.15}$$

where θ_0 is a constant and e is the eccentricity of an ellipse whose semilatus rectum p is given by

$$p = (h^2/\mu) \tag{2.16}$$

and h is the magnitude of the orbital angular momentum of the satellite. That the equation of the orbit is an ellipse is Kepler's first law of planetary motion.

2.3 Kepler's Three Laws of Planetary Motion

Johannes Kepler (1571–1630) was a German astronomer and scientist who developed his three laws of planetary motion by careful observations of the behavior of the planets in the solar system over many years, with help from some detailed planetary observations by the Hungarian astronomer Tycho Brahe. Kepler's three laws are:

1. The orbit of any smaller body about a larger body is always an ellipse, with the center of mass of the larger body as one of the two foci.
2. The orbit of the smaller body sweeps out equal areas in equal time (see Figure 2.5).
3. The square of the period of revolution of the smaller body about the larger body equals a constant multiplied by the third power of the semimajor axis of the orbital ellipse. That is $T^2 = (4\pi^2 a^3)/\mu$ where T is the orbital period, a is the semimajor axis of the orbital ellipse, and μ is Kepler's constant. If the orbit is circular, then a becomes distance r, defined as before, and we have Eq. (2.6).

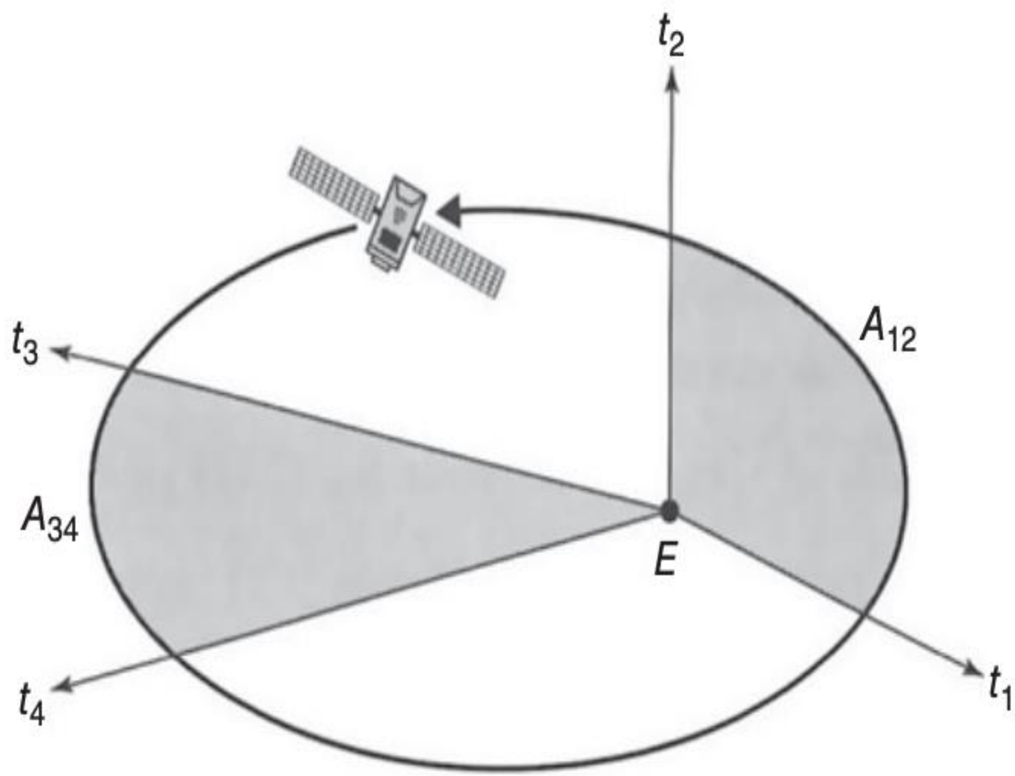
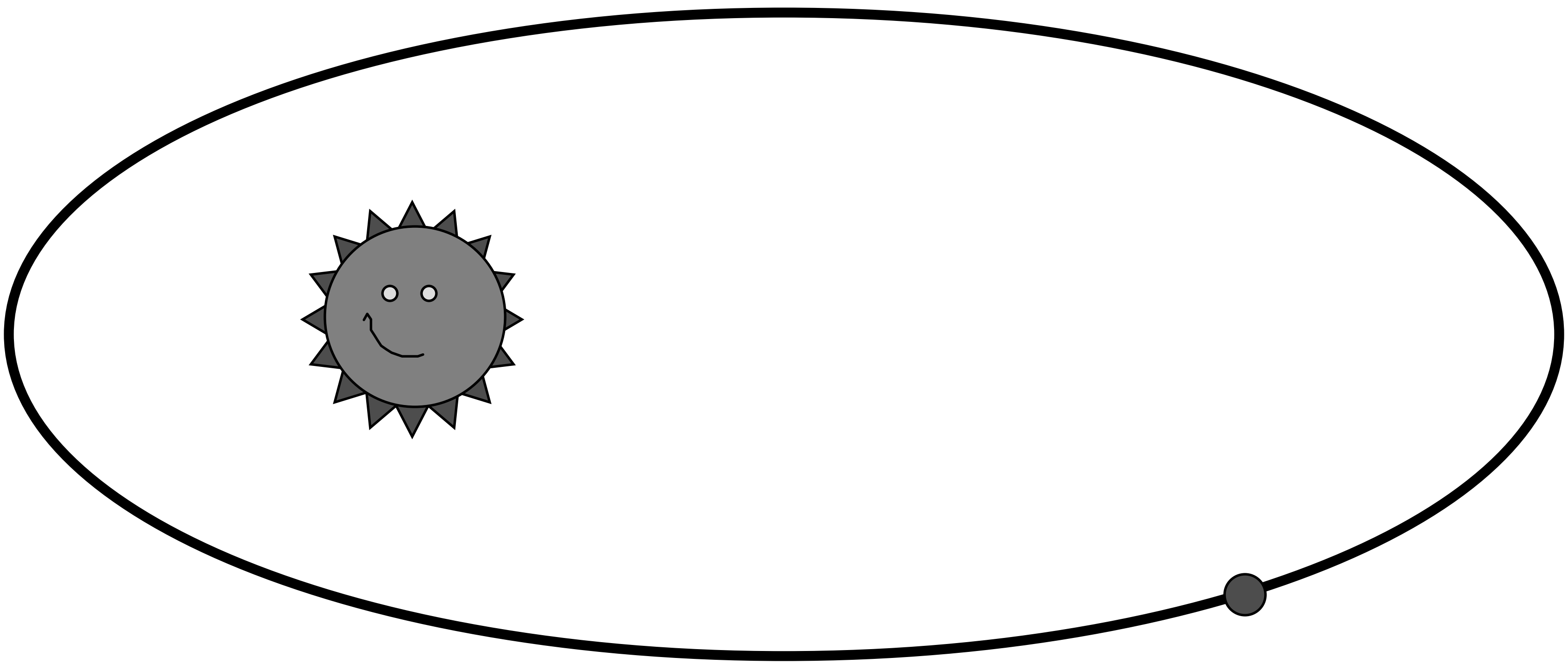


Figure 2.5 Illustration of Kepler's second law of planetary motion. A satellite is in orbit around the planet earth, *E*. The orbit is an ellipse with a relatively high eccentricity, that is, it is far from being circular. The figure shows two shaded portions of the elliptical plane in which the orbit moves, one is close to the earth and encloses the perigee while the other is far from the earth and encloses the apogee. The perigee is the point of closest approach to the earth while the apogee is the point in the orbit that is furthest from the earth. While close to perigee, the satellite moves in the orbit between t_1 and t_2 and sweeps out an area denoted by A_{12}. While close to apogee, the satellite moves in the orbit between times t_3 and t_4 and sweeps out an area denoted by A_{34}. If $t_1 - t_2 = t_3 - t_4$ then $A_{12} = A_{34}$.

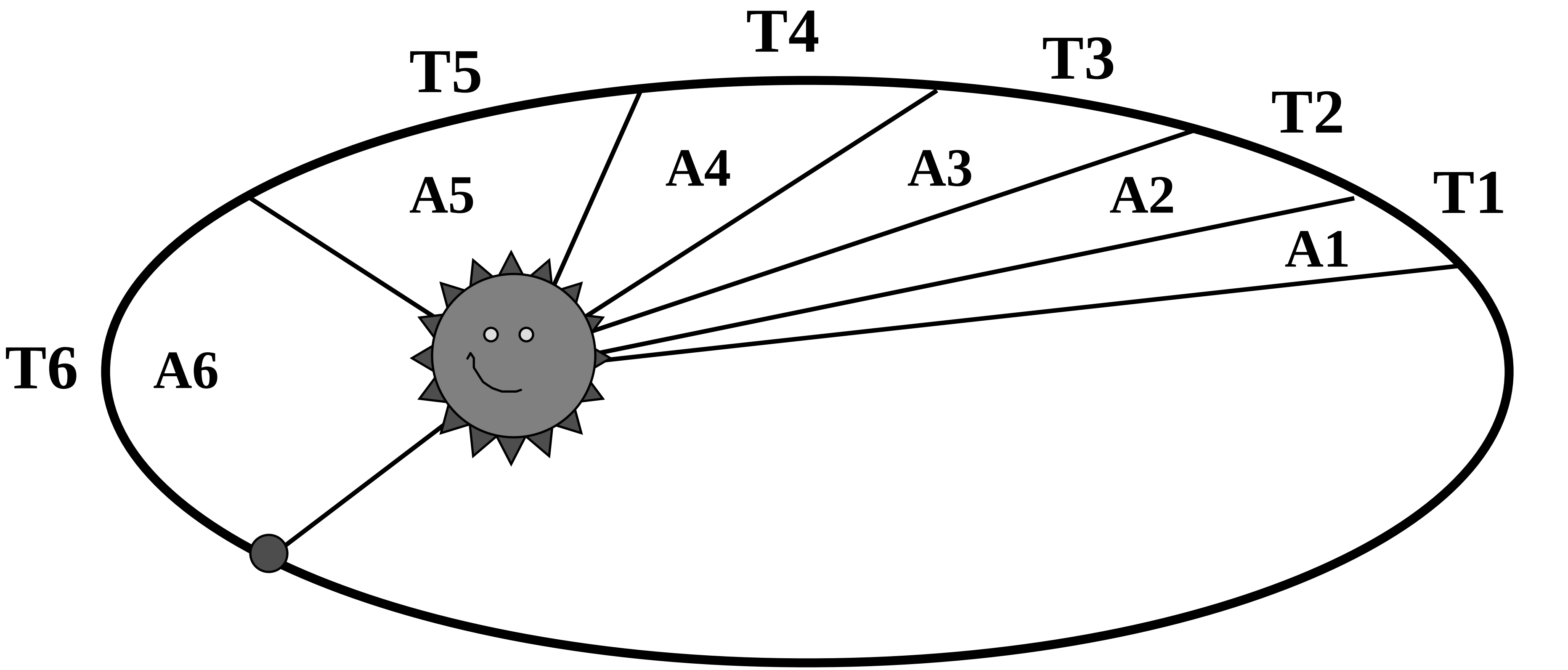
Kepler's 1st Law: Law of Ellipses



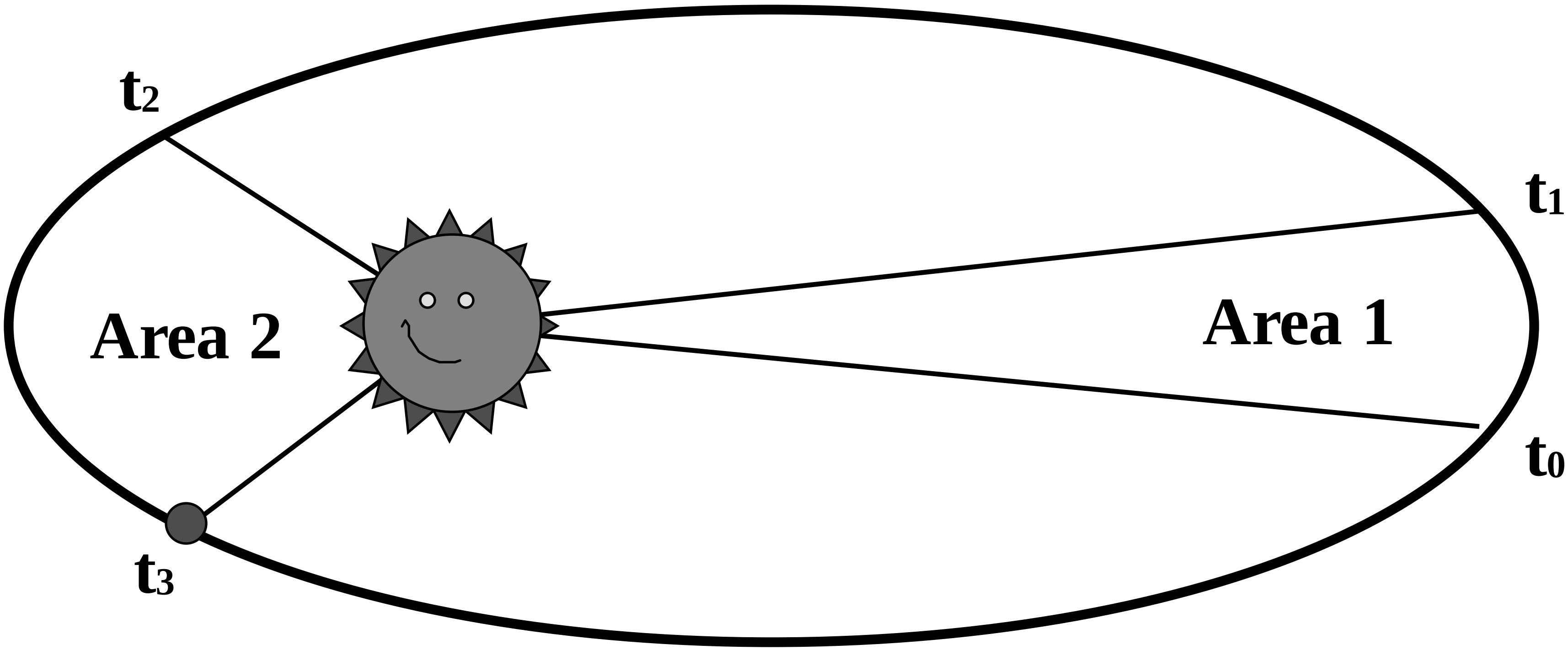
The orbits of the planets are ellipses with the sun at one focus

Kepler's 2nd Law: Law of Equal Areas

The line joining the planet to the center of the sun sweeps out equal areas in equal times



Kepler's 2nd Law: Law of Equal Areas



$t_1 - t_0 = t_3 - t_2$

Area 1 = Area 2

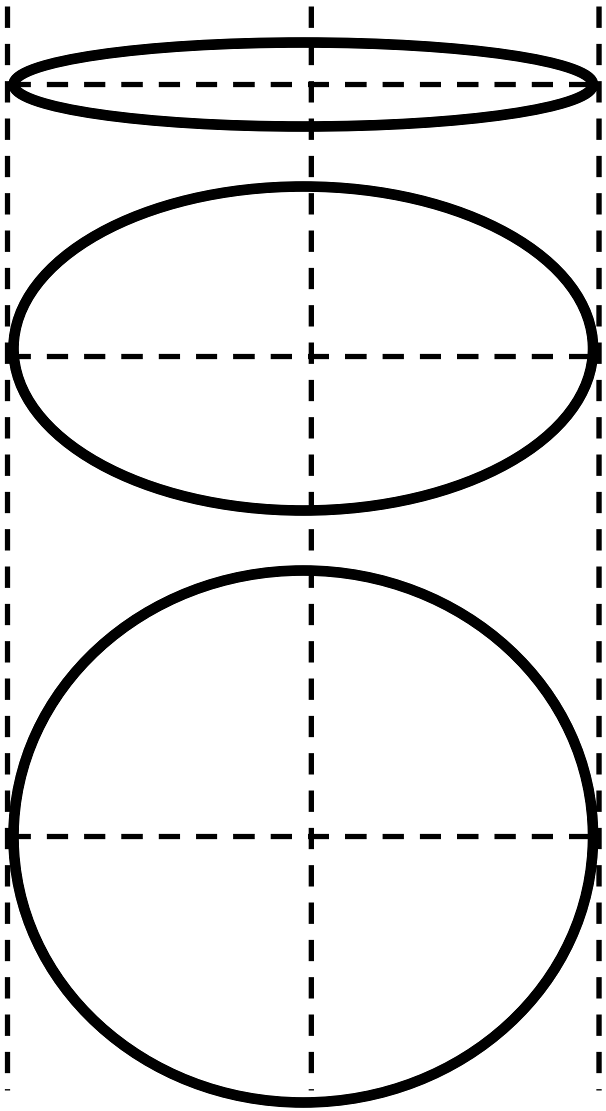
Satellite travels at varying speeds

Kepler's 3rd Law: Law of Harmonics

The squares of the periods of two planets' orbits are proportional to each other as the cubes of their semi-major axes:

$T_1^2/T_2^2 = a_1^3/a_2^3$

"Orbits with the same semi-major axis will have the same period"



Example 2.2

Question: A satellite in an elliptical orbit around the earth has an apogee of 39 152 km and a perigee of 500 km. What is the orbital period of this satellite? Give your answer in hours. Note: Assume the average radius of the earth is 6378.137 km and Kepler's constant has the value $3.986\,004\,418 \times 10^5\ \mathrm{km^3/s^2}$.

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Answer

The mathematical formulation of the third law is $T^2 = (4\pi^2 a^3)/\mu$, where T is the orbital period, a is the semimajor axis of the orbital ellipse, and μ is Kepler's constant.

The perigee of a satellite is the closest distance in the orbit to the earth; the apogee of a satellite is the furthest distance in the orbit from the earth.

For the last part, draw a diagram to illustrate the geometry.

The semimajor axis of the ellipse = $(39\,152 + (2 \times 6378.137) + 500)/2 = 26\,204.137$ km

The orbital period is

$$T^2 = (4\pi^2 a^3)/\mu = (4\pi^2 (26\,204.137)^3)/3.986\,004\,418 \times 10^5 = 1\,782\,097\,845.0\,\text{s}^2$$

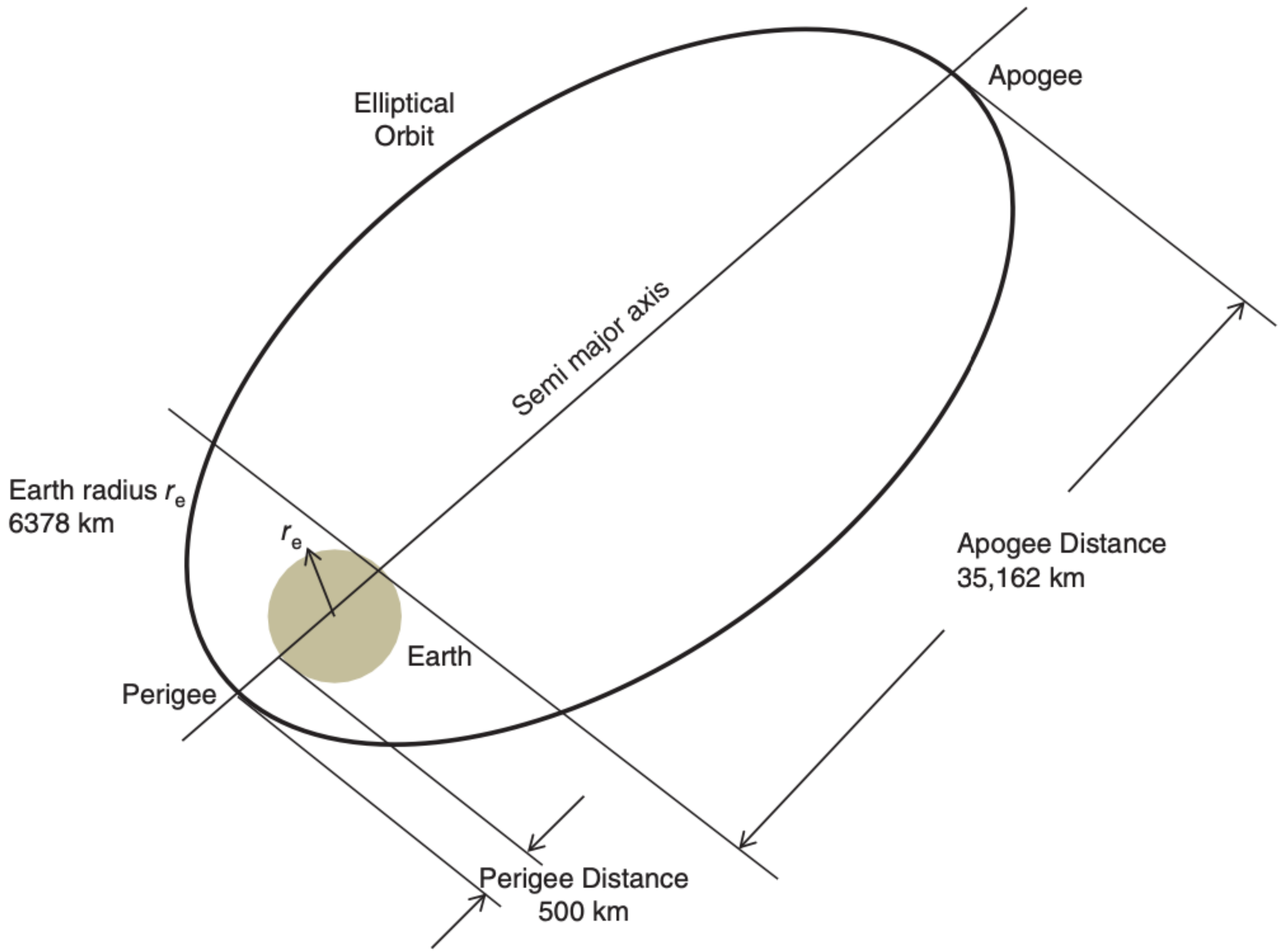

Figure Ex 2.2 The elliptical orbit of the satellite in Example 2.2. This is a Molniya orbit.

Therefore, $T = 42\,214.900\,75$ seconds $= 11.726\,361\,32$ hours $=$ (11 hours 43 minutes 34.9 seconds)

What we have found above is the orbital period of a *Molniya* satellite of the former Soviet Union as shown in Figure Ex 2.2. Describing the orbit of a satellite enables us to develop Kepler's second two laws.