EENG 470
Satellite Communications

Lecture # 5
2.3 Orbit control and Launching Methods

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Chapter 2- (orbital mechanics and launchers) part III

2.5 LAUNCHES AND LAUNCH VEHICLES

A satellite cannot be placed into a stable orbit unless two parameters that are uniquely coupled together—the velocity vector and the orbital height—are simultaneously correct. There is little point in obtaining the correct height and not having the appropriate velocity component in the correct direction to achieve the desired orbit. A geostationary satellite, for example, must be in an orbit at a height of 35,786.03 km above the surface of the earth (42,164.17-km radius from the center of the earth) with an inclination of zero degrees, an ellipticity of zero, and a velocity of 3074.7 m/s tangential to the earth in the plane of the orbit, which is the earth's equatorial plane. The further out from the earth the orbit is, the greater the energy required from the launch vehicle to reach that orbit. In any earth satellite launch, the largest fraction of the energy expended by the rocket is used to accelerate the vehicle from rest until it is about 20 miles (32 km) above the earth. To make the most efficient use of the fuel, it is common to shed excess mass from the launcher as it moves upward on launch: this is called staging. Figure 2.15 gives a schematic of a Proton launch from the Russian Baikonur complex at Kazakhstan, near Tyuratam.

Most launch vehicles have multiple stages and, as each stage is completed, that portion of the launcher is expended until the final stage places the satellite into the desired trajectory. Hence the term: *expendable launch vehicle (ELV)*. The Space Shuttle, called the *Space Transportation System (STS)* by NASA, is partially reusable. The solid rocket boosters are recovered and refurbished for future missions and the shuttle vehicle itself is flown back to earth for refurbishment and reuse. Hence the term: *reusable launch vehicle (RLV)* for such launchers. More advanced launch vehicles are being developed that would provide both *single stage to orbit (SSTO)* and RLV capabilities. The NASA series of X-33 and X-34 test vehicles form the public portion of this quest (see the NASA home page[4]).

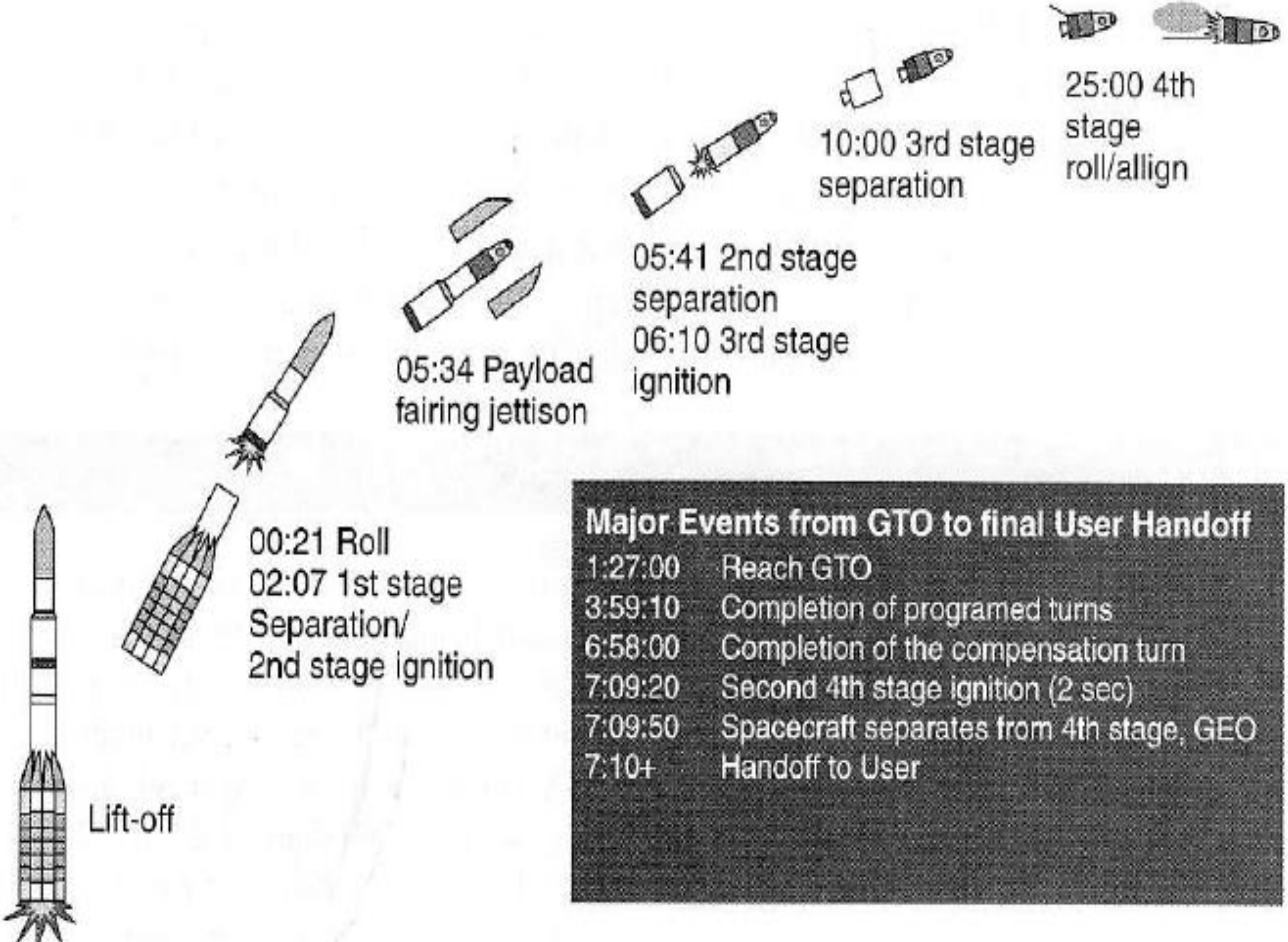


FIGURE 2.15 Schematic of a Proton launch (after reference 5).

- **Spacecraft Development**: The second decade of the 21st century witnessed a surge in spacecraft development, particularly SmallSats, alongside advancements in launch vehicles.

- **Launch Statistics**: In 2017, a total of 345 satellites were launched, with 212 being commercially procured CubeSats for earth observation and meteorology. US entities dominated with over a third of commercial launches.

- **Evolution of Launch Vehicles**: Launch vehicles evolved not only to reliably place satellites into orbit but also to recover and reuse major elements, reducing costs.

- **Pegasus**: The first successful privately developed launch vehicle, launched from a Lockheed 1011 TriStar at around 40,000 ft. It was operational since 1990 and remains significant despite its last launch in 2016. **Cost Comparison**: Pegasus launched 93 satellites into LEO at approximately $40M per launch, compared to SpaceX Falcon 9's advertised cost of $62M for larger payloads.

- **New Air-Launch Proposals**: Stratolaunch, the world's largest aircraft, and a Boeing 747 retired by Virgin Galactic are new proposals for air-launching rockets into LEO. Stratolaunch may be restricted to specific airfields, while Virgin Galactic's concept could utilize various airfields globally.

- **Routine Launches**: Satellite launches have become routine, though success requires coordination of numerous factors.

- **Launch Vehicles**: Most rockets operate in multiple stages, with each stage expended until the final stage places the satellite into the desired trajectory.

- **ELVs and EELVs**: Common terms for such rockets are Expendable Launch Vehicles (ELVs) and Evolved Expendable Launch Vehicles (EELVs).

- **Launch Site Logistics**: Russia desired both manufacturing and launch operations within its borders, leading to the shift away from Baikonur, Kazakhstan.

- **Equatorial Launch Advantage**: Equatorial launches benefit from the Earth's rotational velocity, reducing energy needs by about 6%. ESA chose Kourou, French Guiana, for its launch site due to its proximity to the equator.

- **Sea Launch Concept**: Sea Launch utilized a floating platform towed to the equator for launches, later purchased by Russia with plans to launch Zenit rockets.

- **Payload Limitations**: Payload capabilities decrease as orbital inclination increases from equatorial orbits.

- **Orbital Direction**: Orbits in the same direction as Earth's spin are prograde, while those in the opposite direction are retrograde.

•**Latitude and Orbit Inclination**: A satellite launched into a prograde orbit from a latitude of Φ degrees will enter an orbit with an inclination of Φ degrees to the equator.

•**Velocity Increment for Geostationary Orbit**: To place a satellite from a non-equatorial orbit into a geostationary orbit, a significant velocity increment is required to reorient the orbit into the Earth's equatorial plane.

•**Example**: A satellite launched from Cape Canaveral at 28.5°N latitude requires a velocity increment of 366 m/s to attain an equatorial orbit from a geosynchronous orbit plane of 28.5°.

•**Impact of Launch Site Latitude**: Launch sites closer to the equator, such as the Guiana Space Center in French Guiana (latitude about 5°N) and Sea Launch from the equator, require less fuel for inclination changes by the apogee kick motor (AKM).

•**Significance of Equatorial Launch**: Launching close to the equator for a satellite intended for a geostationary orbit significantly reduces the energy needed to change the orbit's plane from a non-zero inclination to zero.

•**Fuel Efficiency**: Changing the plane of orbit requires approximately 10 times more fuel than changing velocity in the same plane for a given angular change.

Expendable Launch Vehicles (ELVs)

1998 was an important year for ELVs: it was the year when the number of commercial launches in the United States surpassed the number of government launches for the first time[9]. The gap between commercial and government launches will continue to grow. The Teal Group estimated in mid-1999 that 1447 satellites would be launched worldwide between 2000 and 2009 on 850 to 900 launch vehicles[10]. At an average cost of $100 M per launch, this represents a business worth about $ 90 B over 10 years. Of these 1447 satellites, 893 were considered commercial ventures with the remainder split between military and civilian government spacecraft. There is therefore a healthy market for ELVs and a number of companies, consortia, and national entities are seeking to enter this expanding field. Reference 15 contains a good survey of the ELVs being developed for the twenty-first century. Figure 2.16 presents a rough comparison between the main launch vehicles used for Geostationary Transfer Orbit (GTO) injection during the 1990s, plus the Ariane 5 launcher. The 1996 pricing of these vehicles is shown in Figure 2.17. Not included in these data are the advanced Chinese launch vehicles being developed for both unmanned and manned missions in the twenty-first century. The largest of these Chinese launch vehicles rivals the Ariane 5 vehicle with a geostationary transfer orbit capability of 26,000 lb.

- **Commercial Dominance**: In 1998, commercial launches in the United States surpassed government launches for the first time, marking a significant shift in the space industry (Dekok 1999).

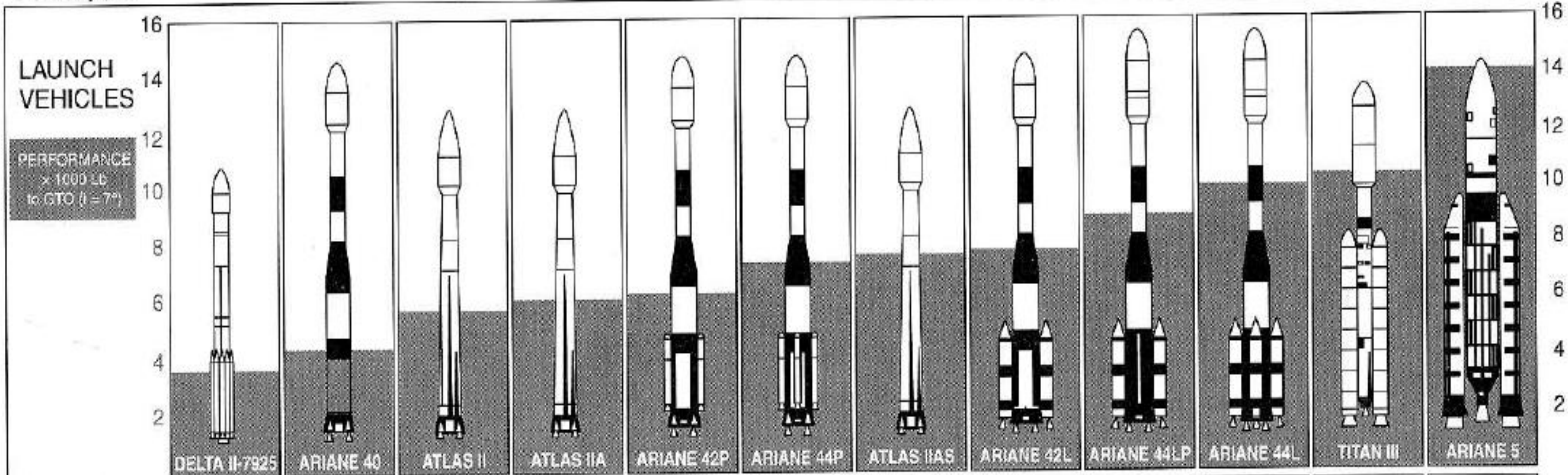
- **Global Satellite Launches**: A total of 81 countries have successfully launched satellites into Low Earth Orbit (LEO), though the majority of launches are conducted by a few countries such as the United States, Russia, and the European Space Agency (ESA).

- **National Space Programs**: Only 11 countries have independently developed and launched their own satellites and rockets, with New Zealand being the most recent addition as of July 2018 (Wikipedia 2018c).

- **Satellite Persistence**: While most satellites have re-entered the Earth's atmosphere, some, like the first successful US satellite, Explorer 1 launched in 1958, remain in orbit.

- **Satellite Launch Statistics**: Estimates suggest that over 12,230 satellites have been launched historically, with around 4,635 currently in Earth orbit according to the United Nations Office of Outer Space Affairs (UNOOSA) as of November 2017 (Pixialytics.com 2018).

- **Shift Towards SmallSats**: Adoption of electric propulsion and digital payloads for SmallSats has reduced their average mass to less than 50 kg, driving a healthy market for Expendable Launch Vehicles (ELVs). **Market Expansion**: Companies, consortia, and national entities worldwide, particularly in the United States, are seeking to enter the growing ELV market (Klotz 2019).



	DELTA II-7925	ARIANE 40	ATLAS II	ATLAS IIA	ARIANE 42P	ARIANE 44P	ATLAS IIAS	ARIANE 42L	ARIANE 44LP	ARIANE 44L	TITAN III	ARIANE 5
1996 Market Prices, $M	$50	$60	$80	$90	$75	$95	$105	$85	$95	$110	–	$120
First Flight	1990	1990	1991	1992	1991	1991	1993	1993	1988	1989	1989	1996
Performance Kg (lb)												
GTO, i = 7 deg 200 × 35,786 Km (108 × 19,323 NM)	1,820 (1) (4,010)	2,050 (4,520)	2,810 (2) (6,200)	3,050 (2) (6,710)	2,840 (6,260)	3,320 (7,320)	3,700 (2) (8,150)	3,380 (7,450)	4,060 (8,950)	4,520 (9,965)	5,000 (1) (11,000)	5,000-6,700 (11-13,900)
Payload Max Diameter m (ft)	2.54 (8.3)	3.65 (12)	3.65 (12)	3.65 (12)	3.65 (12)	3.65 (12)	3.65 (12)	3.65 (12)	3.65 (12)	3.65 (12)	3.65 (12)	4.57 (15)
Launch Site (Projected)	CCAS (VAFB)	KOUROU	CCAS (VAFB)	CCAS (VAFB)	KOUROU	KOUROU	CCAS (VAFB)	KOUROU	KOUROU	KOUROU	CCAS	KOUROU

(1) DELTA & TITAN @ i = 28 deg
(2) ATLAS GTO REFERENCE ORBIT IS 167 × 35,788 KM (90 × 19,324 NM), i = 27 deg

FIGURE 2.16 Representative ELVs (after reference 5). CCAS, Cape Canaveral Air Station; VAFB, Vandenburg Air Force Base.

- **Rising Rocket Launch Sites**: There are currently 22 active rocket launch sites in the United States, with the number expected to increase, especially with the emergence of smaller rockets for SmallSat launches (FAA.gov 2018a).

- **Air-launched Rockets**: Several airfields will also be utilized for air-launched rockets, further adding to the complexity of air traffic control over the United States. **Air Traffic Control Challenges**: The increasing number of rocket launches complicates air traffic control, requiring real-time monitoring of aircraft likely to fly close to scheduled launches. In 2018, there were 42,000 Federal Aviation Authority (FAA)-controlled aircraft flights per day, not including smaller aircraft from non-towered airports (FAA.gov 2018b; AOPA.org 2018).

- **Payload Categories**: Space launch systems are categorized into small lift (<2000 kg), medium lift (>2000 and <22,000 kg), heavy lift (>22,000 and <40,000 kg), and super heavy lift (>40,000 kg).

- **Rocket Details**: Tables 2.3 to 2.6 provide details on various rockets used or proposed for launching satellites, while Table 2.7 focuses on air-launched vehicles, and Table 2.8 lists sub-orbital tourist rockets. Table 2.9 offers a price comparison of launch vehicles for Low Earth Orbit (LEO) satellites.

- **High Altitude Platforms (HAPs)**: Proposals for deploying HAPs for emergency communications over disaster-stricken areas are also noted, involving tethered balloons or semi-rigid inflatable craft in HALO orbits (Tables 2.4–2.8).

Table 2.3 Small lift launchers

Rocket	Height	Payload to LEO	Cost per launch	First launch
ISRO PSLV rocket	144 ft. 44 m	3 800 kg 8 400 lb	US$21M to US$31M per launch	Latest version flew 22 October 2008
Rocket Labs Electron rocket	56 ft. 17 m	100–225 kg 220–496 lb	US$5–6M per launch	21 January 2018
Vega	98 ft. 30 m	1 500 kg	US$37M per launch	13 February 2002
Minotaur C[a] (Taurus before)	92 ft. 28 m	1 590 kg 3 500 lb	US$40M to US$55M	13 March 1994
SS-520-S[b]	9.54 m 31.3 ft	3 kg 66 lb	~US$1M	3 February 2018

[a] The Minotaur C is a vertically launched version of the winged Pegasus launch vehicle.
[b] The SS-520-S is a converted sounding rocket that is launched along a rail. The first flight achieved orbit in less than 4.5 minutes.

Table 2.4 Medium lift launchers

Rocket	Height	Payload to LEO	Cost per launch	First launch
Ariane 5[a]	179 ft. 54.7 m	21 000 kg 46 297 lb	US$165M	9 March 2008
Ariane 6	207 ft. 63 m	21 500 kg 47 400 lb	US$100M	First launch scheduled for 2020
Soyuz[b]	150 ft. 45.6 m	6 450 kg 14 220 lb	US$81M	28 November 1966
Zenit 2[c]	187 ft. 57 m	13 740 kg 30 290 lb	~US$55M	13 April 1985

[a] There were four variants before Ariane 5, starting with Ariane 1, first launched 24 December 1974.
[b] The Soyuz rocket is the launch vehicle used to send astronauts (US, Russian, and other nations) to the ISS. The price per astronaut varies but was US$75M in mid-2018. The payload capability will increase for a Soyuz launch from Kourou.
[c] The price and payload capability given is for a Zenit 2 launched from Baikonur.

Table 2.5 Heavy lift launchers

Rocket	Height	Payload to LEO	Cost per launch	First launch
Falcon 9[a]	233 ft. 71 m	22 800 kg 50 300 lb	US$62M	7 June 2010
Proton M	191 ft. 58.2 m	23 000 kg 51 000 lb	US$65M	9 March 2008
Delta heavy	236 ft. 72 m	28 970 kg 63 470 lb	US$350M	21 December 2004

[a] There are a number of *blocks* of Falcon 9 rockets; the most recent (2018) is Block 5. This is the version slated to fly the Falcon Crew capsule. The Block 5 rockets are designed to fly 10 times. The first recovery of a Falcon 9 first stage took place on 21 December 2015.

Table 2.6 Super heavy lift launchers

Rocket	Height	Payload to LEO	Cost per launch	First launch
Falcon heavy	230 ft. 70 m	63 800 kg 140 700 lb	US$90M	6 February 2018
New glenn 3-stage	312 ft. 72 m	45 000 kg 99 000 lb	Not available	Scheduled for 2020
Space launch system B2	365 ft. 95 m	130 000 kg 286 601 lb	~US$500M	Scheduled for 2020
Saturn 5	363 ft. 110.6 m	140 000 kg 310 000 lb	US$1.16B	7 November 1967
Long March 9[a]	331 ft. 101 m	140 000 kg 310 000 lb	US$40M to US$55M	Scheduled for 2020
BFR[b]	348 ft. 106 m	250 000 kg 550 000 lb	Not available	First sub-orbital test flights scheduled for 2019

[a] There has been a long series of Long March rockets. The latest, Long March 11 will be all solid fueled and the complete rocket can be stored for long periods, leading to speculation that it is designed for rapid response.
[b] The BFR is either a two stage vehicle (data for which are in the table above) or it can be just a single stage. In a lightly loaded version, it can achieve orbit without the booster stage, leading to a single-stage-to-orbit rocket. The two stage version can also be configured to carry 200 passengers anywhere on the earth in 90 minutes.

Table 2.7 Aircraft launchers

Aircraft	Rocket	Payload to LEO	Cost per launch	First launch
VOX Space[a]	Launcher one	~500 kg ~1 100 lb	Not known but competitive	Launcher one has yet to be tested or flown
Stratolaunch[b]	Not yet available	Small lift to medium lift	Not known but competitive	First taxi run on 21 December 2017

[a]Virgin Orbit X (VOX) consists of a Boeing 747 mother ship that carries a two stage rocket, *Launcher One*, under one wing.
[b]Stratolaunch, founded in 2011 by Paul Allen, is being built by Scaled Composites, a Northrop Grumman subsidiary. It has two bodies and six engines. It is being designed to carry up to three small launchers (similar to *Pegasus*) for small lift satellites, and one larger launcher to orbit medium lift satellites.

Table 2.8 Sub-orbital tourist vehicles

Aircraft	Rocket	Payload to LEO	Cost per launch	First launch
Blue shepard[a]	~22 m (with capsule)	Six passengers to more than 75 miles (48 km)	~US$200 000 per passenger	29 April 2015
Virgin galactic VSS unity[b]	Air-dropped from White Knight mother ship	2 pilots and 6 passengers to >50 miles (80 km)	US$250 000 per passenger	6 April 2018

[a]Blue Shepard is a fully reusable, single stage rocket. It was the first rocket to successfully soft land back at the launch site.
[b]VSS Unity is the second SpaceShip Two to be completed; the first crashed in February 2016.

Table 2.9 Comparison of the price per kg to launch a satellite into LEO

Launch vehicle	Price per kg to LEO	Price per kg to GTO
SS-520-S[a]	US$333 300	Not capable of GTO
Rocket labs electron rocket[b]	US$26 650 to US$50 000	Not capable of GTO
Minotaur C (taurus before)	US$25 150 to US$34 590	Not capable of GTO
Vega	US$24 650	US$67 790
Soyuz	US$12 560	US$34 540
Delta heavy	US$12 080	US$33 220
ISRO PSLV rocket	US$5 525–8 150	US$15 190–22 410
Ariane 5	US$7 850	US$21 590
New glenn 3-stage[c]	US$5 555	US$15 280
Ariane 6	US$4 650	US$12 790
Zenit 2	US$4 000	US$11 000
Space launch system B2	US$3 850	US$10 590
Falcon 9	US$2 720	US$7 480
Proton M	US$2 825	US$7 770
Falcon heavy	US$1 410	US$3 880
BFR[c]	US$1 000	US$2 750
Long March 9[c]	US$535	US$1 470

[a] The launch cost is only US$1 000 000 but the 3 kg payload drives up the per kg cost.
[b] The launch cost is only US$5–6M, but the payload is quite small, hence the high cost per kg.
[c] Assumed US$250 000 000 per launch.

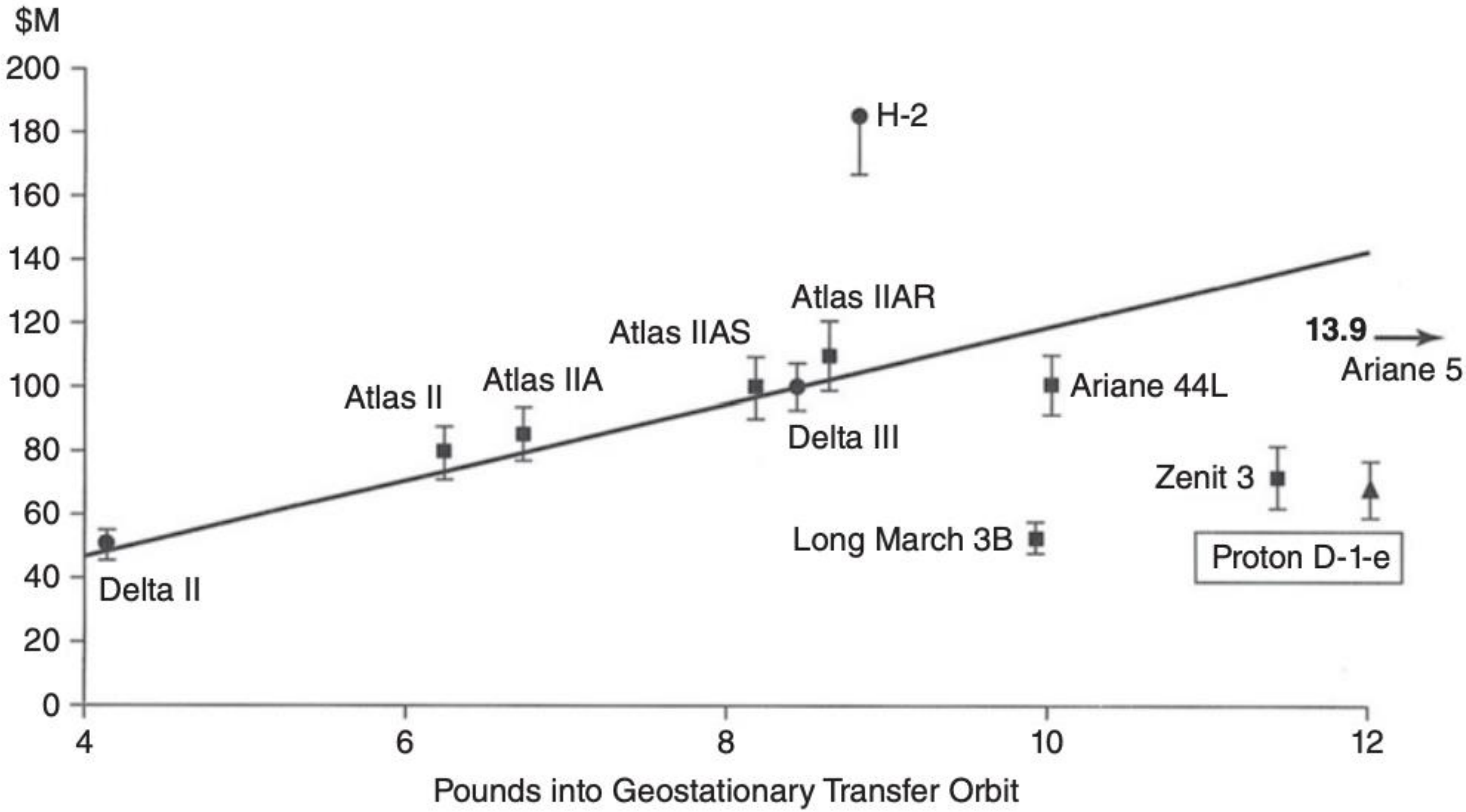


Figure 2.16 Launch vehicle market price versus performance, 1996 prices. After (Walsh and Groves 1997). The launch vehicles have been normalized to a launch into geostationary transfer orbit at an inclination of 28°. The trend line is at US$12 000 per pound. Note that Long march, Zenit, and Proton are well below this trend line, mainly due to aggressive pricing objectives to break into a market long dominated by US and European launchers.

Table 2.10 Some launch vehicle selection factors (Walsh and Groves 1997)

Price/cost
Reliability
Recent launch success/failure history
Dependable launch schedule
Urgency of your launch requirements
Performance
Spacecraft fit to launcher (size, acoustic, and vibration environment)
Flight proven (see recent launch history)
Safety issues
Launch site location
Availability
What is the launcher backlog of orders?
What is the launch site backlog of launchers?
Market issues
What will the market bear at this particular time?

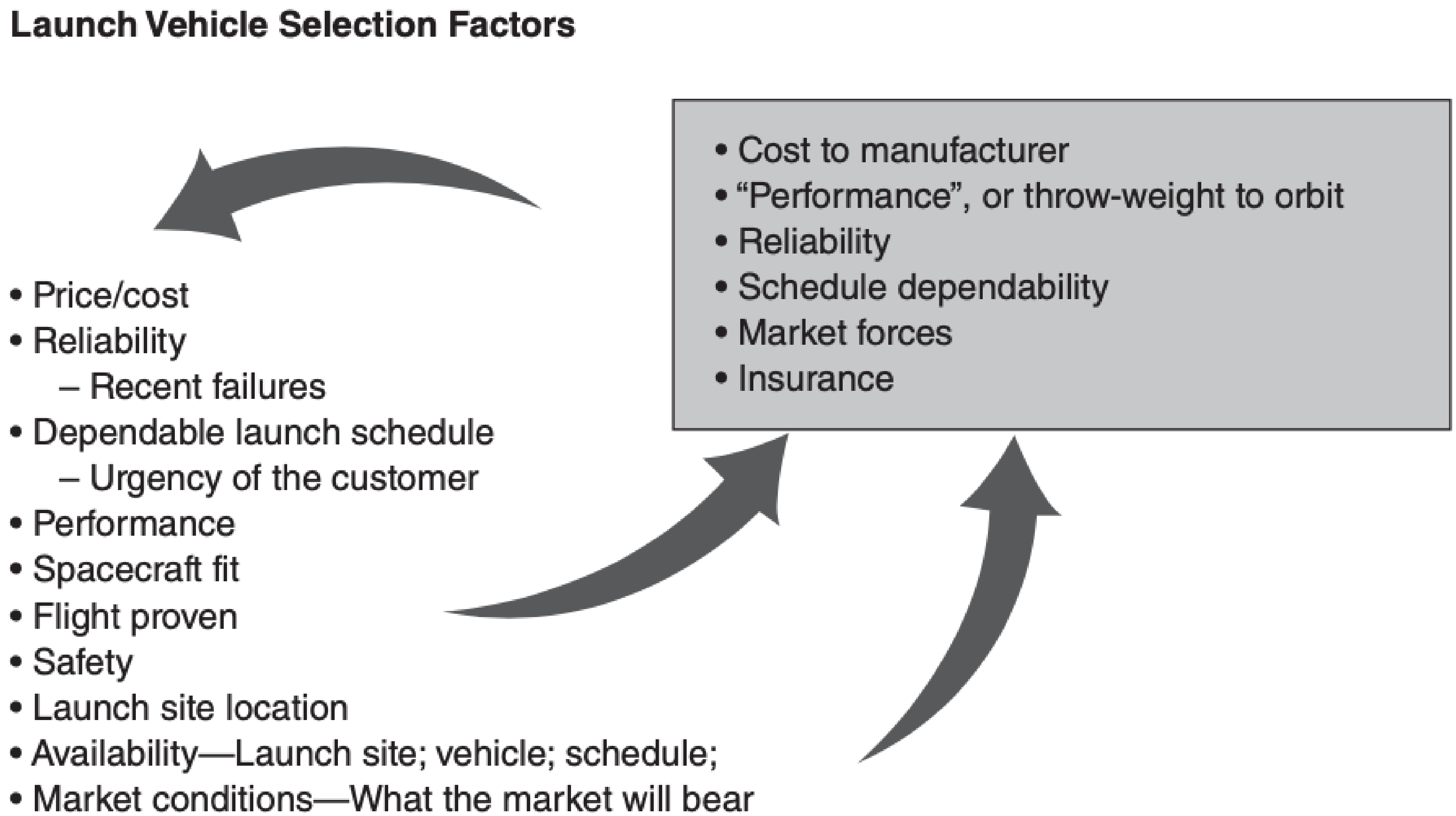


Figure 2.17 Schematic of the decision-making process to select a rocket for a given satellite requirement. After (Walsh and Groves 1997).

2.12 Placing Satellites Into Geostationary Orbit

2.12.1 Geostationary Transfer Orbit and AKM

The initial approach to launching geostationary satellites was to place the spacecraft, with the final rocket stage still attached, into LEO. After a couple of orbits, during which the orbital elements are measured, the final stage is re-ignited and the spacecraft is launched into a GTO. The GTO has a perigee that is the original LEO orbit altitude and an apogee that is the GEO altitude. Figure 2.18 illustrates the process. The position of the apogee point is close to the orbital longitude that would be the in-orbit test location of the satellite prior to it being moved to its operational position. Again, after a few orbits in the GTO while the orbital elements are measured, a rocket motor (usually contained within the satellite itself) is ignited at apogee and the GTO is raised until it is a circular, geostationary orbit. Since the rocket motor fires at apogee, it is commonly referred to as the AKM. The AKM is used both to circularize the orbit at GEO and to remove any inclination error so that the final orbit of the satellite is very close to geostationary.

The first successful GEO satellite was Syncom, launched in 1963. Hughes Corporation built the satellite and the spacecraft was spin stabilized while it was in GTO. In this way, the satellite was correctly aligned for the apogee motor firing. The apogee motor was fairly powerful and the apogee burn was only for a few minutes. During this apogee burn, all of the satellite's deployable elements (e.g., solar panels, antennas) were stowed and locked in place to avoid damage while the AKM accelerated the satellite to GEO. Hughes patented the technique of spin stabilizing the spacecraft in GTO. To avoid infringing this patent, other satellite manufacturers developed a new way to achieve GEO, known as a slow orbit raising technique.

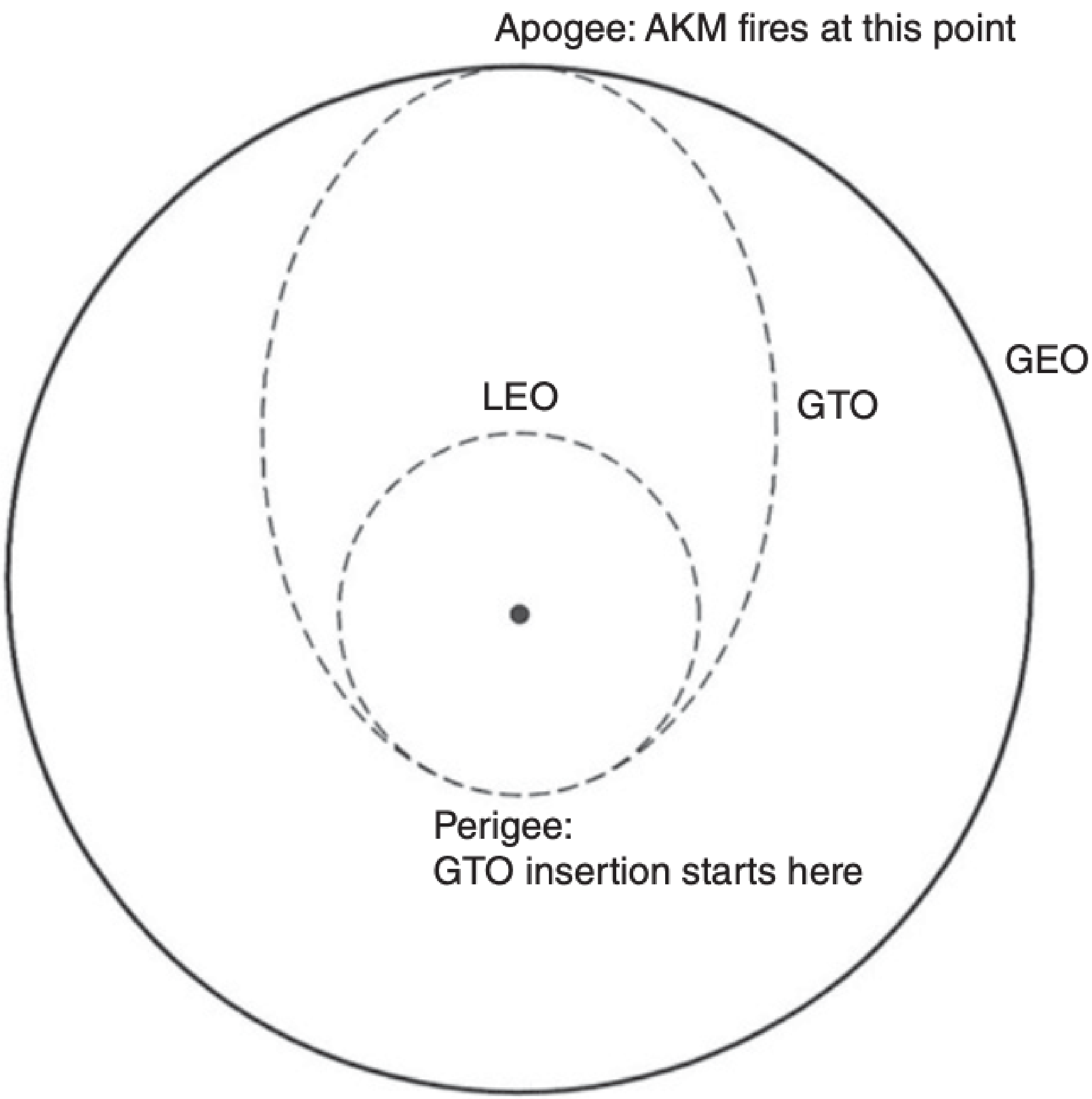


Figure 2.18 Illustration of transfer to geostationary orbit using an apogee kick motor (AKM). (Not to scale.) The spacecraft and final rocket stage are placed in low earth orbit (LEO). After careful orbital determination measurements, the final rocket stage is fired and the satellite placed in an elliptical geostationary transfer orbit (GTO) with apogee at geostationary altitude. The spacecraft is then separated from the rocket casing. After further careful orbital determination measurements, the AKM is fired several times to make the orbit circular, in the earth's equatorial plane, and at the correct altitude. The satellite is now in geostationary orbit (GEO).

2.12.2 Geostationary Transfer Orbit With Slow Orbit Raising

In this procedure, rather than employ an AKM that imparts a vigorous acceleration over a few minutes, the spacecraft thrusters are used to raise the orbit from GTO to GEO over a number of burns. Since the spacecraft cannot be spin stabilized during the GTO (so as not to infringe the Hughes patent), many of the satellite elements are deployed while in GTO, including the solar panels. The satellite normally has two power levels of thrusters: one for more powerful orbit raising maneuvers and one for on-orbit (low thrust) maneuvers. Since the thrusters take many hours of operation to achieve the geostationary orbit, the perigee of the orbit is gradually raised over successive thruster firings. The thruster firings occur symmetrically about the apogee although they could occur at the perigee as well. The burns are typically 60–80 minutes long on successive orbits and up to six orbits can be used. Figure 2.19 illustrates the process.

In the above two cases, AKM and Slow Orbit Raising, the GTO may be a modified orbit with the apogee well above the required altitude for GEO. The excess energy of the orbit due to the higher-than-necessary altitude at apogee can be traded for energy required to raise the perigee. The net energy to circularize the orbit at GEO is therefore less and the satellite can retain more fuel for on-orbit operations. The use of an initial orbit insertion well above that needed for GEO occurs when the launch vehicle has the ability to add additional fuel at launch (due to a lighter satellite or the rocket has increased efficiency due to developments since the original launch agreement was signed).

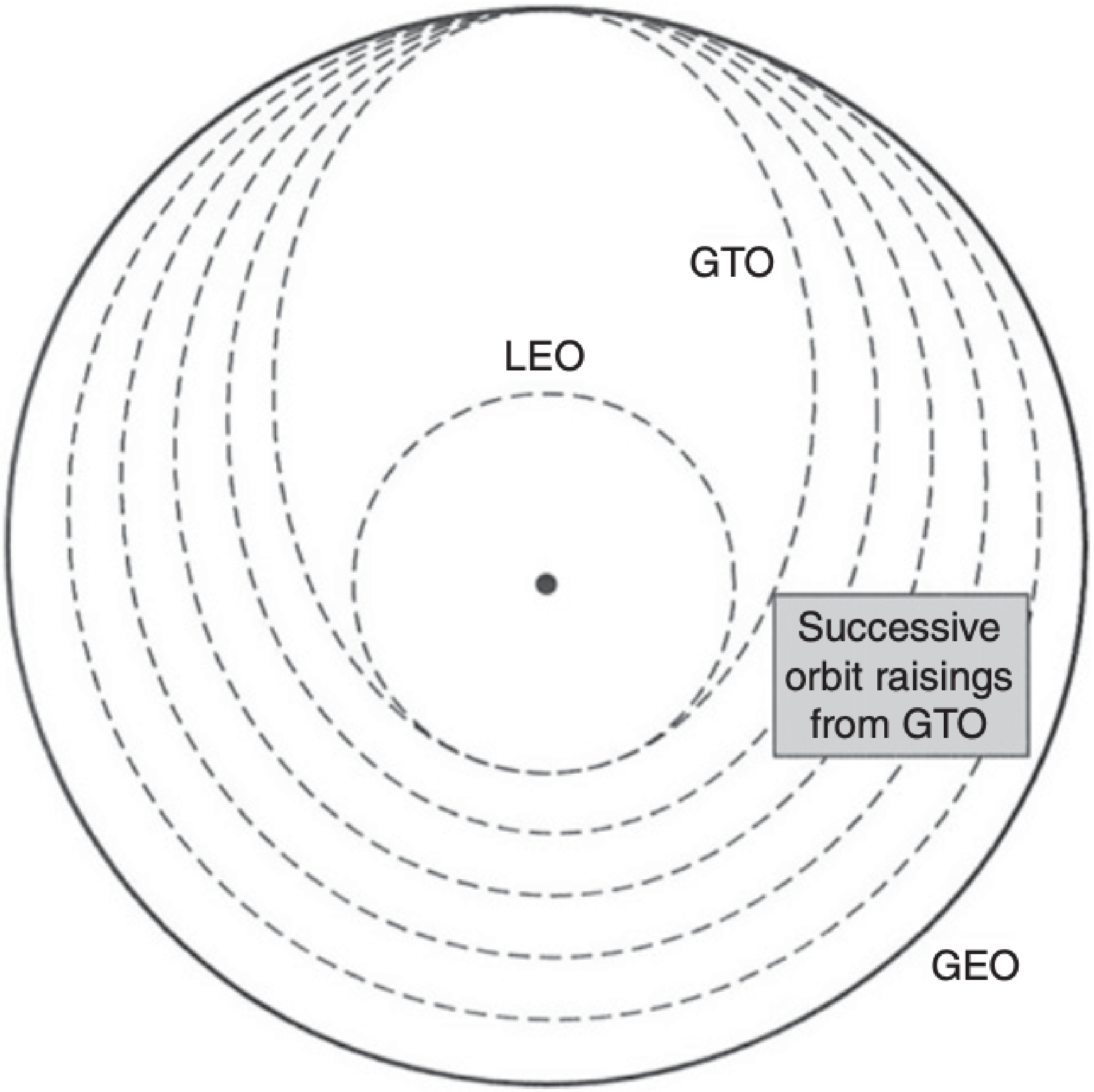


Figure 2.19 Illustration of slow orbit raising technique to geostationary orbit using an ion thrusters. (Not to scale.) The spacecraft and final rocket stage are placed in low earth orbit (LEO) and the satellite is separated from its rocket. The solar panels, antennas, and momentum wheels are deployed so that the satellite can be set to its correct attitude to generate solar power. Ion thrusters are then used to slowly increase the altitude of the satellite until geostationary altitude is achieved. At the same time, other ion thrusters are used to move the satellite's orbit into the equatorial plane. The process may take several months, but significantly reduces the weight of chemical fuel that the satellite has to carry.

Geostationary Transfer Orbit with Slow Orbit Raising In this procedure, rather than employ an apogee kick motor that imparts a vigorous acceleration over a few minutes, the spacecraft thrusters are used to raise the orbit from GTO to GEO over a number of burns. Since the spacecraft cannot be spin-stabilized during the GTO (so as not to infringe the Hughes patent), many of the satellite elements are deployed while in GTO, including the solar panels. The satellite has two power levels of thrusters: one for more powerful orbit raising maneuvers and one for on-orbit (low thrust) maneuvers. Since the thrusters take many hours of operation to achieve the geostationary orbit, the perigee of the orbit is gradually raised over successive thruster firings. The thruster firings occur symmetrically about the apogee although they could occur at the perigee as well. The burns are typically 60 to 80 min long on successive orbits and up to six orbits can be used. Figure 2.20 illustrates the process.

In the first two cases, AKM and slow orbit raising, the GTO may be a modified orbit with the apogee well above the required altitude for GEO. The excess energy of the orbit due to the higher-than-necessary altitude at apogee can be traded for energy required to raise the perigee. The net energy to circularize the orbit at GEO is therefore less and the satellite can retain more fuel for on-orbit operations.

Direct Insertion to GEO This is similar to the GTO technique but, in this case, the launch service provider contracts to place the satellite into GEO. The final stages of the rocket are used to place the satellite directly into GEO rather than the satellite using its own propulsion system to go from GTO to GEO.

Example 2.8

Question: What is the difference, or are the differences, between a *geosynchronous* satellite and a *geostationary* satellite orbit? What is the period of a geostationary satellite? What is the name given to this orbital period? What is the velocity of a geostationary satellite in its orbit? Give your answer in km/s.

A particular launch from Cape Canaveral released a TDRSS satellite into a circular low orbit, with an orbital height of 270 km. At this point, the TDRSS orbit was inclined to the earth's equator by approximately 28°. The TDRSS satellite needed to be placed into a GTO once released from the launch adaptor, with the apogee of the GTO at geostationary altitude and the perigee at the height of the original circular orbit.

(i) What was the eccentricity of the GTO?
(ii) What was the period of the GTO?
(iii) What was the difference in velocity of the satellite in GTO between when it was at apogee and when it was at perigee?

Note: Assume the average radius of the earth is 6378.137 km and Kepler's constant has the value $3.986\,004\,418 \times 10^5$ km^3/s^2.

Answer

A *geostationary* satellite orbit is one that has zero inclination to the equatorial plane, is perfectly circular (eccentricity is zero), and is at the correct orbital height to remain apparently stationary in orbit as viewed from the surface of the earth. A *geosynchronous* satellite orbit has most of the attributes of a geostationary orbit, but is either not exactly circular, not in the equatorial plane, or not at exactly the correct orbital height.

From Table 2.1, the orbital period of a geostationary satellite is 23 hours, 56 minutes, and 4.1 seconds.

The orbital period of a geostationary satellite is called a sidereal day.

From Table 2.1, the velocity of a geostationary satellite is 3.0747 km/s.

(i) The GTO will have an apogee of 35 786.03 km (the geostationary altitude) and a perigee of 270 km (the release altitude of the TDRSS).
The semimajor axis

$$a = (2r_e + h_p + h_a)/2 = (2 \times 6378.137 + 270 + 35\,786.03)/2 = 24\,406.152 \text{ km}$$

From Eq. (2.27) and Example 2.5, $r_0 = r_e + h_p$ and the eccentric anomaly $E = 0$ when the satellite is at perigee. From Eq. (2.27) $r_0 = a(1 - e\cos E)$, with $\cos E = 1$. Therefore, $r_e + h_p = a(1 - e)$ and, rearranging the equation,

$$e = 1 - (r_e + h_p)/a = 1 - (6378.137 + 270)/24\,406.152 = 0.727\,604.$$

The eccentricity of the GTO is therefore 0.728.

(ii) The orbital period

$$T = ((4\pi^2 a^3)/\mu)^{1/2} = ((4\pi^2 \times 24\,406.152^3)/3.986\,004\,418 \times 10^5)^{1/2}$$
$$= 37\,945.471\,02 \text{ seconds} = 10 \text{ hours } 32 \text{ minutes } 25.47 \text{ seconds.}$$

(iii) Orbital velocities. Eq. (2.5) gives the orbital velocity of a satellite as $v = (\mu/r)^{1/2}$. The perigee value of r = 270 + 6378.137 = 6648.137 km and the apogee value of r = 35 786 + 6478.137 = 42 164.137 km. Using these values in Eq. (2.5) yields a perigee velocity of 7.743117 km/s and an apogee velocity of 3.074660 km/s. The difference in velocity between perigee and apogee is 4.67 km/s.