EENG 470
Satellite Communications

Lecture # 4
2. Orbit control and Launching Methods

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Lecture(4)

2.8 Look Angle Determination

- Navigation Precision Improvement:

- Introduction of latitude and longitude grid lines significantly enhanced the accuracy of navigation across the Earth's oceans.

- Definition of Latitude and Longitude:

- Latitude, measured in degrees, indicates the angular distance north or south from the equator, facilitating vertical positioning.

- Longitude, also measured in degrees, denotes the angular distance east or west from a specified longitudinal line, enabling horizontal positioning.

- Historical Context:

- England and France, as major seafaring powers, competed for maritime dominance during the period when the latitude and longitude grid system gained popularity.

- England, opting for Greenwich as the zero-reference longitude, and France, selecting Paris, reflected their respective geopolitical influences.

- The strategic decision by the British Admiralty to distribute maps for free contrasted with the French practice of charging for theirs, contributing to Greenwich's rapid adoption as the dominant reference point.

- Influence of Geometry:

- 90° per quadrant on the map. (a quadrant refers to one-fourth of the Earth's surface, divided by the equator and prime meridian (or Greenwich Meridian)).

- Measurement Units:

- 360° of longitude measured from the Greenwich Meridian.
- ±90° of latitude (North Pole and South Pole).

- **Role of GEO Satellite Systems:**

  - GEO satellite systems, registered in Geneva, specify their subsatellite locations in degrees east from the Greenwich Meridian, mitigating confusion. (subsatellite locations : the location of satellites over the equator, measured in degrees east from the Greenwich Meridian.)

- **Earth Stations and Coordinates:**

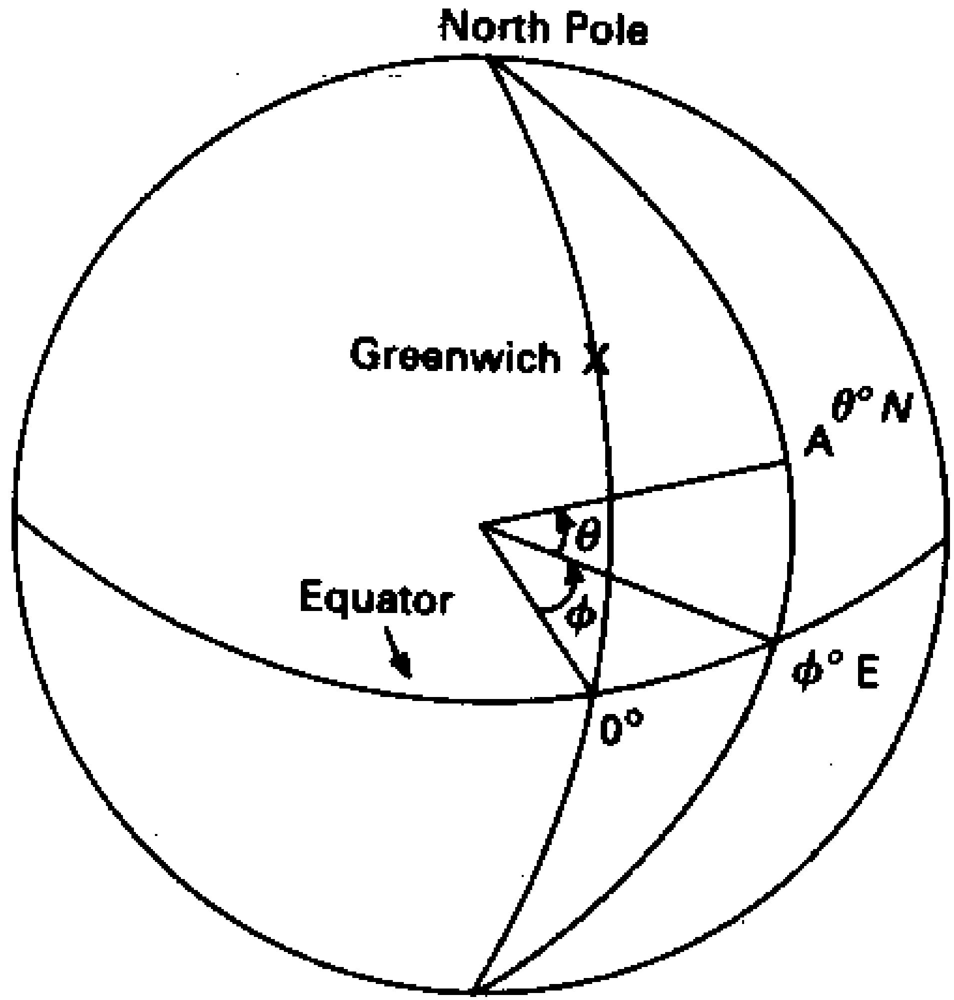
  - Earth stations communicate with satellites using pointing coordinates derived from their geographic latitude and longitude, ensuring accurate tracking of satellite motion relative to their positions on Earth.

- **Latitude:** Angular distance, measured in degrees, north or south of the equator.

  L from -90 to +90 (or from 90S to 90N)

- **Longitude:** Angular distance, measured in degrees, from a given reference longitudinal line (Greenwich, London).

  l from 0 to 360E (or 180W to 180E)



Latitude (θ°N) and longitude (φ°E) of a point A.

(Source: M.Richaria, Satellite Communication Systems, Fig.2.9)

- Look Angles:

- Coordinates for earth station antenna alignment with a satellite for communication purposes.

- Commonly Used Angles:

- Most commonly expressed as azimuth (Az) and elevation (El).
- Other pairs like right ascension and declination exist, particularly for radio astronomy antennas.

- **Azimuth (Az):**
    - Measured eastward (clockwise) from geographic north to the projection of the satellite path on a locally horizontal plane at the earth station.
- **Elevation (El) :**
    - Angle measured upward from the local horizontal plane at the earth station to the satellite path.

- **Importance of Satellite Location:**
    - Precise satellite location crucial for determining accurate look angles.
- **Subsatellite Point:**
    - Key location for many instances in determining satellite position relative to the Earth's surface.

- **Azimuth:** Measured eastward (clockwise) from geographic north to the projection of the satellite path on a (locally) horizontal plane at the earth station.

- **Elevation Angle:** Measured upward from the local horizontal plane at the earth station to the satellite path.

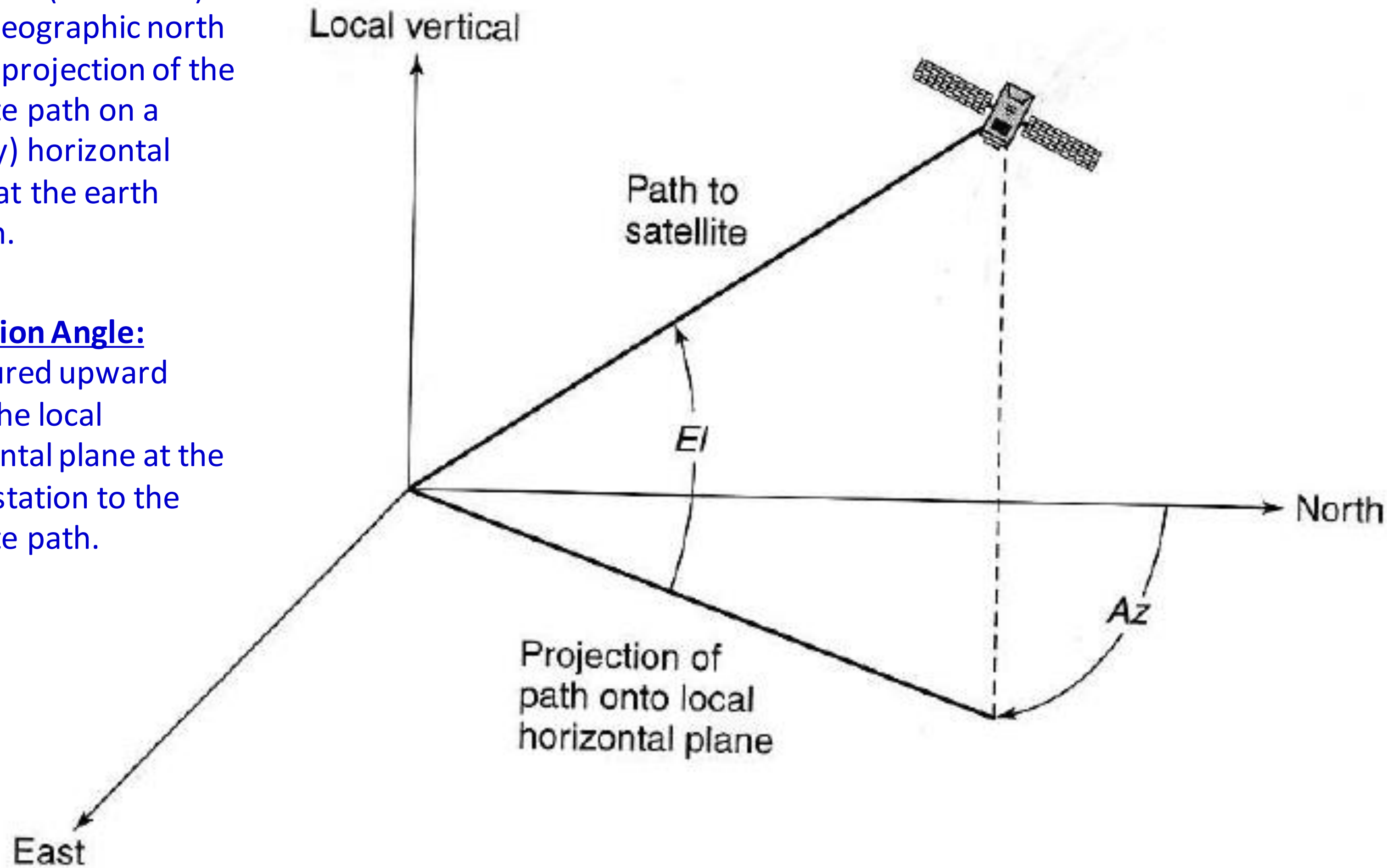


FIGURE 2.10 The definition of elevation (*El*) and azimuth (*Az*). The elevation angle is measured upward from the local horizontal at the earth station and the azimuth angle is measured from true north in an eastward direction to the projection of the satellite path onto the local horizontal plane.

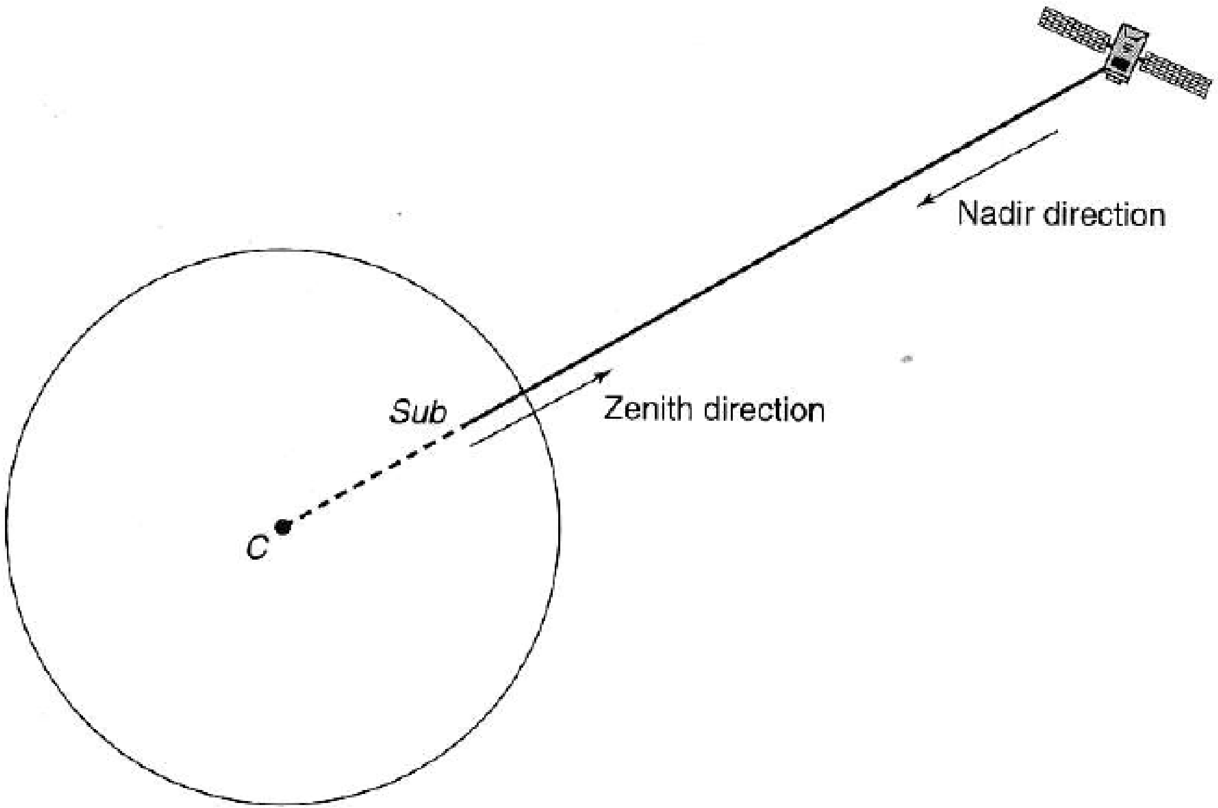


FIGURE 2.11 Zenith and nadir pointing directions. The line joining the satellite and the center of the earth, *C*, passes through the surface of the earth at point *Sub*, the subsatellite point. The satellite is directly overhead at this point and so an observer at the subsatellite point would see the satellite at zenith (i.e., at an elevation angle of 90°). The pointing direction from the satellite to the subsatellite point is the nadir direction from the satellite. If the beam from the satellite antenna is to be pointed at a location on the earth that is not at the subsatellite point, the pointing direction is defined by the angle away from nadir. In general, two off-nadir angles are given: the number of degrees north (or south) from nadir; and the number of degrees east (or west) from nadir. East, west, north, and south directions are those defined by the geography of the earth.

Elevation Angle Calculation (2.8.2):

Figure 2.12 illustrates the geometry involved in calculating the elevation angle.

• Three vectors are involved:

(1) rs (from the center of the Earth to the satellite),

(2) re (from the center of the Earth to the earth station),

(3) d (from the earth station to the satellite). These vectors form a triangle in the same plane.

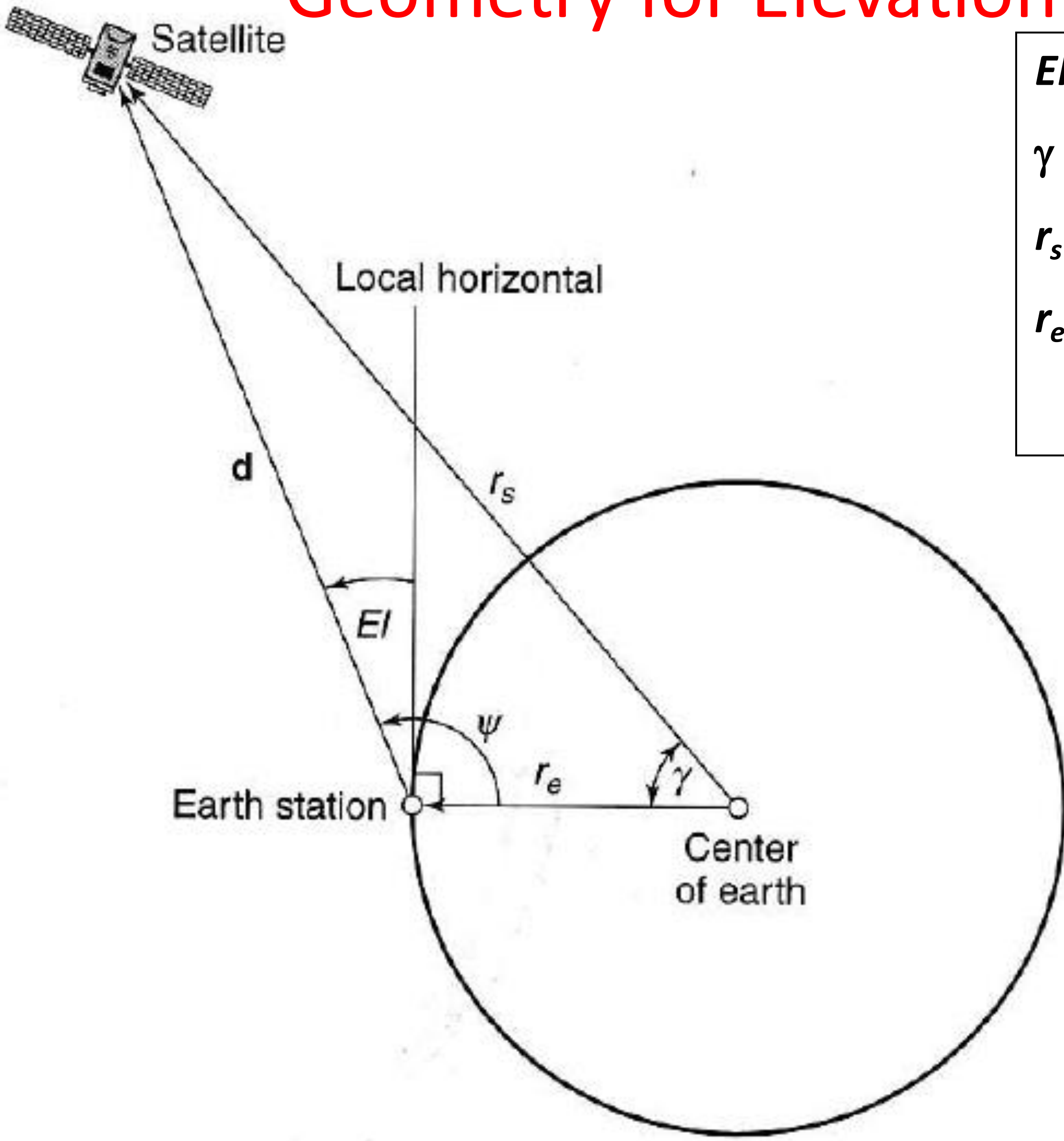
• The central angle γ between re and rs represents the angle between the earth station and the satellite, while ψ is the angle measured from re to d within the triangle.

• γ is non-negative and related to the earth station's north latitude Le (number of degrees north from the equator) and west longitude le (number of degrees west).

- SUB-SATELLITE POINT
  - Latitude $\mathbf{L_s}$
  - Longitude $\mathbf{l_s}$
- EARTH STATION LOCATION
  - Latitude $\mathbf{L_e}$
  - Longitude $\mathbf{l_e}$
- Calculate γ, ANGLE AT EARTH CENTER

Between the line that connects the earth-center to the satellite and the line from the earth-center to the earth station.

Geometry for Elevation Calculation



$El = \psi - 90^o$

γ = **central angle**

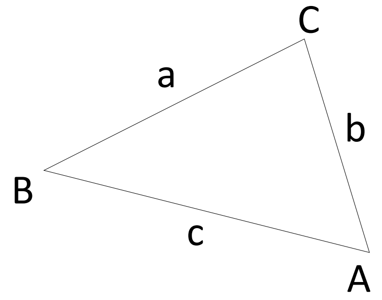
r_s = **radius to the satellite**

r_e = **radius of the earth**

FIGURE 2.12 The geometry of elevation angle calculation. The plane of the paper is the plane defined by the center of the earth, the satellite, and the earth station. The central angle is γ. The elevation angle *El* is measured upward from the local horizontal at the earth station.

Slant path geometry

- Review of plane trigonometry
  - Law of Sines
  - Law of Cosines
  - Law of Tangents



$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

$$c^2 = a^2 + b^2 - 2ab\cos C$$

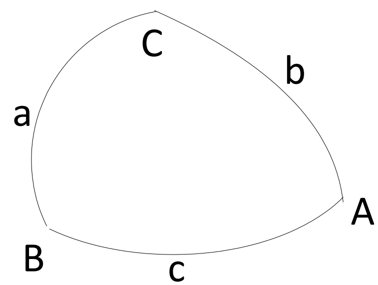
$$\tan\frac{C}{2} = \sqrt{\frac{(d-a)(d-b)}{d(d-c)}}, d = \frac{a+b+c}{2}$$

- Review of spherical trigonometry
  - Law of Sines
  - Law of Cosines for angles
  - Law of Cosines for sides

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

$$\cos a = \cos b\cos c + \sin b\sin c\cos A$$

$$\cos A = -\cos B\cos C + \sin B\sin C\cos a$$



Elevation Angle Calculation

Figure 2.12 shows the geometry of the elevation angle calculation. In Figure 2.12, $\boldsymbol{r}_s$ is the vector from the center of the earth to the satellite; $\boldsymbol{r}_e$ is the vector from the center of the earth to the earth station; and $\boldsymbol{d}$ is the vector from the earth station to the satellite. These three vectors lie in the same plane and form a triangle. The central angle γ measured between $\boldsymbol{r}_e$ and $\boldsymbol{r}_s$ is the angle between the earth station and the satellite, and ψ is the angle (within the triangle) measured from $\boldsymbol{r}_e$ to $\boldsymbol{d}$. Defined so that it is nonnegative, γ is related to the earth station north latitude L_e (i.e., L_e is the number of degrees in latitude that the earth station is north from the equator) and west longitude l_e (i.e., l_e is the number of degrees in longitude that the earth station is west from the Greenwich meridian) and the subsatellite point at north latitude L_s and west longitude l_s by

$$\cos(\gamma) = \cos(L_e)\cos(L_s)\cos(l_s - l_e) + \sin(L_e)\sin(L_s) \tag{2.31}$$

The law of cosines allows us to relate the magnitudes of the vectors joining the center of the earth, the satellite, and the earth station. Thus

$$d = r_s\left[1 + \left(\frac{r_e}{r_s}\right)^2 - 2\left(\frac{r_e}{r_s}\right)\cos(\gamma)\right]^{1/2} \tag{2.32}$$

Since the local horizontal plane at the earth station is perpendicular to $\mathbf{r}_e$, the elevation angle El is related to the central angle ψ by

$$El = \psi - 90^\circ \tag{2.33}$$

By the law of sines we have

$$\frac{r_s}{\sin(\psi)} = \frac{d}{\sin(\gamma)} \tag{2.34}$$

Combining the last three equations yields

$$\begin{aligned} \cos(El) &= \frac{r_s \sin(\gamma)}{d} \\ &= \frac{\sin(\gamma)}{\left[1 + \left(\frac{r_e}{r_s}\right)^2 - 2\left(\frac{r_e}{r_s}\right)\cos(\gamma)\right]^{1/2}} \end{aligned} \tag{2.35}$$

Equations (2.35) and (2.31) permit the elevation angle El to be calculated from knowledge of the subsatellite point and the earth station coordinates, the orbital radius r_s, and the earth's radius r_e. An accurate value for the average earth radius is 6378.137 km[1] but a common value used in approximate determinations is 6370 km.

Azimuth Angle Calculation

Because the earth station, the center of the earth, the satellite, and the subsatellite point all lie in the same plane, the azimuth angle *Az* from the earth station to the satellite is the same as the azimuth from the earth station to the subsatellite point. This is more difficult to compute than the elevation angle because the exact geometry involved depends on whether the subsatellite point is east or west of the earth station, and in which of the hemispheres the earth station and the subsatellite point are located. The problem simplifies somewhat for geosynchronous satellites, which will be treated in the next section. For the general case, in particular for constellations of LEO satellites, the tedium of calculating the individual look angles on a second-by-second basis has been considerably eased by a range of commercial software packages that exist for predicting a variety of orbital dynamics and intercept solutions (see reference 13 for a brief review of 10 software packages available in early 2001).

Specialization to Geostationary Satellites

For most geostationary satellites, the subsatellite point is on the equator at longitude l_s, and the latitude L_s is 0. The geosynchronous radius r_s is 42,164.17 km[1]. Since L_s is zero, Eq. (2.31) simplifies to

$$\cos(\gamma) = \cos(L_e)\cos(l_s - l_e) \tag{2.36}$$

Substituting r_s = 42,164.17 km and r_e = 6,378.137 km in Eqs. (2.32) and (2.35) gives the following expressions for the distance d from the earth station to the satellite and the elevation angle El at the earth station

$$d = 42{,}164.17[1.02288235 - 0.30253825\cos(\gamma)]^{1/2}\ \text{km} \tag{2.37}$$

$$\cos(El) = \frac{\sin(\gamma)}{[1.02288235 - 0.30253825\cos(\gamma)]^{1/2}} \tag{2.38}$$

For a geostationary satellite with an orbital radius of 42,164.17 km and a mean earth radius of 6378.137 km, the ratio $r_s/r_e = 6.6107345$ giving

$$El = \tan^{-1}[(6.6107345 - \cos\gamma)/\sin\gamma] - \gamma \tag{2.39}$$

To find the azimuth angle, an intermediate angle α must first be found. The intermediate angle α permits the correct 90° quadrant to be found for the azimuth since the azimuthal angle can lie anywhere between 0° (true north) and clockwise through 360° (back to true north again). The intermediate angle is found from

$$El = \tan^{-1}\left[\frac{\left(\cos\gamma - \frac{r_e}{r_s}\right)}{\sin\gamma}\right]$$

$$\alpha = \tan^{-1}\left[\frac{\tan|(l_s - l_e)|}{\sin(L_e)}\right] \tag{2.40}$$

Having found the intermediate angle α, the azimuth look angle Az can be found from:

Case 1: Earth station in the Northern Hemisphere with

(a) Satellite to the SE of the earth station: $Az = 180° - \alpha$ (2.41a)

(b) Satellite to the SW of the earth station: $Az = 180° + \alpha$ (2.41b)

Case 2: Earth station in the Southern Hemisphere with

(c) Satellite to the NE of the earth station: $Az = \alpha$ (2.41c)

(d) Satellite to the NW of the earth station: $Az = 360° - \alpha$ (2.41d)

Example : GEOSTATIONARY SATELLITES

We will concentrate on the **GEOSTATIONARY CASE**
This will allow some simplifications in the formulas

- SUB-SATELLITE POINT
  (Equatorial plane, Latitude $\mathbf{L_s} = 0^o$
  Longitude $\mathbf{l_s}$)
- EARTH STATION LOCATION
  Latitude $\mathbf{L_e}$
  Longitude $\mathbf{l_e}$

THE CENTRAL ANGLE γ - GEO

The original calculation previously shown:

$\cos(\gamma) = \cos(Le)\cos(L_s)\cos(l_s - l_e) + \sin(L_e)\sin(L_s)$

Simplifies using $L_s = 0^o$ since the satellite is over the equator:

$\cos(\gamma) = \cos(Le)\cos(l_s - l_e)$

ELEVATION CALCULATION – GEO 1

Using r_s = 42,164 km and r_e = 6,378.14 km gives

d = 42,164 [1.0228826 - 0.3025396 cos(γ)]$^{1/2}$ km

$$\cos(El) = \frac{\sin(\gamma)}{[1.0228826 - 0.3025396\cos(\gamma)]^{1/2}}$$

NOTE: These are slightly different numbers than those given in equations (2.67) and (2.68), respectively, due to the more precise values used for r_s and r_e

Visibility Test

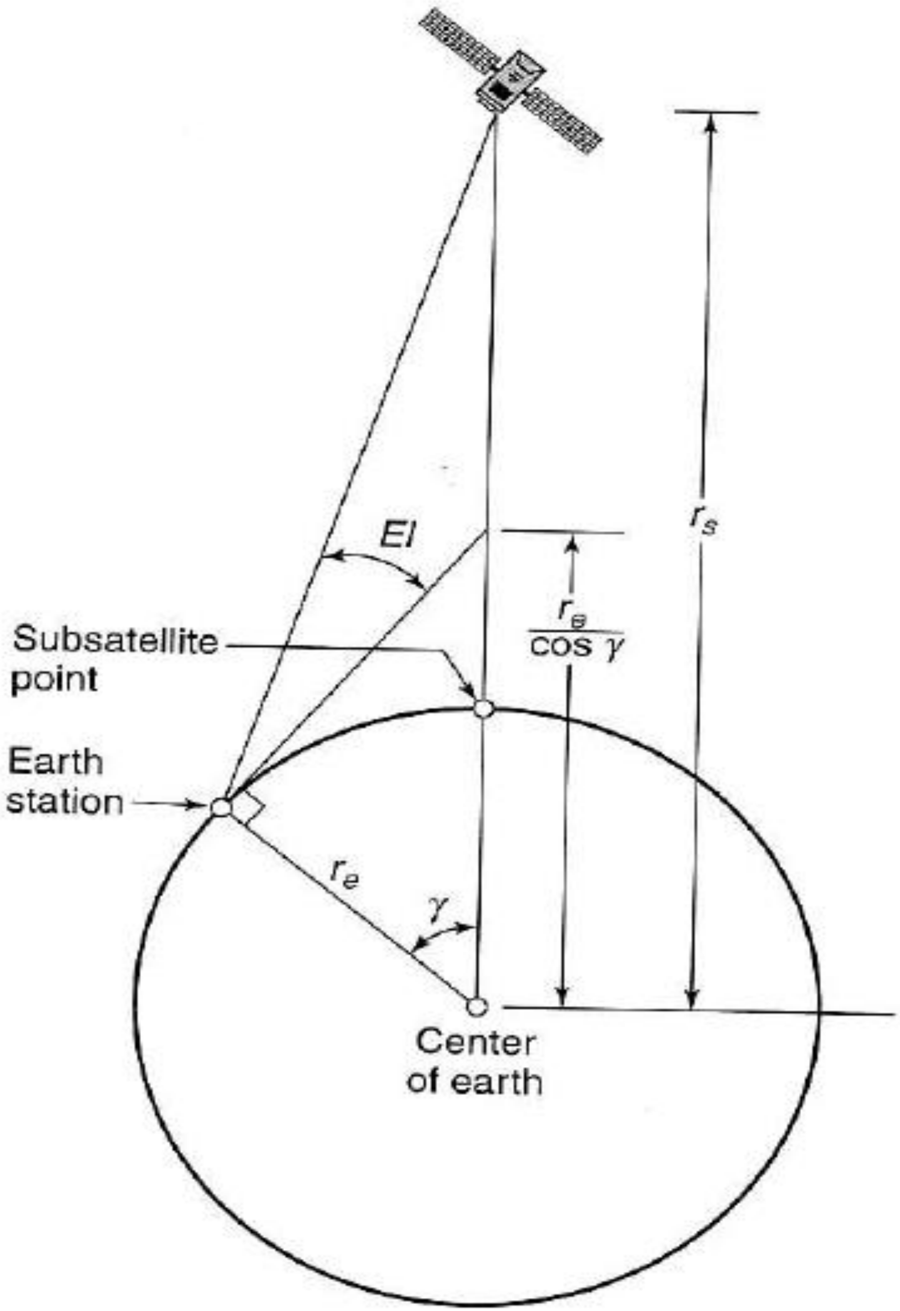


FIGURE 2.13 The geometry of the visibility calculation. The satellite is said to be visible from the earth station if the elevation angle *El* is positive. This requires that the orbital radius r_s be greater than the ratio $r_e/\cos(\gamma)$ where r_e is the radius of the earth and γ is the central angle.

Visibility Test

For a satellite to be visible from an earth station, its elevation angle *El* must be above some minimum value, which is at least 0°. A positive or zero elevation angle requires that (see Figure 2.13)

$$r_s \geq \frac{r_e}{\cos(\gamma)} \tag{2.42}$$

This means that the maximum central angular separation between the earth station and the subsatellite point is limited by

$$\gamma \leq \cos^{-1}\left(\frac{r_e}{r_s}\right) \tag{2.43}$$

For a nominal geostationary orbit, the last equation reduces to $\gamma \leq 81.3°$ for the satellite to be visible.

EXAMPLE 2.2.1 ***Geostationary Satellite Look Angles***

An earth station situated in the Docklands of London, England, needs to calculate the look angle to a geostationary satellite in the Indian Ocean operated by Intelsat. The details of the earth station site and the satellite are as follows:

Earth station latitude and longitude are 52.0° N and 0°.
Satellite longitude (subsatellite point) is 66.0° E

Step 1: Find the central angle γ

$$\cos(\gamma) = \cos(L_e)\cos(l_s - l_e)$$
$$= \cos(52.0)\cos(66.0) = 0.2504$$

yielding $\gamma = 75.4981°$
The central angle γ is less than 81.3° so the satellite is visible from the earth station.

Step 2: Find the elevation angle *El*

$$El = \tan^{-1}[(6.6107345 - \cos\gamma)/\sin\gamma] - \gamma$$
$$= \tan^{-1}[(6.6107345 - 0.2504)/\sin(75.4981)] - 75.4981$$
$$= 5.847°$$

Step 3: Find the intermediate angle α

$$\alpha = \tan^{-1}\left[\frac{\tan|(l_s - l_e)|}{\sin(L_e)}\right]$$
$$= \tan^{-1}[(\tan(66.0 - 0))/\sin(52.0)]$$
$$= 70.667°$$

Step 4: Find the azimuth angle
The earth station is in the Northern Hemisphere and the satellite is to the southeast of the earth station. From Eq. (2.41a), this gives

$$Az = 180° - \alpha = 180 - 70.667 = 109.333°\text{(clockwise from true north)}$$ ■

EXAMPLE OF A GEO LOOK ANGLE ALCULATION (1)

FIND the **Elevation** and **Azimuth** Look Angles for the following case:

Earth Station Latitude	52^o N	London, England Dockland region
Earth Station Longitude	0^o	
Satellite Latitude	0^o	Geostationary INTELSAT IOR Primary
Satellite Longitude	66^o E	

EXAMPLE OF A GEO LOOK ANGLE ALCULATION (2)

Step 1. Find the central angle γ

$\cos(\gamma) = \cos(L_e)\ \cos(l_s\text{-}l_e) = \cos(52)\ \cos(66) = 0.2504$

yielding $\gamma = 75.4981^o$

Step 2. Find the elevation angle *El*

$$El = \tan^{-1}\left[\frac{\left(\cos\gamma - \frac{r_e}{r_s}\right)}{\sin\gamma}\right]$$

$El = \tan^{-1}[\ (0.2504 – (6378.14\ /\ 42164))\ /\ \sin\ (75.4981)\ ]$
$= 5.85^o$

EXAMPLE OF A GEO LOOK ANGLE ALCULATION (3)

Step 3. Find the intermediate angle, α

$$\alpha = \tan^{-1}\left[\frac{\tan\left|\left(l_s - l_e\right)\right|}{\sin\left(L_e\right)}\right]$$

$= \tan^{-1}$ [(tan (66 - 0)) / sin (52)]

= 70.6668

The earth station is in the Northern hemisphere and the satellite is to the South East of the earth station. This gives

$Az = 180^o - \alpha$

$= 180 - 70.6668 = 109.333^o$ (clockwise from true North)

ANSWER: The look-angles to the satellite are

Elevation Angle = 5.85^o

Azimuth Angle = 109.33^o