

EENG 470 Satellite Communications

# Lecture # 8 Chapter 4 : Link Budget

### Prof. Mohab A. Mangoud

Professor of Wireless Communications (Networks, IoT and AI) University of Bahrain, College of Engineering Department of Electrical and Electronics Engineering P. O. Box 32038- Kingdom of Bahrain

> <u>mmangoud@uob.edu.bh</u> <u>http://mangoud.com</u>

# SATELLITE LINK DESIGN

CHAPTER

The design of a satellite communication system is a complex process requiring compromises between many factors to achieve the best performance at an acceptable cost. We will first consider geostationary satellite systems, since GEO satellites carry the vast majority of the world's satellite traffic.

## INTRODUCTION

Figure 2.17 of Chapter 2 shows that the cost to build and launch a GEO satellite is about \$25,000 per kg. Weight is the most critical factor in the design of any satellite, since the heavier the satellite the higher the cost, and the capital cost of the satellite must be recovered over its lifetime by selling communication services. The overall dimensions of the satellite are critical because the spacecraft must fit within the confines of the launch vehicle. When stowed for launch, the diameter of the spacecraft typically must be less than 3.5 m. Most large GEO satellites use deployable solar panels and antennas, but the antenna reflectors require accurate surfaces and are not folded for launch. This limits the maximum aperture dimension to about 3.5 m. As in most radio systems antennas are a limiting factor in the capacity and performance of the communication system.

The weight of a satellite is driven by two factors: the number and output power of the transponders on the satellite and the weight of station-keeping fuel. As much as half the total weight of satellites intended to remain in service for 15 years may be fuel. High power transponders require lots of electrical power, which can only be generated by solar cells. Increasing the total output power of the transponders raises the demand for electrical power and the dimensions of the solar cells, adding more weight to the satellite. Three other factors influence system design: the choice of frequency band, atmospheric propagation effects, and multiple access technique. These factors are all related, with the frequency band often being determined by what is available. Tables 4.1 and 4.2 tabulate Regions I, II, and III are regions of the earth's surface defined in the International Telecommunication Union's Radio Regulations. Region I covers Europe, Africa, and northern Asia. Region II covers North and South America, and Region III covers the remainder of Asia.

## TABLE 4.1 Major Frequency Allocations for Fixed Satellite Service and Broadcasting Satellites

Frequency	Fixed satellite service	Broadcasting satellites			
2320-2345 MHz		Radio broadcasting	17.7–18.6	Down	oh
2500-2535		Down region III	18.1–18.6	Down	
2500-2655	Down region II	Down region II	18.6-18.8	Down regions Land III	
2055-2090	Down region I	Down region II	18.8–19.7	Down	
3400–3700	Down	Up region II, III	27.0-27.5	Up regions II and III	
3700-4200	Down		27.5-29.5	Up	
4500-4800	Down, up		30.0-31.0	Up	
5/25-5850 5850-5925	Up region		37.5-39.5	Down	
5925-7075			39.5-40.5	Down	
7250-7450	Down, government	No.	40.5-42.5	Down	
74507550	Down, government		42.5-43.5	Up	i Sari
7550–7750	Down, government	1	47.2-50.2	Up	
8215-8400	Up, government		50.4-51.4	Up	
10.7–11.7 GHz	Down	Up region I	71.0-75.5	Up	
11.7-12.2	Down region II	Down regions I and III	81–84	Down	
12.2-12.7		Down regions I and II,	84-86	Down	
12.50-12.75	Up region Land IL down region L	U.S. Direct Broadcast Satellite T	92–95	Up	
12.75–13.25	e program und n, down region i	lln	102-105	Down	
14.00–14.25	Up	θþ	149–164	Down	
14.25-14.50	Up		202-217	Up	
14.5-14.8		Up	231-241	Down	
17.3–17.7 17.7–18.6	Dour	Up	265-275	Up	
11.1-10.0	Down			- 1-	

Table 4.1	Major frequency	allocations for fixed,	, mobile, and broadcast satell	ites

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Fixed satellite service (FSS) GHz		Mobile sate Gi	ellite service Hz	Broadcast satellite service (BS GHz	
Uplink	Downlink	Uplink	Downlink	Uplink	Downlink
			312-315 MHz		
			387-390 MHz		
		455-460 MHz			
					1.452-1.492 (DAB)
			1.518-1.530		
			1.535-1.559		
		1.610-1.675			
		1.980-2.100			
			2.120-2.170		
					2.320-2.345 (DAB)
					2.250-2.535
			2.485 - 2.500		
					2.655-2.670
	2.670-2.690				
	3.400-3.500				
	3.600-4.200				
	4.500-4.800				
5.725 - 5.850					
5.850-7.075					
7.250-7.750					
7.900-8.400					
	10.7–11.7				11.7–12.7
	11.7–12.2 (II)				
12.75 - 13.25					
13.75 - 14.8					
				17.3–17.8	
17.8–18.1 (II)					
18.11 - 21.2					
	17.8-20.2				
	21.4–22.0 (I, III)				
24.75 - 25.25					
27.50-31.0					
		29.5-29.9			
	38.0-42.0				
42.5-43.5					
		43.5-47.0			
47.2 - 50.2					
50.4-51.4					
	71.0–76.0				
81.0-86.0					

Frequency	Aeronautical mobile	Maritime mobile	Land mobile and other services				
137–138 MHz 148–149.9 149.9–150.05			Down, shared Up, shared Up, shared	1660.0–1660.5 2483.5–2500	Down	Down	Up Down
399.9-400.05 400.15-401 406-406.1 890-896			Up Down, shared Emergency beacons Region II (limited use) Shared with cellular radio	5.00–5.25 GHz 7.30–7.75 15.4–15.7 20.2–21.2 29 5–31 0	Up Up, government Down Down Up	Up Up, government Down Down Un	Up Up, government Down Down Un
1559–1610 1530–1535 1535–1544		Down Down	Navigation satellite, down Down region I only	39.5–40.5 43.5–45.5	Down Up, government	Down Up, government	Down Up, government
1544–1545 1545–1555 1555–1559 1559–1610 1610–1625.5	Down	Down	Down Navigation satellite, down Navigation satellite, up	45.5–47.0 66.0–71.0 71.0–74.0 81.0–84.0	Up Down Up Down	Up Down Up Down	Up Down Down Down
1625.5–1631.5 1631.5–1634.5 1634.5–1645.5 1645.5–1646.5	Up	Up Up Up	Up Shared Up	95.0–100 134–142 190–200 252–265			
1646.5–1656.5 1656.5–1660 1660.0–1660.5	Up		Up Up	Regions I, II, and Telecommunication	III are regions of the e ons Union. (See Table	earth's surface defined 4.1 for an explanation	by the International of their geographic lo

#### TABLE 4.2 Major Frequency Allocations for Mobile Satellite Services

Telecommunications Union. (See Table 4.1 for an explanation of their geographic locations.)

	Navigation satellites			
ISLs GHz	Uplinks GHz	Downlinks GHz		
		399.9–400.05 MHz		
		1.164-1.215		
		1.212-1.240		
	1.240-1.300			
		1.559-1.610		
	1.610-1.626			
		2.483-2.500		
	5.000-5.010			
		5.010-5.030		
	14.3–14.4			
22.55-23.55				
24.65-24.75	24.65-24.75			
25.25-25.5				
25.5-27.0				
32.3–33.0				
	43.5–47.0			
54.2-58.2				
59.0-71.0	66.0-71.0			
	95.0-100.0			

 Table 4.2
 Major frequency bands for inter-satellite links (ISLs) and navigational satellites

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the most important frequencies allocated for satellite communications. The major bands are the 6/4 GHz, 14/11 GHz, and 30/20 GHz bands. (The uplink frequency is quoted first, by convention.) However, over much of the geostationary orbit there is already a satellite using both 6/4 GHz and 14/11 GHz every 2°. This is the minimum spacing used for satellites in GEO to avoid interference from uplink earth stations. Additional satellites can only be accommodated if they use another frequency band, such as 30/20 GHz. Rain in the atmosphere attenuates radio signals. The effect is more severe as the frequency increases, with little attenuation at 4 and 6 GHz, but significant attenuation above 10 GHz. Attenuation through rain (in decibels) increases roughly as the square of frequency, so a satellite uplink operating at 30 GHz suffers four times as much attenuation as an uplink at 14 GHz.

Low earth orbit (LEO) and medium earth orbit (MEO) satellite systems have similar constraints to GEO satellite systems, but require more satellites which each serve a smaller area of the earth's surface. Although the satellites are much closer to the earth than GEO satellites and therefore produce stronger signals, this advantage is usually lost since the earth terminals need low gain omnidirectional antennas because the position of the satellite is continually changing. LEO and MEO satellites use multiple beam antennas to increase the gain of the satellite antenna beams, and also to provide frequency reuse. Mobile satellite terminals must operate with low gain antennas at the mobile unit, and at as low a RF frequency as can be obtained. The link between the satellite and the major earth station (often called a hub station) is usually in a different frequency band as it is a fixed link. Figure 4.1 shows an illustration of a maritime satellite communication system using a GEO satellite and L-band links to mobiles, with C-band links to a fixed hub station.

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Maritime satellite 6-GHz uplink 1.6-GHz uplinks from ships 4-GHz downlink Interconnection to terrestrial Shore network station 90-ft dish Q 1.5-GHz downlinks to ships 2-m dish

FIGURE 4.1 A maritime satellite communication system.



Figure 4.1 Illustration of a maritime satellite system using a GEO satellite. Ships are equipped with a steerable, gyro stabilized antennas. Aircraft have phased array antennas.

All communication links are designed to meet certain performance objectives usually a bit error rate (BER) in a digital link or a signal-to-noise ratio (S/N) in an analog link, measured in the baseband channel. The baseband channel is where an information carrying signal is generated or received; for example, a TV camera generates a baseband video signal, and a TV receiver delivers a baseband video signal to the picture tube to form the images that the viewer watches. Digital data are generated by computers at baseband, and BER is measured at baseband. The baseband channel BER or S/N ratio is determined by the carrier-to-noise ratio (C/N) at the input to the demodulator in the receiver. In most satellite communications applications, the C/N ratio at the demodulator input must be greater than 6 dB for the BER or S/N objective to be achieved. Digital links operating at C/N ratios below 10 dB must use error correction techniques to improve the BER delivered to the user. Analog links using frequency modulation (FM) require wideband FM to achieve a large improvement in S/N ratio relative to C/N ratio.

The C/N ratio is calculated at the input of the receiver, at the output terminals (or port) of the receiving antenna. RF noise received along with the signal and noise generated by the receiver are combined into an equivalent noise power at the input to the receiver, and a noiseless receiver model is used. In a noiseless receiver, the C/N ratio is constant at all points in the RF and IF chain, so the C/N ratio at the demodulator is equal to the C/N ratio at the receiver input. In a satellite link there are two signal paths: an uplink from the earth station to the satellite, and a downlink from the satellite to the earth station. The overall C/N at the earth station receiver depends on both links, and both, therefore must achieve the required performance for a specified percentage of time. Path attenuation in the earth's atmosphere may become excessive in heavy rain, causing the C/N ratio to fall below the minimum permitted value, especially when the 30/20 GHz band is used, leading to a link outage.

Designing a satellite system therefore requires knowledge of the required performance of the uplink and downlink, the propagation characteristics and rain attenuation for the frequency band being used at the earth station locations, and the parameters of the satellite and the earth stations. Additional constraints may be imposed by the need to conserve RF bandwidth and to avoid interference with other users. Sometimes, all of this information is not available and the designer must estimate values and produce tables of system performance based on assumed scenarios. It is usually impossible to design a complete satellite communication system at the first attempt. A trial design must first be tried, and then refined until a workable compromise is achieved. This chapter sets out the basic procedures for the design of satellite communication links, and includes design examples for a digital TV link using a GEO satellite and quadrature phase shift keying (QPSK) modulation, and a LEO satellite system for personal communication.

#### 4.2 BASIC TRANSMISSION THEORY

The calculation of the power received by an earth station from a satellite transmitter is fundamental to the understanding of satellite communications. In this section, we discuss two approaches to this calculation: the use of flux density and the *link equation*.



FIGURE 4.2 Flux density produced by an isotropic source.

Consider a transmitting source, in free space, radiating a total power  $P_t$  watts uniformly in all directions as shown in Figure 4.2. Such a source is called isotropic; it is an idealization that cannot be realized physically because it could not create transverse electromagnetic waves. At a distance *R* meters from the hypothetical isotropic source transmitting RF power  $P_t$  watts, the flux density crossing the surface of a sphere with radius *R* is given by

$$F = \frac{P_{\rm t}}{4\pi R^2} \,{\rm W/m^2}$$

All real antennas are directional and radiate more power in some directions than in others. Any real antenna has a gain  $G(\theta)$ , defined as the ratio of power per unit solid angle radiated in a direction  $\theta$  to the average power radiated per unit solid angle

$$G(\theta) = \frac{P(\theta)}{P_0/4\pi}$$

where

 $P(\theta)$  is the power radiated per unit solid angle by the antenna  $P_0$  is the total power radiated by the antenna  $G(\theta)$  is the gain of the antenna at an angle  $\theta^{-1}$ 

The reference for the angle  $\theta$  is usually taken to be the direction in which maximum power is radiated, often called the boresight direction of the antenna. The gain of the antenna is then the value of  $G(\theta)$  at angle  $\theta = 0^\circ$ , and is a measure of the increase in flux density radiated by the antenna over that from an ideal isotropic antenna radiating the same total power. For a transmitter with output  $P_t$  watts driving a lossless antenna with gain  $G_t$ , the flux density in the direction of the antenna boresight at distance Rmeters is

$$F = \frac{P_{\rm t}G_{\rm t}}{4\pi R^2} \,\mathrm{W/m^2}$$

The product  $P_tG_t$  is often called the *effective isotropically radiated power* or EIRP, and it describes the combination of transmitter power and antenna gain in terms of an equivalent isotropic source with power  $P_tG_t$  watts, radiating uniformly in all directions.



If we had an ideal receiving antenna with an aperture area of  $A \text{ m}^2$ , as shown in Figure 4.3, we would collect power  $P_r$  watts given by

$$P_{\rm r} = F \times A$$
 watts

A practical antenna with a physical aperture area of  $A_r$  m<sup>2</sup> will not deliver the power given in Eq. (4.4). Some of the energy incident on the aperture is reflected away from the antenna, and some is absorbed by lossy components. This reduction in efficiency is described by using an *effective aperture*  $A_e$  where

$$A_{\rm e} = \eta_{\rm A} A_{\rm r}$$

and  $\eta_A$  is the *aperture efficiency* of the antenna<sup>2</sup>. The aperture efficiency  $\eta_A$  accounts for all the losses between the incident wavefront and the antenna output port: these include illumination efficiency or aperture taper efficiency of the antenna, which is related to the energy distribution produced by the feed across the aperture, and also other losses due to spillover, blockage, phase errors, diffraction effects, polarization, and mismatch losses. For parabolodial reflector antennas,  $\eta_A$  is typically in the range 50 to 75%, lower for small antennas and higher for large Cassegrain antennas. Horn antennas can have efficiencies approaching 90%. Thus the power received by a real antenna with a physical receiving area  $A_r$  and effective aperture area  $A_e$  m<sup>2</sup> is



Note that this equation is essentially independent of frequency if  $G_t$  and  $A_e$  are constant within a given band; the power received at an earth station depends only on the EIRP of the satellite, the effective area of the earth station antenna, and the distance R.

A fundamental relationship in antenna theory<sup>2</sup> is that the gain and area of an antenna are related by

$$G = 4\pi A_{\rm e}/\lambda^2$$

where  $\lambda$  is the wavelength (in meters) at the frequency of operation. Substituting for  $A_e$  in Eq. (4.6) gives

$$P_{\rm r} = \frac{P_{\rm t}G_{\rm t}G_{\rm r}}{\left(4\pi R/\lambda\right)^2} \text{ watts}$$

This expression is known as the *link equation*, and it is essential in the calculation of power received in any radio link. The frequency (as wavelength,  $\lambda$ ) appears in this equation for received power because we have used the receiving antenna gain, instead of effective area. The term  $[4\pi R/\lambda]^2$  is known as the *path loss*,  $L_p$ . It is not a loss in the sense of power being absorbed; it accounts for the way energy spreads out as an electromagnetic wave travels away from a transmitting source in three-dimensional space.

Collecting the various factors, we can write

Power received = 
$$\frac{\text{EIRP} \times \text{Receiving antenna gain}}{\text{Path loss}} \text{ watts}$$
(4.9)

In communication systems, decibel quantities are commonly used to simplify equations like Eq. (4.9). In decibel terms, we have

$$P_{\rm r} = \rm EIRP + G_{\rm r} - L_{\rm p} \, dBW \tag{4.10}$$

where

EIRP = 
$$10 \log_{10}(P_t G_t) dBW$$
  
 $G_r = 10 \log_{10}(4\pi A_e/\lambda^2) dB$   
Path loss  $L_p = 10 \log_{10}[(4\pi R/\lambda)^2] = 20 \log_{10}(4\pi R/\lambda) dB$ 

Equation (4.10) represents an idealized case, in which there are no additional losses in the link. It describes transmission between two ideal antennas in otherwise empty space. In practice, we will need to take account of a more complex situation in which we have losses in the atmosphere due to attenuation by oxygen, water vapor, and rain, losses in the antennas at each end of the link, and possible reduction in antenna gain due to mispointing. All of these factors are taken into account by the *system margin* but need to be calculated to ensure that the margin allowed is adequate. More generally, Eq. (4.10) can be written

$$P_{\rm r} = \rm EIRP + G_{\rm r} - L_{\rm p} - L_{\rm a} - L_{\rm ta} - L_{\rm ra} \, \rm dBW$$

(4.11)

## where

- $L_{\rm a}$  = attenuation in the atmosphere
- $L_{ta}$  = losses associated with the transmitting antenna
- $L_{ra}$  = losses associated with the receiving antenna

The conditions in Eq. (4.11) are illustrated in Figure 4.4. The expression dBW means decibels greater or less than 1 W (0 dBW). The units dBW and dBm (dB greater or less than 1 W and 1 mW) are widely used in communications engineering. EIRP, being the product of transmitter power and antenna gain is often quoted in dBW.

Note that once a value has been calculated in decibels, it can readily be scaled if one parameter is changed. For example, if we calculated  $G_r$  for an antenna to be 48 dB, at a frequency of 4 GHz, and wanted to know the gain at 6 GHz, we can multiply  $G_r$  by  $(6/4)^2$ . Using decibels, we simply add 20 log(6/4) or 20 log(3) - 20 log(2) = 9.5 - 6 = 3.5 dB. Thus the gain of our antenna at 6 GHz is 51.3 dB.

Appendix A gives more information on the use of decibels in communications engineering.



#### EXAMPLE 4.2.1

A satellite at a distance of 40,000 km from a point on the earth's surface radiates a power of 10 W from an antenna with a gain of 17 dB in the direction of the observer. Find the flux density at the receiving point, and the power received by an antenna at this point with an effective area of 10 m<sup>2</sup>.

Using Eq. (4.3)

$$F = P_t G_t / (4\pi R^2) = 10 \times 50 / (4\pi \times (4 \times 10^7)^2) = 2.49 \times 10^{-14} \, \text{W/m}^2$$

The power received with an effective collecting area of 10 m<sup>2</sup> is therefore

 $P_{\rm r} = 2.49 \times 10^{-13} \,{\rm W}$ 

The calculation is more easily handled using decibels. Noting that  $10 \log_{10} 4\pi = 11.0 \text{ dB}$ *F* in dB units =  $10 \log_{10}(P_tG_t) - 20 \log_{10}(R) - 11.0$ 

$$= 27.0 - 152.0 - 11.0$$
  
= -136.0 dB(W/m<sup>2</sup>)<sup>o</sup>  
$$P_{r} = -136.0 + 10.0 = -126 \text{ dBW}$$

Then

Here we have put the antenna effective area into decibels greater than  $1 \text{ m}^2$  ( $10 \text{ m}^2 = 10 \text{ dB}$  greater than  $1 \text{ m}^2$ ).

#### EXAMPLE 4.2.2

The satellite in Example 4.2.1 operates at a frequency of 11 GHz. The receiving antenna has a gain of 52.3 dB. Find the received power.

Using Eq. (4.10) and working in decibels  $P_{\rm r} = {\rm EIRP} + G_{\rm r} - L_{\rm p} \, {\rm dBW}$  (4.10)  ${\rm EIRP} = 27.0 \, {\rm dBW}^{\circ}$   $G_{\rm r} = 52.3 \, {\rm dB}^{\circ}$ Path loss =  $(4\pi R/\lambda)^2 = 20 \log_{10}(4\pi R/\lambda) \, {\rm dB}$   $= 20 \log_{10}[(4\pi \times 4 \times 10')/(2.727 \times 10^{-2})] \, {\rm dB} = 205.3 \, {\rm dB}$  $P_{\rm r} = 27.0 + 52.3 - 205.3 = -126.0 \, {\rm dBW}$ 

We have the same answer as in Example 4.2.1 because the figure of 52.3 dB is the gain of a 10 m<sup>2</sup> aperture at a frequency of 11 GHz.

Equation (4.10), with other parameters for antenna and propagation losses, is commonly used for calculation of received power in a microwave link and is set out as a link power budget in tabular form using decibels. This allows the system designer to adjust parameters such as transmitter power or antenna gain and quickly recalculate the received power.

The received power,  $P_r$ , calculated by Eqs. (4.6) and (4.8) is commonly referred to as carrier power, C. This is because most satellite links use either frequency modulation for analog transmission or phase modulation for digital transmission. In both of these modulation systems, the amplitude of the carrier is not changed when the data are modulated onto the carrier, so received carrier power C is always equal to received power  $P_r$ .

## Book Chapter – new edition

#### 4.2 Transmission Theory

The calculation of the power received by an earth station from a satellite transmitter is fundamental to the understanding of satellite communications. In this section, we discuss two approaches to this calculation: the use of *flux density* and the *link equation*.

Consider a transmitting source, in free space, radiating a total power  $P_t$  watts uniformly in all directions as shown in Figure 4.2. Such a source is called *isotropic*; it is an idealization that cannot be realized physically because it could not create transverse electromagnetic (EM) waves. At a distance R meters from the hypothetical isotropic source transmitting RF power  $P_t$  watts, the flux density crossing the surface of a sphere with radius R m is given by

$$F = \frac{P_{\rm r}}{4\pi R^2} \,\mathrm{W/m^2} \tag{4.1}$$



**Figure 4.2** Calculation of flux density from an isotropic source with EIRP  $P_t$  watts. The flux density is measured over a 1 m<sup>2</sup> section of a sphere at a distance *R* meters from the source.

Flux density F watts/m<sup>2</sup>

All real antennas are directional and radiate more power in some directions than in others. Any real antenna has a gain  $G(\theta)$ , defined as the ratio of power per unit solid angle radiated in a direction  $\theta$  to the average power radiated per unit solid angle (Silver 1949, p. 2).

$$G(\theta) = \frac{P(\theta)}{P_{\rm o}/4\pi} \,\mathrm{W/m^2} \tag{4.2}$$

where

 $P(\theta)$  is the power radiated per unit solid angle by the antenna  $P_{\rm o}$  is the total power radiated by the antenna  $G(\theta)$  is the gain of the antenna at an angle  $\theta$ 

The reference for the angle  $\theta$  is usually taken to be the direction in which maximum power is radiated, called the *boresight* direction of the antenna or the *antenna electrical axis*. The gain of the antenna is then the value of  $G(\theta)$  at angle  $\theta = 0^\circ$ , and is a measure of the increase in flux density radiated by the antenna over that with an ideal isotropic antenna radiating the same total power. See Appendix B for more details of antennas and their properties. For a transmitter with output  $P_t$  watts driving a lossless antenna with gain  $G_t$ , the flux density in the direction of the antenna boresight at distance R meters is

$$F = \frac{P_{\rm t}G_{\rm t}}{4\pi R^2} \,\mathrm{W/m^2} \tag{4.3}$$

The product  $P_t G_t$  is often called the *effective isotropically radiated power* (EIRP), and describes the combination of transmitter power and antenna gain in terms of an equivalent isotropic source with power  $P_t G_t$  watts, radiating uniformly in all directions.

If we had an ideal receiving antenna with an aperture area of  $A \text{ m}^2$ , as shown in Figure 4.3, we would collect power  $P_r$  watts given by

$$P_{\rm r} = F \times A \text{ watts} \tag{4.4}$$

A practical antenna with a physical aperture area of  $A_r m^2$  will not deliver the power given in Eq. (4.4). Some of the energy incident on the aperture is reflected away from the antenna, referred to as scattering, and some is absorbed by lossy components. This reduction in efficiency is described by using an *effective aperture*  $A_e$  where

$$A_{\rm e} = \eta_{\rm A} A_{\rm r} {\rm m}^2 \tag{4.5}$$

and  $\eta_A$  is the *aperture efficiency* of the antenna (Stutzman and Thiele 2013, p. 363). The aperture efficiency  $\eta_A$  accounts for all the losses between the incident wavefront and the antenna output port: these include *illumination efficiency* or *aperture taper efficiency* of



Receiving antenna with Area  $A m^2$ , gain  $G_r$ 

**Figure 4.3** Calculation of received power by an antenna with gain  $G_r$ from a source with EIRP  $P_t G_t$ watts. *F* is the flux density incident on the receiving antenna.  $P_r$  is the power delivered to the receiver. the antenna, which is related to the energy distribution produced by the feed across the aperture, and also losses due to spillover, blockage, phase errors, diffraction effects, polarization, and mismatch losses. For parabolodial reflector antennas,  $\eta_A$  is typically in the range 50–75%, lower for small antennas and higher for large Cassegrain and Gregorian antennas. Horn antennas can have efficiencies approaching 80%. (See Appendix B for an explanation of the aperture efficiency of antennas.)

Thus the power received by a real antenna with a physical receiving area  $A_r$  and effective aperture area  $A_e$  m<sup>2</sup> at a distance R from the transmitter is

$$P_{\rm r} = \frac{P_{\rm t}G_{\rm t}A_{\rm e}}{4\pi R^2} \text{ watts}$$
(4.6)

Note that this equation is essentially independent of frequency if  $G_t$  and  $A_e$  are constant within a given band; the power received at an earth station depends only on the EIRP of the satellite, the effective area of the earth station antenna, and the distance *R*.

A fundamental relationship in antenna theory is that the gain and area of an antenna are related by (Stutzman and Thiele 2013, p. 363)

$$G = 4\pi A_{\rm e}/\lambda^2 \tag{4.7}$$

where  $\lambda$  is the wavelength (in meters for  $A_{\rm e}$  in square meters) at the frequency of operation.

Substituting for  $A_{\rm e}$  in Eq. (4.6) gives

$$P_{\rm r} = \frac{P_{\rm t}G_{\rm t}G_{\rm r}}{(4\pi R/\lambda)^2} \text{ watts}$$
(4.8)

This expression is known as the *link equation*, and it is essential in the calculation of power received in any radio link. The frequency (as wavelength,  $\lambda$ ) appears in this equation for received power because we have used the receiving antenna gain, instead of effective area. The term  $(4\pi R/\lambda)^2$  is known as the *path loss*,  $L_p$ . It is not a loss in the sense of power being absorbed; it accounts for the way energy spreads out as an EM wave travels away from a transmitting source in three-dimensional (3-D) space.

Collecting the various factors, we can write

0

$$P_{\rm r} = \frac{\rm EIRP \times \rm Receiving \ antenna \ gain}{\rm Path \ Loss} \ watts$$
(4.9)

In communication systems, decibel quantities are commonly used to simplify equations like (4.9). In decibel terms, we have

$$P_{\rm r} = {\rm EIRP} + G_{\rm r} - L_{\rm p} \, {\rm dBW} \tag{4.10}$$

where

 $\begin{aligned} \text{EIRP} &= 10 \ log_{10} \ (P_{\text{t}} \ G_{\text{t}}) \ \text{dBW} \\ G_{\text{r}} &= 10 \ log_{10} \ (4\pi A_{\text{e}}/\lambda^2) \ \text{dB} \end{aligned}$ 

Path loss  $L_{\rm p}$  is given by

$$L_{\rm p} = 10 \, \log_{10} \left[ \left( 4\pi R/\lambda \right)^2 \right] = 20 \, \log_{10} \left( 4\pi R/\lambda \right) \, {\rm dB} \tag{4.11}$$

If you are unfamiliar with decibels, read Appendix A, which discusses how decibels are used in the analysis of radio communication systems. Equation (4.10) represents an idealized case, in which there are no additional losses in the link. It describes
transmission between two ideal antennas in otherwise empty space. In practice, we will need to take account of a more complex situation in which we have losses in the atmosphere due to attenuation by oxygen, water vapor, and rain, losses in the antennas at each end of the link, and possible reduction in antenna gain due to mispointing. All of these factors are taken into account by the *system margin* but need to be calculated to ensure that the margin allowed is adequate. More generally, Eq. (4.10) can be written

$$P_{\rm r} = {\rm EIRP} + G_{\rm r} - L_{\rm p} - L_{\rm a} - L_{\rm ta} - L_{\rm ra} \, {\rm dBW}$$
 (4.12)

where

 $L_{a}$  = attenuation in the atmosphere  $L_{ta}$  = losses associated with the transmitting antenna  $L_{ra}$  = losses associated with the receiving antenna

The conditions in Eq. (4.12) are illustrated in Figure 4.4. The expression dBW means decibels greater or less than 1 W (0 dBW). The units dBW and dBm (dB greater or less than 1 W and 1 mW) are widely used in communications engineering. EIRP, being the product of transmitter power and antenna gain is normally quoted in dBW.

Note that once a value has been calculated in decibels, it can readily be scaled if one parameter is changed. For example, if we calculated  $G_r$  for an antenna to be 48 dB at a frequency of 4 GHz, and wanted to know the gain at 6 GHz, we can multiply  $G_r$  by  $(6/4)^2$ .

Using decibels, we simply add 20 log (6/4) (or 20 log (3) – 20 log (2)) = 9.5 - 6 = 3.5 dB. Thus the gain of our antenna at 6 GHz is 51.3 dB.

Appendix A gives more information on the use of decibels in communications engineering.



**Figure 4.4** Calculation of received power from a satellite with EIRP  $P_t G_t$  watts including losses. Loss  $L_{ta}$  is an off-axis loss deducted from the satellite antenna on-axis gain when calculating the antenna gain in the direction of the receiving earth station. Atmospheric loss  $L_a$  includes clear air loss caused by gases in the atmosphere and any additional loss from clouds and rain. Receiving antenna losses  $L_{ra}$  include ohmic losses in the waveguide between the antenna feed and the LNA, and an off-axis loss if the receiving antenna does not point directly at the satellite.

#### Example 4.1

A satellite at a distance of 40 000 km from a point on the earth's surface radiates a power of 10 W from an antenna with a gain of 17 dB in the direction of the observer. Find the flux density at the receiving point, and the power received by an earth station antenna at this point with an effective area of  $10 \text{ m}^2$ .

#### Answer

Using Eq. (4.3)

$$F = P_{\rm t}G_{\rm t}/(4\pi R^2) = 10 \times 50/[4\pi \times (4 \times 10^7)^2] = 2.49 \times 10^{-14}$$

The power received with an effective collecting area of 10 m<sup>2</sup> is therefore

$$P_{\rm r} = 2.49 \times 10^{-13} {\rm W}$$

The calculation is more easily handled using decibels. Noting that 10  $\log_{10} 4\pi \,{\approx}\, 11.0\, dB$ 

F in dB units = 10 
$$\log_{10} (P_t G_t) - 20 \log_{10} (R) - 11.0$$
  
= 27.0 - 152.0 - 11.0  
= -136.0 dBW/m<sup>2</sup>

Then

$$P_{\rm r} = F \, {\rm dBW}/{\rm m}^2 + A_{\rm e} \, {\rm dBm}^2$$
  
 $P_{\rm r} = -136.0 + 10.0 = -126 \, {\rm dBW} \, {\rm or} - 96 \, {\rm dBm}$ 

Here we have put the antenna effective area into decibels greater than  $1 \text{ m}^2$  ( $10 \text{ m}^2 = 10 \text{ dB}$  greater than  $1 \text{ m}^2$ ) and also given the answer in dBW and dBm, decibels above 1 watt and 1 milliwatt.

#### **Example 4.2**

The satellite in Example 4.1 operates at a frequency of 11 GHz. The receiving antenna has a gain of 52.3 dB. Find the received power at the earth station in dBW and dBm. It is common practice to quote transmit power in dBW and received power in dBm.

#### Answer

Using Eq. (4.10) and working in decibels

 $P_{\rm r} = {\rm EIRP} + G_{\rm r} - {\rm path \ loss \ dBW}$ EIRP = 27.0 dBW  $G_{\rm r} = 52.3 \ {\rm dB}$ Path loss  $L_{\rm p} = (4\pi R/\lambda)^2 = 20 \ \log_{10} (4\pi R/\lambda) \ {\rm dB}$ = 20 log<sub>10</sub>[(4\pi × 4 × 10<sup>7</sup>)/(2.727 × 10<sup>-2</sup>)] = 205.3 \ {\rm dB}  $P_{\rm r} = 27.0 + 52.3 - 205.3 = -126.0 \ {\rm dBW}$ 

The received power in dBm units is numerically 30 dB greater than in dBW.

Hence

 $P_{\rm r} = 126.0 + 30 = -96.0 \, \rm dBm$ 

We have the same answer as in Example 4.1 because the figure of 52.3 dB is the gain of a  $10 \text{ m}^2$  aperture at a frequency of 11 GHz.

Equation (4.12) is commonly used for calculation of received power in a microwave link and is set out as a link power budget in tabular form using decibels. This allows the system designer to adjust parameters such as transmitter power or antenna gain and quickly recalculate the received power.

The received power,  $P_r$  calculated by Eqs. (4.6) and (4.8) is commonly referred to as carrier power, *C*. This is because satellite links typically use phase modulation for digital transmission where the amplitude of the carrier is not changed when the data is modulated onto the carrier, so received carrier power *C* watts is always equal to received power  $P_r$  watts.

## Link Budget

- Adapted from
- Dr. Joe Montana (George mason University)
- Dr. James W. LaPean course notes
- Dr. Jeremy Allnutt course notes
- And some internet resources + Tim Pratt book

# Link Power Budget



#### Illustration of the various propagation loss mechanisms on a typical earthspace path



### Radio Propagation: Atmospheric Attenuation

- Rain is the main cause of atmospheric attenuation (hail, ice and snow have little effect on attenuation because of their low water content).
- Total attenuation from rain can be determined by:

## $A = \alpha L [dB]$

- where α [dB/km] is called the specific attenuation, and can be calculated from specific attenuation coefficients in tabular form that can be found in a number of publications;
- where L [km] is the effective path length of the signal through the rain; note that this differs from the geometric path length due to fluctuations in the rain density.

### Signal Polarisation: Cross-Polarisation Discrimination

- Depolarisation can cause interference where orthogonal polarisation is used to provide isolation between signals, as in the case of frequency reuse.
- The most widely used measure to quantify the effects of polarisation interference is called Cross-Polarisation Discrimination (XPD):

### **XPD = 20 log (E\_{11}/E\_{12})**



Source: *Satellite Communications*, Dennis Roddy, McGraw-Hill

- To counter depolarising effects circular polarising is sometimes used.
- Alternatively, if linear polarisation is to be used, polarisation tracking equipment may be installed at the antenna.

### Atmospheric attenuation



elevation of the satellite

- Link-power budget calculations take into account all the gains and losses from the transmitter, through the medium to the receiver in a telecommunication system. Also taken into the account are the attenuation of the transmitted signal due to propagation and the loss or gain due to the antenna.
- The decibel equation for the received power is:

### $- [P_R] = [EIRP] + [G_R] - [LOSSES]$

Where:

- $[P_R]$  = received power in dBW
- [EIRP] = equivalent isotropic radiated power in dBW
- $[G_R]$  = receiver antenna gain in dB
- [LOSSES] = total link loss in dB
- [LOSSES] = [FSL] + [RFL] + [AML] + [AA] + [PL], where:
  - [FSL] = free-space spreading loss in  $dB = P_T/P_R$  (in watts)
  - [RFL] = receiver feeder loss in dB
  - [AML] = antenna misalignment loss in dB
  - [AA] = atmospheric absorption loss in dB
  - [PL] = polarisation mismatch loss in dB
- The major source of loss in any ground-satellite link is the freespace spreading loss.

# Translating to dBs

• The transmission formula can be written in dB as:

$$P_r = EIRP - L_{ta} - L_p - L_a - L_{pol} - L_{ra} - L_{other} + G_r - L_r$$

- This form of the equation is easily handled as a spreadsheet (additions and subtractions!!)
- The calculation of <u>received signal</u> based on <u>transmitted power</u> and all losses and gains involved until the receiver is called "Link Power Budget", or "Link Budget".
- The <u>received power Pr</u> is commonly referred to as <u>"Carrier</u>
  <u>Power", C.</u>

- Demonstrated formula assumes idealized case.
- Free Space Loss (L<sub>p</sub>) represents spherical spreading only.
- Other effects need to be accounted for in the transmission equation:
  - $L_a = Losses due to attenuation in atmosphere$
  - L<sub>ta</sub> = Losses associated with transmitting antenna
  - $L_{ra} =$  Losses associates with receiving antenna
  - L<sub>pol</sub> = Losses due to polarization mismatch
  - L<sub>other</sub> = (any other known loss as much detail as available)
  - Lr = additional Losses at receiver (after receiving antenna)

$$P_{r} = \frac{P_{t}G_{t}G_{r}}{L_{p}L_{a}L_{ta}L_{ra}L_{pol}L_{other}L_{r}}$$

### Simple Link Power Budget

Parameter	Value	Totals	Units
Frequency	11.75		GHz
Transmitter			
Transmitter Power	40.00		dBm
Modulation Loss	3.00		dB
Transmission Line Loss	0.75		dB
Transmitted Power		<mark>36.25</mark>	<mark>dBm</mark>
Transmit Antenna			
Diameter	0.5		m
Aperture Efficiency	0.55		none
Transmit Antenna Gain		<mark>33.18</mark>	<mark>dBi</mark>
Slant Path			
Satellite Altitude	35,786		km
Elevation Angle	14.5		degrees
Slant Range	41,602		km
Free-space Path Loss	206.22		dB
Gaseous Loss	0.65		dB
Rain Loss (allocated)	3.50		dB
Path Loss		<b>210.37</b>	dB

Parameter	Value	Totals	Unit
<b>Receive Antenna</b>			
Radome Loss	0.50		dB
Diameter	1.5		m
Aperture Efficiency	0.6		none
Gain	43.10		dBi
Polarization Loss	0.20		dB
Effective RX Ant. Gain		42.40	dB
<b>Received Power</b>		<mark>-98.54</mark>	dBm
Summary			
Transmitted Power	36.25		dBm
Transmit Anntenna Gain	33.18		dBi
EIRP		69.43	dBm
Path Loss		210.37	dB
Effective RX Antenna Gain		42.4	dBi
Received Power		-98.54	dBm

# Why calculate Link Budgets?

- System performance tied to operation thresholds.
- Operation thresholds C<sub>min</sub> tell the minimum power that should be received at the demodulator in order for communications to work properly.
- Operation thresholds depend on:
  - Modulation scheme being used.
  - Desired communication quality.
  - Coding gain.
  - Additional overheads.
  - Channel Bandwidth.
  - Thermal Noise power.

# Closing the Link

• We need to calculate the Link Budget in order to verify if we are "closing the link".

$$P_r \ge C_{min}$$
 $\rightarrow$  Link Closed $P_r < C_{min}$  $\rightarrow$  Link not closed

 Usually, we obtain the <u>"Link Margin"</u>, which tells how tight we are in closing the link:

$$Margin = P_r - C_{min}$$

• Equivalently:

Margin > 0 Margin < 0

- → Link Closed
- → Link not closed

# **Carrier to Noise Ratios**

- C/N: carrier/noise power in RX BW (dB)
- C/N<sub>o</sub>: carrier/noise p.s.d. (dbHz) .....  $\{N_o = KT\}$

## System Figure of Merit

- <u>G/T\_s</u>: RX antenna gain/system temperature
  - Also called the System Figure of Merit,  $G/T_s$
  - Easily describes the sensitivity of a receive system
  - Must be used with caution:
    - Some (most) vendors measure G/T<sub>s</sub> under ideal conditions only
    - G/T<sub>s</sub> degrades for most systems when rain loss increases
      - This is caused by the increase in the sky noise component
      - This is in addition to the loss of received power flux density

### System Noise Power

#### 4.3 SYSTEM NOISE TEMPERATURE AND G/T RATIO

#### **Noise Temperature**

Noise temperature is a useful concept in communications receivers, since it provides a way of determining how much thermal noise is generated by active and passive devices in the receiving system. At microwave frequencies, a black body with a physical temperature,  $T_p$  degrees kelvin, generates electrical noise over a wide bandwidth.

The noise power is given by<sup>3</sup>

$$P_{\rm n} = kT_{\rm p}B_{\rm n}$$

k = Boltzmann's constant =  $1.39 \times 10^{-23}$  J/K = -228.6 dBW/K/Hz  $T_{\rm p}$  = physical temperature of source in kelvin degrees  $B_{\rm p}$  = noise bandwidth in which the noise power is measured, in hertz

 $P_{\rm n}$  is the available noise power (in watts) and will be delivered only to a load that is impedance matched to the noise source. The term  $kT_{\rm p}$  is a noise power spectral density, in watts per hertz. The density is constant for all radio frequencies up to 300 GHz.

# System Noise Power - 1

- Performance of system is determined by C/N ratio.
- Most systems require C/N > 10 dB. (Remember, in dBs: C - N > 10 dB)
- Hence usually: C > N + 10 dB
- We need to know the noise temperature of our receiver so that we can calculate N, the noise power (N = P<sub>n</sub>).
- T<sub>n</sub> (noise temperature) is in Kelvins (symbol K):

$$T[K] = T[^{0}C] + 273$$

$$T[K] = (T[^{0}F] - 32)\frac{5}{9} + 273$$

# System Noise Power - 2

- System noise is caused by thermal noise sources
  - External to RX system
    - Transmitted noise on link
    - Scene noise observed by antenna
  - Internal to RX system
- The power available from thermal noise is:

$$N = kT_s B$$
 (dBW)

where k = Boltzmann's constant = 1.38x10<sup>-23</sup> J/K(-228.6 dBW/HzK),

 $T_s$  is the effective system noise temperature, and B is the effective system bandwidth

We will see more on calculating Ts next class.

# **Noise Spectral Density**

• N = K.T.B  $\rightarrow$  N/B = N<sub>0</sub> is the noise spectral density (density of noise power per hertz):

$$N_0 = \frac{N}{B} = \frac{kT_sB}{B} = kT_s \text{ (dBW/Hz)}$$

- N<sub>0</sub> = noise spectral density is constant up to 300GHz.
- All bodies with T<sub>p</sub> >0K radiate microwave energy.

# System Noise Temperature

- 1) System noise power is proportional to system noise temperature
- 2) Noise from different sources is uncorrelated (AWGN)

Additive White Gaussian Noise (AWGN)

- Therefore, we can
  - Add up noise powers from different contributions
  - Work with noise temperature directly
- So:

$$T_{s} = T_{transmitted} + T_{antenna} + T_{LNA} + T_{lineloss} + T_{RX}$$

- But, we must:
  - Calculate the effective noise temperature of each contribution
  - Reference these noise temperatures to the same location

### Noise Model

Calculation of System Noise Temperature





### Noise Model

Calculation of System Noise Temperature



- Components are replaced by equivalent noiseless blocks with same gain and noise generators at the input to each block such that the block produces the same noise at its output as the device it replaces
- Noise at component input must be multiplied by corresponding gain to reference at component output
- System Noise Temperature, T<sub>s</sub>, is the noise temperature of a noise source, located at the input of a noiseless receiver, which gives the same noise power as the original receiver measured at the output of the receiver and usually includes noise from the antenna

## Noise Model for a Cascaded System

Calculation of System Noise Temperature



1/3/2009

## Book's approach to calculate T<sub>s</sub>

### Noise Model

Calculation of System Noise Temperature



- Use noise model to write P<sub>n</sub>
- $P_n = G_{IF}kT_{IF}B$  (IF) +  $G_{IF}G_mkT_mB$  (LocalOscilator) +  $G_{IF}G_mG_{RF}kB(T_{RF} + T_{in})$  (Front-End: RF+Input)

### Noise Model

Calculation of System Noise Temperature

• Use noise model to write  $P_n$   $P_n = G_{IF}kT_{IF}B$  (IF)  $+G_{IF}G_mkT_mB$  (LocalOscilator)  $+G_{IF}G_mG_{RF}kB(T_{RF}+T_{in})$  (Front-End: RF+Input)

Equation can be written as

$$P_n = G_{IF}G_m G_{RF} kB \left[ T_{RF} + T_{in} + \frac{T_m}{G_{RF}} + \frac{T_{IF}}{G_m G_{RF}} \right]$$

### Equivalent Noise Model of Receiver

Calculation of System Noise Temperature



### Equivalent Noise Model of Receiver

Calculation of System Noise Temperature



• Note that equivalent noise model gives  $P_n = G_{IF}G_mG_{RF}kT_sB$ 

### Calculating System Noise Temperature

$$P_n = G_{IF}G_mG_{RF}kB \left[ T_{RF} + T_{in} + \frac{T_m}{G_{RF}} + \frac{T_{IF}}{G_mG_{RF}} \right]$$

**Equation 1** 

$$P_n = G_{IF} G_m G_{RF} k T_s B$$

**Equation 2** 

• Equate to obtain

$$T_{\rm S} = \left[ T_{\rm RF} + T_{\rm in} + \frac{T_{\rm m}}{G_{\rm RF}} + \frac{T_{\rm IF}}{G_{\rm m}G_{\rm RF}} \right]$$

## Calculating System Noise Temperature

$$T_{\rm S} = \left[T_{\rm RF} + T_{\rm in} + \frac{T_{\rm m}}{G_{\rm RF}} + \frac{T_{\rm IF}}{G_{\rm m}G_{\rm RF}}\right]$$

- Succeeding stages of the receiver contribute less and less noise to the total system
- When RF amplifier in receiver frontend has high gain, noise contributed by IF amplifier and later stages can be ignored
- In such a case

T<sub>s</sub>=T <sub>antenna</sub> +T <sub>LNA</sub>

Values of gains in above equation are all linear and not decibels

### **Reducing Noise Power**

- Make B as small as possible just enough bandwidth to accept all of the signal power (C).
- Make T<sub>s</sub> as small as possible
  - Lowest T<sub>RF</sub>
  - Lowest T<sub>in</sub> (How?)
  - High  $G_{RF}$
- If we have a good low noise amplifier (LNA), i.e., low  $T_{RF}$ , high  $G_{RF}$ , then rest of receiver does not matter that much.

$$T_{\rm S} = \left[T_{RF} + T_{in} + \frac{T_m}{G_{RF}} + \frac{T_{IF}}{G_m G_{RF}}\right] \cong T_{RF} + T_{in}$$

### **Reducing Noise Power**

#### **Low Noise Amplifier**

Parametric amplifier (older technology, complex and expensive):

Cooled (thermo-electrically or liquid nitrogen or helium):

- 4 GHz : 30 K
- 11 GHz: 90 K

Uncooled:

- 4 GHz : 40 K
- 11 GHz: 100 K
- Ga AS FET (Galium Arsenide Field-Effect Transistor): Cooled (thermo-electrically):
  - 4 GHz : 50 K
  - 11 GHz: 125 K

Uncooled:

- 4 GHz : 50 K
- 11 GHz: 125 K

### **Reducing Noise Power**

Discussion on T<sub>in</sub>

- Earth Stations: Antennas looking at space which appears cold and produces little thermal noise power (about 50K).
- Satellites: antennas beaming towards earth (about 300 K):
  - Making the LNA noise temperature much less gives diminishing returns.
  - Improvements aim reduction of size and weight.
#### **Antenna Noise Temperature**

- Contributes for T<sub>in</sub>
- Natural Sources (sky noise):
  - Cosmic noise (star and inter-stellar matter), decreases with frequency, (negligible above 1GHz). Certain parts of the sky have punctual "hot sources" (hot sky).
  - − Sun (T  $\cong$  12000 f<sup>-0.75</sup> K): point earth-station antennas away from it.
  - Moon (black body radiator): 200 to 300K if pointed directly to it.
  - Earth (satellite)
  - Propagation medium (e.g. rain, oxygen, water vapor): noise reduced as elevation angle increases.
- Man-made sources:
  - Vehicles, industrial machinery
  - Other terrestrial and satellite systems operating at the same frequency of interest.

# Noise from Active Devices

- Active devices produce noise from:
  - Dissipative losses in the active device
  - Dissipative losses in the supporting circuits
  - Electrical noise caused by the active device
- The effective temperature of active devices is specified by the manufacturer
  - Can be measured by a couple of methods
  - Can be (somewhat laboriously) calculated
  - Assumes specific impedance matches
- The effective temperature is (almost) always specified at the <u>input</u> of the device
- The noise is often given as a **noise figure (**see later)

## Noise Model For a Lossy Device



- In some circumstances, we need to use a different model to deal with the noise that reaches the receiver after passing through lossy medium e.g waveguide transmission line and rain losses
- Noise emission is modeled as noise source placed at "output" of atmosphere

- Till now in this model, all noise sources in receiver were replaced by single noise source at the receiver input
- This assumes that all noise comes in from the antenna or is internally generated at the receiver



### Noise Model For a Lossy Device

It is important to remember that the 'gain' of the individual subsystems can be greater or less than 1.0

Lossy element: L = Loss $G=P_{out}/P_{in} = 1/L$  (Note:  $G_{dB} < 0 dB$  because 0 < G < 1 or  $P_{out} < P_{in}$ )

· Noise temperature contribution of a loss is

 $\mathbf{T}_{\mathbf{N}} = \mathbf{T}_{\mathbf{0}}(1 - \mathbf{G})[\mathbf{K}]$ 

- G is the "gain" (smaller than unity), also called transmissivity  $G = 1/Loss = (P_{out}/P_{in})$
- $-T_0$  is the physical temperature of the lossy element
- This temperature is referenced to the <u>output</u> of the lossy element.

The noise model shown in Figure 4.7b replaces all the individual sources of noise

in the receiver by a single noise source at the receiver input. This assumes that all the noise comes in from the antenna or is internally generated in the receiver. In some circumstances, we need to use a different model to deal with noise that reaches the receiver after passing through a lossy medium. Waveguide and rain losses are two examples. When raindrops cause attenuation, they radiate additional noise whose level depends on the attenuation. We can model the noise emission as a noise source placed at the "output" of the atmosphere, which is the antenna aperture. The noise model for an *equivalent output noise source* is shown in Figure 4.7c, and produces a noise temperature  $T_{no}$  given by

$$T_{\rm no} = T_{\rm p} (1 - G_{\rm l}) \tag{4.19}$$

where  $G_1$  is the linear gain (less than unity, not in decibels) of the attenuating device or medium, and  $T_p$  is the physical temperature in degrees kelvin of the device or medium. For an attenuation of A dB, the value of  $G_1$  is given by

$$G_1 = 10^{A/10} \tag{4.20}$$



**FIGURE 4.7** (a) Noise model of receiver. The noisy amplifiers and downconverter have been replaced by noiseless units, with equivalent noise generators at their inputs. (b) Noise model of receiver. All noisy units have been replaced by one noiseless amplifier, with a single noise source  $T_s$  as its input. (c) Noise model for a lossy device. The lossy device has been replaced by a lossless device, with a single noise source  $T_{no}$  at its output.

# Noise from Lossy Elements -1

- All lossy elements reduce the amount of power transmitted through them
  - Carrier or signal power
  - Noise power
- The noise temperature contribution of a loss is:

$$\mathbf{T}_{\mathbf{N}} = \mathbf{T}_{\mathbf{0}}(1 - \mathbf{G}) [\mathbf{K}]$$

$$G = 1/Loss$$

- where **G** is the "gain" (smaller than unit) of the lossy element, also called transmissivity  $(P_{out} / P_{in})$  and  $T_0$  is the physical temperature of the loss.
- Note the temperature is at the <u>output</u> of the loss.

## Noise from Lossy Elements –2

Assume lossy element has gain =  $G_L=1/L$ Notes:  $G_L < 0 \text{ dB}$  (because  $0 < G_L < 1$ )  $T_0$ = physical temperature



## Noise from Lossy Elements –2



### Numerical Examples

#### EXAMPLE 4.3.1

Suppose we have a 4-GHz receiver with the following gains and noise temperatures:

$T_{\rm in} = 25 \ {\rm K}$	$G_{\rm RF} = 23 \ \rm dB$
$T_{\rm RF} = 50 \ {\rm K}$	$G_{\rm IF} = 30 \ \rm dB$
$T_{\rm IF} = 1000  {\rm K}$	
$T_{\rm m} = 500  {\rm K}$	

Calculate the system noise temperature assuming that the mixer has a gain  $G_m = 0$  dB. Recalculate the system noise temperature when the mixer has a 10-dB loss. How can the noise temperature of the receiver be minimized when the mixer has a loss of 10 dB?

The system noise temperature is given by Eq. (4.18)

$$T_{\rm s} = [25 + 50 + (500/200) + (1000/200)] = 87.5 \text{ K}$$

If the mixer had a loss, as is usually the case, the effect of the IF amplifier would be greater. For  $G_{\rm m} = -10$  dB, the linear value is  $G_{\rm m} = 0.1$  as a ratio. Then

$$T_{\rm s} = [25 + 50 + (500/200) + (1000/20)] = 137.5 \text{ K}$$

The lowest system noise temperatures are obtained by using a high gain LNA. Suppose we increase the LNA gain in this example to  $G_{RF} = 50$  dB, giving ratio  $G_{RF} = 10^5$ .

$$T_{\rm s} = [25 + 50 + (500/10^5) + (1000/10^4)] = 75.1 \,{\rm K}$$

The high gain of the RF LNA amplifier has made the system noise temperature almost as low as it can go:  $T_s = T_{in} + T_{RF} = 75$  K in this example. LNAs for use in satellite receivers usually have gains in the range 40–55 dB.

#### **EXAMPLE 4.3.2**

The system illustrated in Example 4.3.1 has an LNA with a gain of 50 dB. A section of lossy waveguide with an attenuation of 2 dB is inserted between the antenna and the RF amplifier. Find the new system noise temperature for a waveguide temperature of 300°K.

The waveguide loss of 2 dB (ratio 1.58) can be treated as a gain,  $G_1$ , that is less than unity: ( $G_1 = 1/1.58 = 0.631$ ). The lossy waveguide attenuates the incoming noise and adds noise

generated by its own ohmic loss. The equivalent noise generator placed at the output of the section of waveguide that represents the noise generated by the waveguide has a noise temperature  $T_{wg}$ , where

$$T_{\rm wg} = T_{\rm p}(1 - G_{\rm l}) = 300(1 - 0.631) = 110.7 \text{ K}$$

The waveguide attenuates the noise from the antenna, so  $T_{in} = 0.63 \times 25 = 15.8 \text{ K}$ The new system noise temperature, referred to the input of the LNA, is

TWA TAL

$$T_{\rm s} = [15.8 + 110.7 + 50 + (500/10^5) + (1000/10^4)] = 176.6 \,\mathrm{K}$$

We can refer the system noise temperature to the antenna output port by dividing the above result by  $G_1$ . This transfers the noise source from the LNA input to the waveguide input.

$$T_{\rm s} = 176.6/0.631 = 279.9 \, {\rm K}$$

The new system noise temperature is 5.7 dB higher than the system noise temperature without the lossy waveguide.

## Numerical Example 1-Noise Temperature

Given: 4 GHz Receiver

$$\begin{array}{ll} T_{in} = T_{a} = 50 \ K & \\ T_{RF} = 50 \ K & \\ T_{m} = 500 \ K & \\ T_{m} = 500 \ K & \\ T_{IF} = 1000 \ K & \\ \end{array} \begin{array}{ll} G_{RF} = 23 \ dB & (=200) \\ G_{m} = 0 \ dB & (=1) \\ G_{IF} = 30 \ dB & (=1000) \end{array}$$

Find: System temperature T<sub>s</sub> at antenna output

System Temperature (T<sub>s</sub>) referred to this point  $T_{in}=50 \text{ K}$   $G_{RF}=200$   $G_{m}=1$   $G_{IF}=1000 \text{ K}$  $G_{IF}=1000 \text{ K}$ 

### Numerical Example 1-Noise Temperature



$$T_{\rm S} = \left[ T_{\rm RF} + T_{\rm in} + \frac{T_{\rm m}}{G_{\rm RF}} + \frac{T_{\rm IF}}{G_{\rm m}G_{\rm RF}} \right]$$

$$= \left[ 50 + 50 + \frac{500}{200} + \frac{1000}{200 \text{ x 1}} \right]$$
$$= \left[ 50 + 50 + 2.5 + 5 \right] = 107.5K$$

### Numerical Example 1-Noise Temperature

(b) If mixer has 10 dB loss

$$G_{\rm m} = -10 \,\mathrm{dB} = 0.1$$
$$T_{\rm s} = \left[50 + 50 + \frac{500}{200} + \frac{1000}{0.1 \times 200}\right] = 152.5 K$$

- Comment:  $G_{RF}G_m$  is too small, so IF amplifier contribution is large

(c) If, in addition,  $G_{RF} = 50 \text{ dB} (=10^5)$ 

$$T_{\rm s} = \left[50 + 50 + \frac{500}{100,000} + \frac{1000}{0.1 \times 100,000}\right] = 100.1K$$

- Now insert a lossy waveguide with L = 2 dB between antenna and LNA
- Find system temperature at LNA input



• Loss of 2 dB, obtain  $G_L$  and  $T_L$ 

$$G_{L} = -2dB = \frac{1}{1.58} = 0.63$$
$$T_{L} = 290(1 - G_{L})$$
$$= 290(1 - 0.63)$$
$$= 107.3K$$

• Input noise power is attenuated by 2 dB. New  $T_{in}$ :

$$T_{\rm in} = T_{\rm a}G_{\rm L} + T_{\rm L}$$
  
= 50 x 0.63 + 107.3 K  
= 138.8 K

$$T_{\rm S} = \begin{bmatrix} T_{in} + T_{RF} + \frac{T_m}{G_{RF}} + \frac{T_{IF}}{G_m G_{RF}} \end{bmatrix}$$
  
=  $\begin{bmatrix} 138.8 + 50 + \frac{500}{200} + \frac{1000}{200 \times 1} \end{bmatrix}$   
=  $\begin{bmatrix} 138.8 + 50 + 2.5 + 5 \end{bmatrix} = 196.3K$   
From Previous  
Example of  
Noise  
Temperature

 So noise temperature (power) increased from 107.5 to 196.3 K at the same reference point

$$\frac{N_2}{N_1} = \frac{KT_{s2}B}{KT_{s1}B} = \frac{T_{s2}}{T_{s1}} = 1.82 \quad \text{or} \quad 2.6 \,\text{dB}$$

- Inserting 2 dB loss at receiver front end decreased carrier power (C) by 2 dB and increased noise temperature by 88.8 K, from 107.5 K to 196.3 K (comparing at the same reference point)
- N has increased by 2.6 dB
- C has decreased by 2 dB
- Net result: C/N has been reduced by 4.6 dB
- Moral:

Losses before LNA must be kept very small

### Noise Figure and Noise Temperature

- Noise figure (NF) is a measure of degradation of the signal to noise ratio (SNR), caused by components in the RF signal chain
- Noise Figure is defined as the ratio of signal to noise ratio at the input to that at the output

• NF= 
$$\frac{[S/N]_{in}}{[S/N]_{out}}$$
  $F_N = \frac{[S/N]_{in}}{[S/N]_{out}} = \frac{N_{out}}{kT_0B_NG}$ 

- Noise Figure can be converted to noise temperature T<sub>d</sub>
- $T_d = T_o(NF-1)$ ,
  - where NF is a linear ratio, not in decibels and where T<sub>o</sub> is reference temperature used to calculate standard noise figure. Usually 290 K

Table 4.3 gives a comparison between noise figure and noise temperature over the range encountered in typical systems.

TABLE 4.3	Comparison	of Noise	Temperature	and Nois	e Figure	

Noise temperature (K) Noise figure (dB)	0 0	20 0.29	40 0.56	60 0.82	80 1.06	100 1.29	120 1.50	150 1.81	200 2.28	290 3.0
Noise temperature (K)	400	600	800	1,000	1,500 7 9	2,000	3,000	5,000 12.6	10,000 15 5	
Noise figure (dB)	3.8	4.9	5.8	6.5	7.9	9.0	10.5	12.6	15.5	

#### G/T Ratio for Earth Station: A Figure of Merit

- Transmitters are characterized by EIRP
- Receivers are characterized by  $G_r/T_s$  or G/T
- G/T describes the sensitivity of a receive system
- Also called the system figure of merit as it specifies the quality of a receiving earth station or a satellite receiving system

#### Link Budget Equation and G/T

Now we have defined C and N, write C/N as

$\left(\frac{C}{N}\right) = \frac{P_r}{kT_sB} = \frac{P_tG_tG_r}{kT_sB} \left[\frac{\lambda}{4\pi R}\right]^2$	
$= \frac{P_t G_t}{kB} \left[ \frac{\lambda}{4\pi R} \right]^2 \left( \frac{G_r}{T_s} \right)$	

• Therefore,

$$\frac{C}{N} \propto \frac{G_r}{T_s}$$

Usually given in dB/K or dBK<sup>-1</sup>

#### Link Budget Summary



#### 4.3.2 Calculation of System Noise Temperature

The equivalent circuits in Figure 4.7a can be used to represent a receiver for the purpose of noise analysis. The noisy devices in the receiver are replaced by equivalent noiseless blocks with the same gain and noise generators at the input to each block such that the block produces the same noise at its output as the device it replaces. The entire receiver is then reduced to a single equivalent noiseless block with the same end-to-end gain as the actual receiver and a single noise source at its input with temperature *T*s, called the *system noise temperature*.



**Figure 4.7a** Noise model of receiver.  $T_{in}$  is the noise temperature of the sky and antenna,  $T_{rf}$  is the noise temperature of the LNA,  $G_{rf}$  is the gain of the LNA,  $T_m$  is the noise temperature of the mixer,  $G_m$  is the gain of the mixer,  $T_{if}$  is the noise temperature of the IF amplifier.  $P_n$  is the noise power at the output of the receiver.

The total noise power at the output of the IF amplifier of the receiver in Figure 4.7a is given by

$$P_{\rm n} = G_{\rm IF} \, kT_{\rm IF} \, B_{\rm n} + G_{\rm IF} \, G_{\rm m} \, kT_{\rm m} \, B_{\rm n} + G_{\rm IF} \, G_{\rm m} \, G_{\rm RF} \, kB_{\rm n} (T_{\rm RF} + T_{\rm in}) \, \text{watts} \quad (4.16)$$

.

where  $G_{\rm RF}$ ,  $G_{\rm m}$ , and  $G_{\rm IF}$  are respectively the gains of the RF amplifier, mixer, and IF amplifier, and  $T_{\rm RF}$ ,  $T_{\rm m}$ , and  $T_{\rm IF}$  are their equivalent noise temperatures.  $T_{\rm in}$  is the noise temperature of the antenna, measured at its output port. Antennas do not usually generate noise unless they have ohmic loss.  $T_{\rm in}$  accounts for noise radiated into the antenna from the signal path through the atmosphere and also any noise radiated from the earth into the sidelobes of the antenna pattern. Initial calculations of system noise temperature are usually made by assuming clear sky conditions and using an assumed value for attenuation on the signal path. Calculation of an exact antenna temperature requires convolution of the 3-D antenna pattern with a model of the 3-D temperature profile of the earth and sky, and is rarely attempted.

Any part of the signal path that incurs a loss by absorption of signal energy results in the generation of thermal noise. This is called an *ohmic loss* to distinguish it from other types of loss such as path loss. The loss in the atmosphere in clear sky conditions (no rain present) is caused mainly by absorption of microwave signal energy by oxygen and water vapor molecules. Because these molecules absorb microwave energy they also radiate thermal noise that is received by the earth station antenna. Thermal noise generated by the atmosphere is characterized by sky noise temperature.

Figure 4.7a shows a model of a noiseless receiver in which each block in the receiver is replaced by a noiseless block followed by an equivalent noise source at the output of the block. The noise source in each case has a noise temperature that results in the same output noise power as the noisy device, measured in the receiver noise bandwidth. In Figure 4.7b, the noise sources are combined into a single equivalent system noise source with a noise temperature  $T_s$  at the input of the receiver, and the receiver is represented as a single noiseless block that has the same end to end gain as the receiver in Figure 4.7a. Figure 4.7c is an alternative configuration to Figure 4.7b with a single equivalent noise source with noise temperature  $T_{no}$  at the output of the receiver.

Equation (4.16) can be rewritten as

$$P_{n} = G_{IF} G_{m} G_{RF} \left[ \frac{k T_{IF} B_{n}}{(G_{m} G_{RF})} + \frac{k T_{m} B_{n}}{G_{RF}} + k B_{n} (T_{RF} + T_{in}) \right]$$
$$= G_{m} G_{IF} G_{RF} k B_{n} \left[ T_{RF} + T_{in} + \frac{T_{m}}{G_{RF}} + \frac{T_{IF}}{G_{m} G_{RF}} \right] watts$$
(4.17)



**Figure 4.7b** Noise model of receiver with a single noise source  $T_s$ , the system noise temperature, at the input to a noiseless receiver with identical gain to the receiver in Figure 4.7a.



**Figure 4.7c** Noise model of receiver with a single noise source  $T_{no}$ , the system noise temperature, at the input to a noiseless receiver with identical gain to the receiver in Figure 4.7a.

The single source of noise shown in Figure 4.7b with noise temperature  $T_s$  generates the same noise power  $P_n$  at its output as the model in Figure 4.7a

$$P_{\rm n} = G_{\rm m} G_{\rm IF} G_{\rm RF} k T_{\rm s} B_{\rm n} \text{ watts}$$
(4.18)

The noise power at the output of the noise model in Figure 4.7b will be the same as the noise power at the output of the noise model in Figure 4.7a if

$$kB_{\rm n} T_{\rm s} = kB_{\rm n} \left[ T_{\rm RF} + T_{\rm in} + \frac{T_{\rm m}}{G_{\rm RF}} + \frac{T_{\rm IF}}{G_{\rm m}G_{\rm RF}} \right] \text{ watts}$$
(4.19)

Hence the equivalent noise source in Figure 4.7b has a system noise temperature  $T_{\rm s}$  where

$$T_{\rm s} = \left[ T_{\rm RF} + T_{\rm in} + \frac{T_{\rm m}}{G_{\rm RF}} + \frac{T_{\rm IF}}{G_{\rm m}G_{\rm RF}} \right] \,\mathrm{K} \tag{4.20}$$

Succeeding stages of the receiver contribute less and less noise to the total system noise temperature. Frequently, when the RF amplifier in the receiver front end has a high gain, the noise contributed by the IF amplifier and later stages can be ignored and the system noise temperature is simply the sum of the antenna noise temperature and the LNA noise temperature, so  $T_s = T_{antenna} + T_{LNA}$ . Note that the values for component gains in Eq. (4.20) must be linear ratios, not in decibels.

The noise model shown in Figure 4.7b replaces all the individual sources of noise in the receiver by a single noise source at the receiver input. This assumes that all the noise comes in from the antenna or is internally generated in the receiver. In some circumstances, we need to use a different model to deal with noise that reaches the receiver after passing through a lossy medium. Waveguide and rain losses are two examples. When raindrops cause attenuation, they radiate additional noise whose level depends on the attenuation. We can model the noise emission as a noise source placed at the output of the atmosphere, which is the antenna aperture. The noise model for an *equivalent output noise source* is shown in Figure 4.7c, and produces a noise temperature  $T_{no}$  given by

$$T_{\rm no} = T_{\rm p}(1 - G_{\rm l}) \,\mathrm{K}$$
 (4.21)

where  $G_l$  is the linear gain (less than unity, not in decibels) of the attenuating device or medium, and  $T_p$  is the physical temperature in degrees kelvin of the device or medium. For an attenuation of A dB, the value of  $G_l$  is given by

$$G_{\rm l} = 10^{-A/10} \tag{4.22}$$

	25 K	
$T_{\rm RF}$	50 K	
T <sub>m</sub>	500 K	
$T_{\rm IF}$	1000 K	
$G_{RF}$	23 dB	(ratio 200)
$G_{ m IF}$	30 dB	(ratio 1000)

Table 4.3 Gain and noise temperature values for 4 GHz receiver example

#### **Example 4.3**

Suppose we have a 4 GHz receiver with the gains and noise temperatures in Table 4.3.

Calculate the system noise temperature assuming that the mixer has a gain  $G_m = 0$  dB. Recalculate the system noise temperature when the mixer has a 10 dB loss. How can the noise temperature of the receiver be minimized when the mixer has a loss of 10 dB?

#### Answer

The system noise temperature is given by Eq. (4.20)

 $T_{\rm s} = [25 + 50 + (500/200) + (1000/200)] = 82.5 \text{ K}$ 

If the mixer had a loss, as is usually the case, the effect of the IF amplifier would be greater. For a mixer with a loss of 10 dB,  $G_m = -10$  dB and the linear value is  $G_m = 0.1$  as a ratio. Then

 $T_{\rm s} = [25 + 50 + (500/200) + (1000/20)] = 127.5 \text{ K}$ 

The lowest system noise temperatures are obtained by using a high gain LNA. Suppose we increase the LNA gain in this example to  $G_{\rm RF} = 50$  dB, giving a ratio  $G_{\rm RF} = 10^5$ . Then

 $T_{\rm s} = [25 + 50 + (500/10^5) + (1000/10^4)] = 75.1 \,\rm K$ 

The high gain of the RF LNA has made the system noise temperature almost as low as it can go. The minimum value of  $T_s$  is given by  $T_{s \min}$  where in this example

$$T_{\rm s\ min} = T_{\rm in} + T_{\rm rf} = 75 \,\rm K$$

The mixer and IF amplifier contribute almost nothing to the system noise temperature. LNAs for use in satellite receivers usually have gains in the range 40–55 dB with the result that system noise temperature can be equated to  $T_{in} + T_{rf}$ .

#### **Example 4.4**

The system illustrated in Example 4.3, Table 4.3, has an LNA with a gain of 50 dB. A section of lossy waveguide with an attenuation of 2 dB is inserted between the antenna and the RF amplifier. Find the new system noise temperature for a waveguide temperature of 300°K.

#### Answer

The waveguide loss of 2 dB (ratio 1.58) can be treated as a gain,  $G_l$  that is less than unity:  $G_l = 1/1.58 = 0.631$ . The lossy waveguide attenuates the incoming noise and adds noise

generated by its own ohmic loss. The equivalent noise generator placed at the output of the section of waveguide that represents the noise generated by the waveguide has a noise temperature  $T_{\rm wg}$ , where

$$T_{\rm wg} = T_{\rm p}(1 - G_{\rm l}) = 300(1 - 0.631) = 110.7 \,\rm K$$

The waveguide attenuates the noise from the antenna, so  $T_{in} = 0.631 \times 25 = 15.8$  K The new system noise temperature, referred to the input of the LNA, is

 $T_{\rm s} = 15.8 + 110.7 + 50 + (500/10^5) + (1000/10^4) = 176.6 \,\rm K$ 

The system noise temperature is  $10 \log_{10} (176.6/75) = 3.7 \text{ dB}$  higher than the original receiver configuration without the 2 dB waveguide loss. In addition, we have lost 2 dB of signal power so the receiver output CNR is reduced by 5.7 dB. Avoiding losses between the antenna and LNA is critical in a low noise receiver, which is why the LNA is mounted immediately behind the antenna feed in virtually all satellite communication receivers. Antennas for GPS receivers typically include an LNA in the antenna base, powered by a DC voltage across the conductors of the coaxial cable connecting the antenna to the GPS receiver. However, the RF filter in a GPS receiver is typically located ahead of the LNA to block interference that could saturate the amplifier. Any loss in the RF filter and the corresponding increase in system noise temperature is accepted in exchange for the reduction in interference.

We can refer the system noise temperature to the antenna output port by dividing the above result by  $G_l$ . This transfers the noise source from the LNA input to the waveguide input.

$$T_{\rm s} = 176.6/0.631 = 280 \,{\rm K}$$

The new system noise temperature is 5.7 dB higher than the system noise temperature without the lossy waveguide, but there is no longer a loss of signal, so we have the same result for the reduction in CNR.

Note that when the system noise temperature is low, each 0.1 dB of attenuation ahead of the RF amplifier will add approximately 6.6 K to the system noise temperature. Using the formula in Example 4.2 with  $T_p = 290$  °K,  $G_l = -0.1$  dB = 0.977 as a ratio gives

 $T_{\rm no} = 290 \times 0.023 = 6.6 \, {\rm K}$ 

This is the reason for placing the front end of the receiver at the output of the antenna feed. Waveguide losses ahead of the LNA can have a disastrous effect on the system noise temperature of low noise receiving systems.

The value of  $T_{in}$  in Examples 4.3 and 4.4 was set to 25 K. This corresponds to an atmospheric path attenuation of approximately  $0.1 \times 25/6.6 = 0.4$  dB, using the above formula and rounding to the nearest tenth of a dB, assuming a noiseless antenna. Note that in the analysis of communication systems, results in decibels are usually quoted to the nearest tenth of a dB. Including an additional decimal place implies that all calculations are correct to 0.01 dB, which is never the case because of the assumptions made at the beginning of the calculation.

I

Table 4.4 Comparison of noise temperature and noise figure

Noise temperature										
(K)	0	20	40	60	80	100	120	150	200	290
Noise figure (dB)	0	0.29	0.56	0.82	1.06	1.29	1.50	1.81	2.28	3.0
Noise temperature (K)	400	600	800	1000	1500	2000	3000	5000	10 000	
Noise figure (dB)	3.8	4.9	5.8	6.5	7.9	9.0	10.5	12.6	15.5	

#### 4.3.3 Noise Figure and Noise Temperature

Noise figure is frequently used to specify the noise generated within a device. The operational noise figure is defined by the following formula (Krauss et al. 1980, p. 26)

$$NF = (SNR)_{\rm in} / (SNR)_{\rm out}$$
(4.23)

where  $(SNR)_{in}$  is the SNR at the input to the device and  $(SNR)_{out}$  is the SNR at the output of the device. Because noise temperature is more useful in satellite communication systems, it is best to convert noise figure to noise temperature,  $T_n$ . The relationship is

$$T_{\rm n} = T_{\rm o}(NF - 1) \,\mathrm{K}$$
 (4.24)

where the noise figure is a linear ratio, not in decibels and where  $T_{\rm o}$  is the reference temperature used to calculate the standard noise figure – usually 290 K. NF is frequently given in decibels and must be converted to a ratio before being used in Eq. (4.24).

Table 4.4 gives a comparison between noise figure and noise temperature over the range encountered in typical systems.

#### 4.3.4 G/T Ratio for Earth Stations

The link equation can be rewritten in terms of CNR at the earth station

$$\frac{C}{N} = \left[\frac{P_t G_t G_r}{kT_s B_n}\right] \left[\frac{\lambda}{4\pi R}\right]^2 = \left[\frac{P_t G_r}{kB_n}\right] \left[\frac{\lambda}{4\pi R}\right]^2 \left[\frac{G_r}{T_s}\right]$$
(4.25)

Thus  $CNR \propto G_r/T_s$ , and the terms in the square brackets are all constants for a given satellite system. The ratio  $G_r/T_s$ , which is usually quoted as simply G/T in decibels with units dBK<sup>-1</sup>, can be used to specify the quality of a receiving earth station or a satellite receiving system, since increasing  $G_r/T_s$  increases the received CNR.

Satellite terminals may be quoted as having a negative G/T, which is below  $0 \, dBK^{-1}$ . This simply means that the numerical value of  $G_r$  is smaller than the numerical value of  $T_s$ .

#### Example 4.5 Earth Station G/T Ratio

An earth station antenna has a diameter of 30 m with an aperture efficiency of 68% and is used to receive a signal at 4150 MHz. At this frequency, the system noise temperature is 60 K when the antenna points at the satellite at an elevation angle of 28°. What is the

earth station G/T ratio under these conditions? If heavy rain causes the sky temperature to increase so that the system noise temperature rises to 88 K, what is the new G/T value?

#### Answer

First calculate the antenna gain. For a circular aperture

$$G_{\rm r} = \eta_{\rm A} 4\pi A/\lambda^2 = \eta_{\rm A} (\pi D/\lambda)^2$$

At 4150 MHz,  $\lambda = 0.0723$  m. Then

$$G = 0.68 \times (\pi 30/0.0723)^2 = 1.16 \times 10^6$$
 or 60.6 dB

Converting  $T_{\rm s}$  into dBK

$$T_{\rm s} = 10 \log_{10} 60 = 17.8 \, \rm dBK$$

$$G/T = 60.6 - 17.8 = 42.8 \text{ dBK}$$

If  $T_s = 88 \text{ K}$  in heavy rain

$$\frac{G}{T} = 60.6 - 19.4 = 41.2 \text{ dB/K}$$
### 4.4 DESIGN OF DOWNLINKS

The design of any satellite communication is based on two objectives: meeting a minimum C/N ratio for a specified percentage of time, and carrying the maximum revenue earning traffic at minimum cost. There is an old saying that "an engineer is a person who can do for a dollar what any fool can do for one hundred dollars." This applies to satellite communication systems. Any satellite link can be designed with very large antennas to achieve high C/N ratios under all conditions, but the cost will be high. The art of good system design is to reach the best compromise of system parameters that meets the specification at the lowest cost. For example, if a satellite link is designed with sufficient margin to overcome a 20-dB rain fade rather than a 3-dB fade, earth station antennas with seven times the diameter are required.

All satellite communications links are affected by rain attenuation. In the 6/4GHz band the effect of rain on the link is small. In the 14/11 GHz (Ku) band, and even more so in the 30/20 GHz (Ka) band, rain attenuation becomes all important. Satellite links are designed to achieve reliabilities of 99.5 to 99.99%, averaged over a long period of time, typically a year. That means the C/N ratio in the receiver will fall below the minimum permissible value for proper operation of the link for between 0.5 and 0.01% of the specified time; the link is then said to suffer an outage. The time period over which the percentage of time is measured can be a month, sometimes the "worst month" in attenuation terms, or a year. Rain attenuation is a very variable phenomenon, both with time and place. Chapter 8 discusses the prediction of path attenuation and provides ways to estimate the likely occurrence of outages on a given link. In this chapter we will simply assume certain rain attenuation statistics to use in examples of link design.

# Link Budgets

C/N ratio calculation is simplified by the use of link budgets. A link budget is a tabular method for evaluating the received power and noise power in a radio link. Link budgets invariably use decibel units for all quantities so that signal and noise powers can be calculated by addition and subtraction. Since it is usually impossible to design a satellite link at the first attempt, link budgets make the task much easier because, once a link budget has been established, it is easy to change any of the parameters and recalculate the result. Tables 4.4a and 4.4b show a typical link budget for a C-band downlink using a global beam on a GEO satellite and a 9-m earth station antenna.

The link budget must be calculated for an individual transponder, and must be repeated for each of the individual links. In a two-way satellite communication link there will be four separate links, each requiring a calculation of C/N ratio. When a bent pipe transponder is used the uplink and downlink C/N ratios must be combined to give an *overall C/N*. In this section we will calculate the C/N ratio for a single link. Later examples in this chapter demonstrate the evaluation of a complete satellite communication system.

Link budgets are usually calculated for a *worst case*, the one in which the link will have the lowest C/N ratio. Factors which contribute to a worst case scenario include: an earth station located at the edge of the satellite coverage zone where the received signal is typically 3 dB lower than in the center of the zone because of the satellite antenna pattern, maximum path length from the satellite to the earth station, a low elevation angle at

The calculation of carrier to noise ratio in a satellite link is based on the two equations for received signal power and receiver noise power that were presented in Sections 4.1 and 4.2. Equation 4.11 gives the received carrier power in dB watts as

$$P_{\rm r} = {\rm EIRP} + G_{\rm r} - L_{\rm p} - L_{\rm a} - L_{\rm r} - L_{\rm t} \,{\rm dBW}$$
 (4.24)

A receiving terminal with a system noise temperature  $T_s K$  and a noise bandwidth  $B_n$  Hz has a noise power  $P_n$  referred to the output terminals of the antenna where

$$P_{\rm n} = kT_{\rm s}B_{\rm n} \text{ watts} \tag{4.25}$$

The receiving system noise power is usually written in decibel units as

$$N = k + T_{\rm s} + B_{\rm n} \,\mathrm{dBW} \tag{4.26}$$

where k is Boltzmann's constant (-228.6 dBW/K/Hz),  $T_s$  is the system noise temperature in dBK, and  $B_n$  is the noise bandwidth of the receiver in dBHz. Note that because we are working in units of power, all decibel conversions are made as  $10 \log_{10}(T_s)$  or  $10 \log_{10}(B_n)$ . The 20  $\log_{10}$  factor used in the calculation of path loss results from the  $(4\pi R/\lambda)^2$  term in the path loss equation.

## Link Budget Example: C-Band Downlink for Earth Coverage Beam

The satellite used in this example (see Tables 4.4a and 4.4b) is in geostationary earth orbit and carries 24 C-band transponders, each with a bandwidth of 36 MHz. The downlink band is 3.7-4.2 GHz and the satellite uses orthogonal circular polarizations to provide an effective RF bandwidth of 864 MHz. The satellite provides coverage of the visible earth, which subtends an angle of approximately 17° from a satellite in a geostationary orbit, by using a global beam antenna. Since antenna beamwidth and gain are linked together [3 dB beamwidth  $\approx \sqrt{(33,000/G)}$  where G is a ratio, not in decibels], the on-axis gain of the global beam antenna is approximately 20 dB. However, we must make the link budget calculation for an earth station at the edge of the coverage zone of the satellite where the effective gain of the antenna is 3 dB lower, at 17 dB. The C/N ratio for the downlink is calculated in clear air conditions and also in heavy rain.

#### TABLE 4.4a C-Band GEO Satellite Link Budget in Clear Air

C-band	satellite parameters	
	Transponder saturated output power	20 W
	Antenna gain, on axis	20 dB
	Transponder bandwidth	36 MHz
	Downlink frequency band	3.7–4.2 GHz
Signal	FM-TV analog signal	
	FM-TV signal bandwidth	30 MHz
	Minimum permitted overall C/N in receiver	9.5 dB
Receivir	ng C-band earth station	
	Downlink frequency	4.00 GHz
	Antenna gain, on axis, 4 GHz	49.7 dB
	Receiver IF bandwidth	27 MHz
	Receiving system noise temperature	75 K
Downlin	nk power budget	
	$P_{\rm t}$ = Satellite transponder output power, 20 W	13.0 dBW
	$B_{\rm o}$ = Transponder output backoff	-2.0 dB
	$G_{t}$ = Satellite antenna gain, on axis	20.0 dB
	$G_{\rm r}$ = Earth station antenna gain	49.7 dB
	$L_{\rm p}$ = Free space path loss at 4 GHz	–196.5 dB
	$L_{ant}$ = Edge of beam loss for satellite antenna	-3.0 dB
	$L_{a}$ = Clear air atmospheric loss	-0.2 dB
	$L_{\rm m}$ = Other losses	<u>-0.5 dB</u>
	$P_{\rm r}$ = Received power at earth station	-119.5 dBW
		- 5 (M)
Down	link noise power budget in clear air	
	k = Boltzmann's constant	-228.6 dBW/K/Hz
	$T_{\rm s}$ = System noise temperature, 75 K	18.8 dBK
	$B_{\rm n}$ = Noise bandwidth, 27 MHz	74.3 dBHz
	N = Receiver noise power	-135.5 dBW
C/N ra	atio in receiver in clear air	

 $C/N = P_r - N = -119.5 \text{ dBW} - (-135.5 \text{ dBW}) = 16.0 \text{ dB}$ 

#### TABLE 4.4b C-Band Downlink Budget in Rain

Pro	a = R	eceived power at earth station in clear air	_`	19.5	dBW
A	= R	ain attenuation		-1.0	dB
Pra	in = R	eceived power at earth station in rain	_'	120.5	dBW
N	<sub>a</sub> = R	eceiver noise power in clear air		135.5	dBW
$\Delta I$	V <sub>rain</sub> = Ir	ncrease in noise temperature due to rain		2.3	dB
N <sub>r</sub>	<sub>ain</sub> = R	eceiver noise power in rain	- '	133.2	dBW
C/N ratio ir	receive	er in rain			
C/	N = P	$P_{rain} - N_{rain} = -120.5 \text{ dBW} - (-133.2 \text{ dBW})$	=	12.7	dB

An antenna with a gain of 20 dB has an effective aperture diameter of 5.6 wavelengths  $[G = \eta (\pi D/\lambda)^2]$ , which gives D = 0.42 m at a frequency of 4 GHz. The calculation of C/N ratio is made at a mid-band frequency of 4 GHz.

The saturated output power of the transponder is 20 W = 13 dBW. We will assume an output back-off of 2 dB, so that the power transmitted by the transponder is 11 dBW. Hence the on-axis EIRP of the transponder and antenna is  $P_tG_t = 11 + 20 = 31$  dBW. The transmitted signal is a single 30-MHz bandwidth analog FM-TV channel in this example. Following common practice for analog TV transmission, the receiver noise bandwidth is set to 27 MHz, slightly less than the 30-MHz bandwidth of the FM-TV signal.

The receiving earth station has an antenna with an aperture diameter of 9 m and a gain of 49.7 dB at 4 GHz, and a receiving system noise temperature of 75 K in clear air conditions. The G/T ratio for this earth station is  $G/T = 49.7 - 10 \log_{10} 75 = 30.9 \text{ dBK}^{-1}$ . The maximum path length for a GEO satellite link is 40,000 km, which gives a path loss of 196.5 dB at 4 GHz ( $\lambda = 0.075$  m). We must make an allowance in the link budget for some losses that will inevitably occur on the link. At C band, propagation losses are small, but the slant path through the atmosphere will suffer a typical attenuation of 0.2 dB in clear air. We will allow an additional 0.5-dB margin in the link design to account for miscellaneous losses, such as antenna mispointing, polarization mismatch, and antenna degradation, to ensure that the link budget is realistic.

The earth station receiver C/N ratio is first calculated for *clear air* conditions, with no rain in the slant path. The C/N ratio is then recalculated taking account of the effects of rain. The minimum permitted overall C/N ratio for this link is 9.5 dB, corresponding to the FM threshold of an analog satellite TV receiver. Table 4.4a shows that we have a downlink C/N of 16.0 dB in clear air, giving a *link margin* of 6.5 dB. This link margin is available in clear air conditions, but will be reduced when there is rain in the slant path.

Heavy rain in the slant path can cause up to 1 dB of attenuation at 4 GHz, which reduces the received power by 1 dB and increases the noise temperature of the receiving system. Using the output noise model discussed in the previous section with a medium temperature of 273 K, and a total path loss for clear air plus rain of 1.2 dB (ratio of 1.32), the sky noise temperature in rain is

$$T_{\rm sky} = 273 \times (1 - 1/1.32) = 66 \,\rm K$$

In clear air the sky noise temperature is about 13 K, the result of 0.2 dB of clear air attenuation. The noise temperature of the receiving system has therefore increased by (66 - 13) K = 53 K to 75 + 53 K = 128 K with 1 dB rain attenuation in the slant path, from a clear air value of 75 K. This is an increase in system noise temperature of 2.3 dB.

We can now adjust the link budget very easily to account for heavy rain in the slant path without having to recalculate the C/N ratio from the beginning. The received carrier power is reduced by 1 dB because of the rain attenuation and the system noise temperature is increased by 2.3 dB. Table 4.4b shows the new downlink budget in rain.

The C/N ratio in rain has a margin of 3.2 dB over the minimum permissible C/N ratio of 9.5 dB for an analog FM-TV transmission. The C/N margin will translate into a higher than needed S/N ratio in the TV baseband signal, and can be traded off against earth station antenna gain to allow the use of smaller (and therefore lower cost) antenna. We should always leave a small margin for unexpected losses if we want to guarantee a particular level of reliability in the link. In this case, we will use a 2-dB margin and examine how the remaining 1.2 dB of link margin can be traded against other parameters in the system.

A reduction in earth station antenna gain of 1.2 dB is a reduction in the gain value, as a ratio, of 1.32. Antenna gain is proportional to diameter squared, so the diameter of the earth station antenna can be reduced by a factor of  $\sqrt{1.32} = 1.15$ , from 9 m to 7.8 m.

We could transmit a QPSK signal from the satellite instead of an analog FM signal. Using the 27-MHz noise bandwidth receiver, we could transmit a digital signal at 54 Mbps using QPSK, but would require a minimum C/N ratio in the receiver of 14.6 dB, allowing a 1-dB implementation margin and a minimum BER of  $10^{-6}$ . The link margin would be -1.9 dB under heavy rain conditions, so we would need to increase the earth station antenna diameter by a factor of 1.55 to 13.9 m to provide a C/N ratio of 14.6 dB under heavy rain conditions. A 54-Mbps digital signal could carry seven digital TV signals

using MPEG-2 compression, a much more attractive proposition than carrying a single analog FM-TV signal, although at the cost of a larger earth station antenna.

Global beam antennas are not widely used, although most Intelsat satellites carry them. Regional TV signal distribution is much more common, so the C-band link in Tables 4.4a and 4.4b is more likely to use a regional antenna, serving the United States, for example, with a 6° by 3° beam. The gain of a typical satellite antenna providing coverage of the 48 contiguous states is 32.0 dB on axis ( $G = 33,000/\theta_1 \times \theta_2$ ), which is 12.0 dB higher than the on-axis gain of a global beam. Using the link budget in Tables 4.4a and 4.4b, we can trade the extra 12-dB gain of a regional coverage satellite antenna for a reduction in earth station antenna dimensions. For the example of a 9.0-m antenna receiving analog FM-TV, we could reduce the antenna diameter by a factor of 4 to 2.25 m (approximately 7 ft 6 inch diameter). This is the smallest size of antenna used by home satellite TV systems operating in C band.

The above examples show how the link budget can be used to study different combinations of system parameters. Most satellite link analyses do not yield the wanted result at the first try, and the designer or analyst must use the link budget to adjust system parameters until an acceptable result is achieved. More examples of link budgets are included later in this chapter.

#### Question #1.

A C-band earth station has an antenna with a transmit gain of 54 dB. The transmitter output power is set to 100 W at a frequency of 6.100 GHz. The signal is received by a satellite at a distance of 37,500 km by an antenna with a gain of 26 dB. The signal is then routed to a transponder with a noise temperature of 500 K, a bandwidth of 36 MHz, and a gain of 110 dB.

a. Calculate the path loss at 6.1 GHz. Wavelength is 0.04918 m.

b. Calculate the power at the output port (sometimes called the output waveguide flange) of the satellite antenna, in dBW.

c. Calculate the noise power at the transponder input, in dBW, in a bandwidth of 36 MHz.

d. Calculate the C/N ratio, in dB, in the transponder.

e. Calculate the carrier power, in dBW and in W, at the transponder output.

**Answer:** Path loss =  $20 \log (4 \pi R / \lambda) = 20 \log (4 \pi \times 37,500 \times 10^3 / 0.04918) dB$ 

 $L_p = 199.6 \, dB$ 

Answer: Uplink power budget gives

 $P_r = P_t + G_t + G_r - L_p dBW$ = 20 + 54 + 26 - 199.6 = -99.6 dBW

**Answer**: N = k T<sub>s</sub> B<sub>N</sub> = -228.6 + 27 + 75.6 = -126.0 dBW

**Answer:**  $C/N = P_r - N = -99.6 + 126.0 = 26.4 dB$ 

**Answer:** The gain of the transponder is 110 dB. Output power is

 $P_t = P_r + G = -99.6 + 110 = 10.4 \text{ dBW} \text{ or } 10^{1.04} = 11.0 \text{ W}.$ 

2. The satellite in Question #1 above serves the 48 contiguous states of the US. The antenna on the satellite transmits at a frequency of 3875 MHz to an earth station at a distance of 39,000 km. The antenna has a 6° E-W beamwidth and a 3° N-S beamwidth. The receiving earth station has an antenna with a gain of 53 dB and a system noise temperature of 100 K and is located at

the edge of the coverage zone of the satellite antenna. (Assume antenna gain is 3 dB lower than in the center of the beam)

Ignore your result for transponder output power in Question 1 above. Assume the transponder carrier power is 10 W at the input port of the transmit antenna on the satellite.

a. Calculate the gain of the satellite antenna in the direction of the receiving earth station. [Use the approximate formula G = 33,000/(product of beamwidths).]

b. Calculate the carrier power received by the earth station, in dBW.

c. Calculate the noise power of the earth station in 36 MHz bandwidth.

d. Hence find the C/N in dB for the earth station.

a. Calculate the gain of the satellite antenna in the direction of the receiving earth station.
[Use the approximate formula G = 33,000/(product of beamwidths).]
Answer: G = 33,000 / (6 x 3) = 1833 or 32.6 dB on axis.
Hence satellite antenna gain towards earth station is 32.6 - 3 = 29.6 dB.

b. Calculate the carrier power received by the earth station, in dBW. **Answer:** Calculate the path loss at 3.875GHz. Wavelength is 0.07742 m. Path loss = 20 log ( $4 \pi R / \lambda$ ) = 20 log ( $4 \pi \times 39,000 \times 10^3 / 0.07742$ ) dB

 $L_p = 196.0 \text{ dB}$ 

Downlink power budget gives

 $P_r = P_t + G_t + G_r - L_p dBW$ = 10 + 29.6 + 53 - 196.0 = -103.4 dBW

c. Calculate the noise power of the earth station in 36 MHz bandwidth. Answer: N = k T<sub>s</sub> B<sub>N</sub> = -228.6 + 20 + 75.6 = -133.0 dBW

d. Hence find the C/N in dB for the earth station.

**Answer:**  $C/N = P_r - N = -103.4 + 133.0 = 29.6 dB$ 

3. A 14/11 GHz satellite communication link has a transponder with a bandwidth of 52 MHz which is operated at an output power level of 20W. The satellite transmit antenna gain at 11 GHz is 30 dB towards a particular earth station. Path loss to this station is 206 dB, including clear air atmospheric loss.

The transponder is used in FDMA mode to send 500 BPSK voice channels with half rate FEC coding. Each coded BPSK signal has a symbol rate of 50 kbps and requires a receiver with a noise bandwidth of 50 kHz per channel. The earth stations used to receive the voice signals have antennas with a gain of 40 dB (1m diameter) and a receiver with  $T_{system} = 150$ K in clear air, and IF noise bandwidth 50 kHz.

a. Calculate the power transmitted by the satellite in one voice channel.

b. Calculate the C/N in clear air for an earth station receiving one BPSK voice signal.

c. What is the margin over a coded BPSK threshold of 6 dB?

a. Calculate the power transmitted by the satellite in one voice channel.

**Answer:** In FDMA, the output power of the transmitter is divided equally between the channels. For  $P_t = 20$  W and 500 channels, power per channels is 20 / 500 = 40 mW/ch.

b. Calculate the C/N in clear air for an earth station receiving one BPSK voice signal.
 Answer: Each channel receiver has a noise bandwidth of 50 kHz or 47 dBHz.
 Path loss at 11GHz is 206.0 dB, including atmospheric loss..

Downlink power budget for one FDMA channel gives

 $P_r = P_t + G_t + G_r - L_p \ dBW$ = -14.0 + 30.0 + 40.0 - 206.0 = -150.0 dBW

The noise power at the input to the receiver is

 $N = k T_s B_N = -228.6 + 21.8 + 47.0 = -159.8 dBW$ Hence C/N = P<sub>r</sub> - N = -150.0 + 159.8 = 9.8 dB.

c. What is the margin over a coded BPSK threshold of 6 dB?

Answer: Margin is receiver C/N - minimum permitted C/N, in dB

Margin = 9.8 - 6.0 = 3.8 dB.

**4.** Geostationary satellites use L, C, Ku and Ka bands. The path length from an earth station to the GEO satellite is 38,500 km. For this range, calculate the path loss in decibels for the following frequencies:

Note: Round all results to nearest 0.1 dB.

a. 1.6 GHz, 1.5 GHz

Wavelengths are: 1.6 GHz,  $\lambda = 0.1875$  m; 1.5 GHz,  $\lambda = 0.200$  m.

a.	1.6 GHz, 1.5 GHz	b.	6.2 GHz, 4.0 GHz
c.	14.2 GHz, 12.0 GHz	d.	30.0 GHz, 20.0 GHz

Answer: Path loss =  $20 \log (4 \pi R / \lambda)$ For 1.6 GHz,  $L_p = 20 \log (4 \pi \times 38,500 \times 10^3 / 0.1875) = 188.2 dB$ For 1.5 GHz,  $L_p = 20 \log (4 \pi \times 38,500 \times 10^3 / 0.200) = 187.7 dB$ 

Path loss at frequency  $f_2$  can be found from path loss at frequency  $f_1$  by scaling:

 $L_p(f_2) = L_p(f_1) + 20 \log (f_2 / f_1)$ . Using the result for 1.6 GHz,  $L_p = 188.2 \text{ dB}$ :

 b. 6.2 GHz, 4.0 GHz
 Answer: At 6.2 GHz, L<sub>p</sub> = 188.2 dB + 20 log (6.2 / 1.6) = 200.0 dB At 4.0 GHz, L<sub>p</sub> = 188.2 dB + 20 log (4.0 / 1.6) = 196.2 dB

 c. 14.2 GHz, 12.0 GHz
 Answer: At 146.2 GHz, L<sub>p</sub> = 188.2 dB + 20 log (14.2 / 1.6) = 207.2 dB At 12.0 GHz, L<sub>p</sub> = 188.2 dB + 20 log (12.0 / 1.6) = 205.7 dB

 d. 30.0 GHz 20.0 GHz
 Answer: At 30.0 GHz, L<sub>p</sub> = 188.2 dB + 20 log (30 / 1.6) = 213.7 dB At 20.0 GHz, L<sub>p</sub> = 188.2 dB + 20 log (20 / 1.6) = 210.1 dB

**Note:** All commercial satellite systems have path losses that fall within the above range, excepting any in the vhf and uhf bands, and above 40 GHz.

5. Low earth orbit satellites use mainly L band, with ranges varying from 1000 km to 2,500 km. Calculate the maximum and minimum path loss from earth to a satellite, in dB, for the uplink frequency of 1.6 GHz, and the downlink frequency of 1.5 GHz.

Answer: Wavelengths are: 1.6 GHz,  $\lambda = 0.1875$  m; 1.5 GHz,  $\lambda = 0.200$  m. Path loss = 20 log ( 4  $\pi$  R /  $\lambda$ ) For 1.6 GHz, Maximum L<sub>p</sub> = 20 log ( 4  $\pi \times 2,500 \times 10^3 / 0.1875$ ) = 64.5 dB For 1.5 GHz, L<sub>p</sub> = 20 log ( 4  $\pi \times 38,500 \times 10^3 / 0.200$ ) = 187.7 dB 6. A geostationary satellite carries a transponder with a 20 watt transmitter at 4 GHz. The transmitter is operated at an output power of 10 watts and drives an antenna with a gain of 30 dB. An earth station is at the center of the coverage zone of the satellite, at a range of 38,500 km. Using decibels for all calculations, find:

- a. The flux density at the earth station in dBW/m<sup>2</sup>.
  b. The power received by an antenna with a gain of 39 dB, in dBW.
- c. The EIRP of the transponder in dBW.

a. The flux density at the earth station in dBW/m<sup>2</sup> Answer: Flux density is given by F = 20 log [ P<sub>t</sub> G<sub>t</sub> / (4  $\pi$  R<sup>2</sup> ) ] dBW/m<sup>2</sup> Hence for R = 38,500 km, f = 4 GHz,  $\lambda$  = 0.075 m F = 10 log P<sub>t</sub> + G<sub>t</sub> - 10 log (4  $\pi$ ) - 20 log (38,500 × 10<sup>3</sup> ) dBW / m<sup>2</sup> = 10.0 +30.0 - 11.0 - 151.7 = -122.7 dBW / m<sup>2</sup>

**b.** The power received by an antenna with a gain of 39 dB, in dBW.

**Answer:** Received power can be calculated from the effective area of the antenna aperture and the incident flux density, but since the antenna gain is given in dB, it is better to use path loss and the link budget.

Path loss  $L_p = 20 \log (4 \pi R / \lambda) = 10 \log (4 \pi \times 38,500 \times 10^3 / 0.075) = 196.2 dB$ Downlink power budget gives

 $P_r = P_t + G_t + G_r - L_p dBW$ = 10.0 + 30.0 + 39.0 - 196.2 = -117.2 dBW

Alternatively, the received power can be found from

 $P_r = F \times A_{eff}$  where  $A_{eff}$  is the effective aperture area of the antenna.

Given G =  $4 \pi A_{eff} / \lambda^2 = 39 \text{ dB}$ , we can find  $A_{eff}$  from

 $A_{eff} = G + 20 \log \lambda - 11.0 dB = 39.0 - 22.5 - 11.0 = 5.5 dB m^2$  $P_r = -122.7 + 5.5 = -117.2 dBW / m^2$ 

c. The EIRP of the transponder in dBW.

Answer: Transponder EIRP =  $P_t + G_t = 10 + 30 = 40 \text{ dBW}$ 

7. A LEO satellite has a multi-beam antenna with a gain of 18 dB in each beam. A transponder with transmitter output power of 0.5 watts at 2.5 GHz is connected to one antenna beam. An earth station is located at the edge of the coverage zone of this beam, where the received power is 3 dB below that at the center of the beam, and at a range of 2,000 km from the satellite. Using decibels for all calculations, find:

a. The power received by an antenna with a gain of +1 dB, in dBW.

b. The noise power of the earth station receiver for a noise temperature of 260 K and an RF channel bandwidth of 20 kHz.

c. The C/N ratio in dB for the LEO signal at the receiver output.

**a.** The power received by an antenna with a gain of +1 dB, in dBW.

Answer: Find the path loss,  $L_p$ , first, for a wavelength of  $\lambda = 0.120$  m:

Path loss  $L_p = 20 \log (4 \pi R / \lambda) = 10 \log (4 \pi \times 2000 \times 10^3 / 0.120) = 166.4 dB$ Downlink power budget gives

 $P_r = P_t + G_t + G_r - L_p - \text{losses dBW}$ = -3.0 + 18.0 + 1.0 - 166.4 - 3.0 = -153.4 dBW

b. The noise power of the earth station receiver for a noise temperature of 260K and an RF channel bandwidth of 20 kHz.

Answer: The noise power at the input to the receiver is

 $N = k T_s B_N = -228.6 + 24.1 + 43.0 = -161.5 dBW$ 

c. The C/N ratio in dB for the LEO signal at the receiver output.

**Answer:**  $C/N = P_r - N = -153.4 + 161.5 = 8.1 dB.$ 

8. A satellite in GEO orbit is a distance of 39,000 km from an earth station. The required flux density at the satellite to saturate one transponder at a frequency of 14.3 GHz is  $-90.0 \text{ dBW/m}^2$ . The earth station has a transmitting antenna with a gain of 52 dB at 14.3 GHz.

Find:

- a. The EIRP of the earth station.
- b. The output power of the earth station transmitter.

a. The EIRP of the earth station

**Answer:** EIRP =  $P_t + G_t = P_t + 52 \text{ dBW}$ 

Flux density is given by F = 20 log [ EIRP /  $(4 \pi R^2)$  ] dBW/m<sup>2</sup>

Hence for R = 39,000 km, f = 14.3 GHz,  $\lambda$  = 0.02010 m

 $F = -90.0 = EIRP - 10 \log (4 \pi) - 20 \log (39,000 \times 10^{3}) dBW / m^{2}$ -90.0 = EIRP - 11.0 - 151.8 dBW / m<sup>2</sup> EIRP = -90.0 + 162.8 = 72.8 dBW

**b.** The output power of the earth station transmitter.

**Answer:** EIRP =  $P_t + G_t = 72.8 \text{ dBW}$ . Hence  $P_t = 72.8 - 52.0 = 20.8 \text{ dBW}$ .

9. A 12 GHz earth station receiving system has an antenna with a noise temperature of 50K, a LNA with a noise temperature of 100 K and a gain of 40 dB, and a mixer with a noise temperature of 1000 K. Find the system noise temperature.

Answer: System noise temperature is calculated from  $T_s = T_{antenna} + T_{LNA} + T_{mixer} / G_{LNA} + \dots$ Hence for  $G_{LNA} = 40 \text{ dB} = 10,000 \text{ as a ratio}$  $T_s = 50 + 100 + 1000 / 10,000 = 150.1 \text{ K}$  **10.** A geostationary satellite carries a C-band transponder which transmits 20 watts into an antenna with an on-axis gain of 30 dB. An earth station is in the center of the antenna beam from the satellite, at a distance of 38,000 km. For a frequency of 4.0 GHz:

a. Calculate the incident flux density at the earth station in watts per square meter and in

 $dBW/m^2$ .

b. The earth station has an antenna with a circular aperture 2 m in diameter and an aperture efficiency of 65%. Calculate the received power level in W and in dBW at the antenna output port.

c. Calculate the on-axis gain of the antenna in dB.

d. Calculate the free space path loss between the satellite and the earth station.

Calculate the power received,  $P_r$ , at the earth station using the link equation:

$$P_{\rm r} = P_{\rm t}G_{\rm t}G_{\rm r}/L_{\rm p}$$

where  $P_tG_t$  is the EIRP of the satellite transponder and  $L_p$  is the path loss.

Make your calculation in dB units and give your answer in dBW.

a. Calculate the incident flux density at the earth station in watts per square meter and in dBW/m<sup>2</sup>.

Answer: Flux density is given by F = 20 log [ EIRP /  $(4 \pi R^2)$  ] dBW/m<sup>2</sup> Hence for R = 38,000 km, f = 4.0 GHz,  $\lambda$  = 0.0750 m, EIRP = 13.0 + 30.0 = 43.0 dBW F = 43.0 - 10 log (4  $\pi$ ) - 20 log (38,000 × 10<sup>3</sup>) dBW / m<sup>2</sup> = 43.0 - 11.0 - 151.6 = -119.6 dBW / m<sup>2</sup>

b. The earth station has an antenna with a circular aperture 2 m in diameter and an aperture efficiency of 65%. Calculate the received power level in watts and in dBW at the antenna output port.

Answer: The effective area of the antenna is

or

 $A_{eff} = \eta_A \pi r^2 = 0.65 \times \pi \times 1 = 2.042 \text{ m or } 3.1 \text{ dBm}^2$ 

For an incident flux density of ~119.6 dBW /  $m^2\,$  or  $\,1.10\times 10^{-12}\; W/m^2$ 

$$P_r = 2.042 \times 1.10 \times 10^{-12} = 2.24 \times 10^{-12} \text{ W or} -116.5 \text{ dBW}$$
  
 $P_r = -119.6 + 3.1 = -116.5 \text{ dBW}$ 

c. Calculate the on-axis gain of the antenna in dB.

Answer: Antenna gain for a circular aperture is given by  $G = \eta_A (\pi D / \lambda)^2$ 

G = 10 log ( 
$$0.65 \times (\pi \times 2 / 0.0750)^2$$
) = 36.6 dB

**d.** Calculate the free space path loss between the satellite and the earth station. Calculate the power received, Pr, at the earth station using the link equation:

$$P_r = P_t G_t G_r / L_p$$

where  $P_t G_t$  is the EIRP of the satellite transponder and  $L_p$  is the path loss.

Make your calculation in dB units and give your answer in dBW.

**Answer:** At a frequency of 4.0 GHz,  $\lambda = 0.075$  m.

Path loss =  $20 \log (4 \pi R / \lambda) = 20 \log (4 \pi \times 38,000 \times 10^3 / 0.075) dB$ 

$$L_p = 196.1 \text{ dB}$$

Downlink power budget gives

$$P_r = P_t + G_t + G_r - L_p dBW$$
  
= 13 + 30.0 + 36.6 - 196.1 = -116.5 dBW

### 4.8 SYSTEM DESIGN EXAMPLES

The following sample system designs demonstrate how the ideas developed in this chapter can be applied to the design of satellite communication systems.

#### **TABLE 4.6 System and Satellite Specification**

Ku-band	satellite parameters		
	Geostationary at 73° W longitude, 28 K	u-band transponders	
	Total RF output power		2.24 kW
	Antenna gain, on axis (transmit and re	ceive)	31 dB
	Beceive system noise temperature		500 K
	Transponder saturated output power:	Ku band	80 W
	Transponder bandwidth:	Ku band	54 MHz
Signal	Compressed digital video signals with transmitted symbol rate of 43.2 Msps		
orginar	Minimum permitted overall (C/N) <sub>0</sub> in r	eceiver	9.5 dB
Transm	itting Ku-band earth station		
	Antenna diameter		5 m
	Aperture efficiency		68%
	Uplink frequency		14.15 GHz
	Required C/N in Ku-band transponder		30 dB
	Transponder HPA output backoff		1 dB
	Miscellaneous uplink losses		0.3 dB
	Location: -2 dB contour of satellite re	ceiving antenna	
# **Receiving Ku-band earth station**

Downlink fr	equency	11.45 GHz
Receiver IF	noise bandwidth	43.2 MHz
Antenna no	ise temperature	30 K
LNA noise t	emperature	110 K
Required ov	verall (C/N), in clear air	17 dB
Miscellaneo	ous downlink losses	0.2 dB
Location: -	3 dB contour of satellite transmitting antenna	
Rain attenuation and	propagation factors	
Ku-band clear air att	enuation	
Uplink	14.15 GHz	0.7 dB
Downlink	11.45 GHz	0.5 dB
Rain attenuation		
Uplink	0.01% of year	6.0 dB
Downlink	0.01% of year	5.0 dB

# System Design Example 4.8.1

This example examines the design of a satellite communication link using a Ku-band geostationary satellite with bent pipe transponders to distribute digital TV signals from an earth station to many receiving stations throughout the United States. The design requires that an overall C/N ratio of 9.5 dB be met in the TV receiver to ensure that the video signal on the TV screen is held to an acceptable level. The uplink transmitter power and the receiving antenna gain and diameter are determined for each system. The available link margins for each of the systems are found and the performance of the systems is analyzed when rain attenuation occurs in the satellite–earth paths. The advantages and disadvantages of implementing uplink power control are considered.

In this example, the satellite is located at 73° W. However, for international registration of this satellite location, the location would be denoted as 287° E. The link budgets developed in the examples below use decibel notation throughout. The satellite and earth stations are specified in Table 4.6, and Figure 4.11 shows an illustration of the satellite television distribution system.

### **Ku-Band Uplink Design**

We must find the uplink transmitter power required to achieve  $(C/N)_{up} = 30$  dB in clear air atmospheric conditions. We will first find the noise power in the transponder for 43.2 MHz bandwidth, and then add 30 dB to find the transponder input power level.

#### Uplink Noise Power Budget

k = Boltzmann's constant	-228.6 dBW/K/Hz
$T_{\rm s} = 500 {\rm K}$	27.0 dBK
B = 43.2  MHz	76.4 dBHz
N = transponder noise power	-125.2 dBW

The received power level at the transponder input must be 30 dB greater than the noise power.

 $P_{\rm r}$  = power at transponder input = -95.2 dBW



Cable TV receive stations



The uplink antenna has a diameter of 5 m and an aperture efficiency of 68%. At 14.15 GHz the wavelength is 2.120 cm = 0.0212 m. The antenna gain is

 $G_{\rm t} = 10 \log[0.68 \times (\pi D/\lambda)^2] = 55.7 \, {\rm dB}$ 

The free space path loss is  $L_{\rm p} = 10 \log \left[ (4\pi R/\lambda)^2 \right] = 207.2 \text{ dB}$ 

### Uplink Power Budget

$P_{t}$	= Earth station transmitter power	$P_{\rm t}  {\rm dBW}$
$G_{t}$	= Earth station antenna gain	55.7 dB
$G_r$	= Satellite antenna gain	31.0 dB
$L_{\pi}$	= Free space path loss	-207.2 dB
L <sub>p</sub>	= E/S on 2 dB contour	-2.0  dB
	= Other losses	-1.0 dB
$\mathcal{P}_{\mathrm{m}}$	= Received power at transponder	$P_{\rm t} = 123.5 \; {\rm dB}$

The required power at the transponder input to meet the  $(C/N)_{up} = 30$  dB objective is -95.2 dBW. Hence

$$P_t - 123.5 \text{ dB} = -95.2 \text{ dBW}$$
  
 $P_t = 28.3 \text{ dBW} \text{ or } 675 \text{ W}$ 

This is a relatively high transmit power so we would probably want to increase the transmitting antenna diameter to increase its gain, allowing a reduction in transmit power.

# **Ku-Band Downlink Design**

The first step is to calculate the downlink  $(C/N)_{dn}$  that will provide  $(C/N)_0 = 17$  dB when  $(C/N)_{up} = 30$  dB. From Eq. (4.43)

$$1/(C/N)_{dn} = 1/(C/N)_0 - 1/(C/N)_{up}$$
 (not in dB)

Thus

$$1/(C/N)_{dn} = 1/50 - 1/1000 = 0.019$$
  
(C/N)<sub>dn</sub> = 52.6  $\Rightarrow$  17.2 dB

We must find the required receiver input power to give  $(C/N)_{dn} = 17.2$  dB and then find the receiving antenna gain,  $G_{r}$ .

# Downlink Noise Power Budget

k	= Boltzmann's constant	-228.6 dBW/K/Hz
$T_{s}$	= 30 + 110  K = 140  K	21.5 dBK
$B_n$	= 43.2 MHz	76.4 dBHz
N	= transponder noise power	-130.7 dBW

The power level at the earth station receiver input must be 17.2 dB greater than the noise power in clear air.

 $P_r$  = power at earth station receiver input = -130.7 dBW + 17.2 dB = -113.5 dBW

We need to calculate the path loss at 11.45 GHz. At 14.15 GHz path loss was 207.2 dB. At 11.45 GHz path loss is

$$L_{\rm p} = 207.2 - 20 \log_{10}(14.15/11.45) = 205.4 \, \rm dB$$

The transponder is operated with 1 dB output backoff, so the output power is 1 dB below 80 W (80 W  $\Rightarrow$  19.0 dBW)

 $P_{\rm t} = 19 \, \rm dBW - 1 \, \rm dB = 18 \, \rm dBW$ 

D	7. 7	n	DI
Down	link	Power	Budget

$P_{t}$	= Satellite transponder output power	18.0 dBW
$G_{t}$	= Satellite antenna gain	31.0 dB
$G_{\rm r}$	= Earth station antenna gain	$G_{r} dB$
$L_{\rm p}$	= Free space path loss	-205.4 dB
$L_{\rm a}$	= E/S on $-3$ dB contour of satellite antenna	-3.0 dB
$L_{\rm m}$	= Other losses	-0.8  dB
$P_{\rm r}$	= Received power at earth station	$G_{\rm r} = 160.2 \; {\rm dB}$

The required power into the earth station receiver to meet the  $(C/N)_{dn} = 17.2$  dB objective is  $P_r = -120.1$  dBW. Hence the receiving antenna must have a gain  $G_r$  where

 $G_{\rm r} - 160.2 \, dB = -113.5 \, dBW$  $G_{\rm r} = 46.7 \, dB \text{ or } 46,774 \text{ as a ratio}$ 

The earth station antenna diameter, D, is calculated from the formula for antenna gain, G, with a circular aperture

$$G_{\rm r} = 0.65 \times (\pi D/\lambda)^2 = 46,774$$

At 11.45 GHz, the wavelength is 2.62 cm = 0.0262 m. Evaluating the above equation to find D gives the required receiving antenna diameter as D = 2.14 m.

## Example 4.6 Link Budget for C-Band Downlink With Earth Coverage Beam

The satellite used in this example is in GEO and carries 24 C-band transponders, each with a bandwidth of 36 MHz. The downlink band is 3.7–4.2 GHz and the satellite uses dual orthogonal circular polarizations to double the number of available channels, thus providing an effective RF bandwidth of 864 MHz. Figure 4.8 illustrates a GEO satellite located at 30°W longitude serving the Atlantic Ocean region.

The satellite provides coverage of the visible earth, which subtends an angle of approximately 17° from a satellite in geostationary orbit, by using a global beam antenna. Antenna beamwidth and gain are linked together by the relationship

 $G \approx \sqrt{30,000/(\text{beamwidth in degrees})^2}$ 

where G is a ratio (not in decibels). The on-axis gain of the global beam antenna is approximately 20 dB. However, we must make a worst case assumption in the link budget calculation, which is for an earth station at the edge of the coverage zone of the satellite where the effective gain of the antenna is 3 dB lower, at 17 dB. The edge of the coverage zone does not necessarily have to be at the -3 dB contour of the satellite antenna



**Figure 4.8** GEO satellite at 30° west longitude with global beam antenna serving the Atlantic Ocean region. Note that most of the energy transmitted by the satellite falls into the ocean; only a small fraction reaches populated areas.

footprint. Coverage extends beyond the -3 dB contour into the sidelobes of the satellite antenna pattern. A larger receiving antenna can be used to compensate for the loss of signal power when operating outside the -3 dB contour.

The CNR for the downlink is calculated in clear air conditions and also in heavy rain. The satellite can connect earth stations in North and South America to earth stations in Europe and Africa using a global beam that covers the visible earth as seen from the satellite. However, most of the signal radiated by the satellite ends up in the ocean and only a small part is available for communications between continents. A satellite antenna with a gain of 20 dB has an effective aperture diameter of 5.6 wavelengths given by

$$G = \eta_{\rm A} \left(\frac{\pi D}{\lambda}\right)^2$$

which gives D = 0.42 m at a frequency of 4 GHz. If the satellite antenna's aperture efficiency is 65%, the physical diameter is 0.52 m. The calculation of CNR is made at a midband frequency of 4 GHz. Appendix B explains the properties of antennas.

The saturated output power of the transponder is 80 W = 19 dBW. Reducing the output power of an amplifier from its maximum value helps to linearize the channel, so we will assume an output *backoff* of 2 dB, which means the power transmitted by the transponder is now 17 dBW.

Hence the on-axis EIRP of the transponder and antenna is

 $P_{\rm t}G_{\rm t} = 17 + 20 = 37 \, \rm dBW$ 

The transmitted signal is a single 30 MHz bandwidth channel carrying a digital signal in this example.

The maximum path length for a GEO satellite link at the edge of coverage is 40 000 km, which gives a path loss of 196.5 dB at 4 GHz ( $\lambda = 0.075$  m). We must make an allowance in the link budget for some losses that will inevitably occur on the link. At C-band, propagation losses are small, but the slant path through the atmosphere will suffer a typical attenuation of 0.2 dB in clear air. We will allow an additional 0.5 dB *margin* in the link design to account for miscellaneous losses, such as antenna mispointing, polarization mismatch, and antenna degradation, to ensure that the link budget is realistic.

Table 4.5a summarizes the parameters of the link and presents a link budget for the downlink from the satellite to a receiving earth station.

C-band satellite parameters		
Transponder saturated output power 80 W	$P_{t \text{ sat}}$	19 dBW
Antenna gain, on axis	$G_{\mathrm{t}}$	20 dB
Transponder bandwidth	$B_{\mathrm{trans}p}$	36 MHz
Downlink frequency band		3.7–4.2 GHz
Digital signal noise bandwidth	B <sub>n</sub>	30 MHz
Minimum permitted overall CNR in receiver	(CNR) <sub>o min</sub>	14.0 dB
Receiving C-band earth station		
Downlink frequency		4.00 GHz
Antenna gain, on axis, 4 GHz	$G_{ m r}$	49.7 dB
Receiver IF bandwidth	B <sub>n</sub>	30 MHz
Receiving system noise temperature	$T_{\rm s}$	45 K
Downlink power budget		
Satellite transponder output power, 80 W	$P_{t}$	19.0 dBW
Transponder output backoff	$B_{\rm out}$	-2.0 dB
Satellite antenna gain, on axis	$G_{\mathrm{t}}$	20.0 dB
Earth station antenna gain	$G_{ m r}$	49.7 dB
Free space path loss at 4 GHz	$L_{ m p}$	-196.5 dB
Edge of beam loss for satellite antenna	$L_{\rm ant}$	-3.0 dB
Clear sky atmospheric loss	$L_{\mathrm{a}}$	-0.2 dB
Other losses (margin)	$L_{ m misc}$	-0.5 dB
Received power at earth station	$P_{\rm r}$	-113.5 dBW
Downlink noise power budget in clear air		
Boltzmann's constant	k	-228.6 dBW/K/Hz
System noise temperature, 58 K	$T_{\rm s}$	17.6 dBK
Noise bandwidth, 30 MHz	B <sub>n</sub>	74.8 dBHz
Receiver noise power	Ν	-136.2 dBW

 Table 4.5a
 Example for a C-band GEO satellite downlink budget in clear air

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The system noise temperature is 58 K because the clear air attenuation of 0.2 dB creates an antenna noise temperature of 13 K, which adds to the LNA noise temperature of 45 K. Hence the CNR in the receiver in clear air is

 $CNR = P_r - N = -113.5 \text{ dBW} - (-136.2 \text{ dBW}) = 22.7 \text{ dB}$ 

The receiving earth station has a gain of 49.7 dB at 4 GHz, and a receiving system noise temperature of 58 K in clear sky conditions. The G/T ratio for this earth station is

$$\frac{G}{T} = 49.7 - 10 \log_{10} 58 = 32.0 \text{ dBK}^{-1}$$

The earth station receiver CNR is first calculated for clear sky conditions, with no rain in the slant path. The CNR is then recalculated taking account of the effects of rain. The minimum permitted overall CNR for this link is 14.0 dB giving a maximum BER of  $10^{-6}$ 

with quadrature phase shift keying (QPSK) modulation and no FEC. A CNR of 22.7 dB in clear air with QPSK modulation gives a BER well below  $10^{-16}$  (Chapter 5 shows how this calculation is made). At a bit rate of 60 Mbps the theoretical time between bit errors is longer than  $3 \times 10^8$  seconds or 9.5 years. The link is said to be *essentially error free*. Since clear sky conditions at 4 GHz prevail in most geographical locations for more than 97% of any given year, the link operates error free for most of the year.

### Example 4.7 C-Band Link CNR in Heavy Rain

The results for the receiving terminal CNR in clear air are used as the starting point for the calculation of CNR with rain in the slant path to the terminal. Table 4.5a shows that we have a downlink CNR of 22.7 dB in clear air, giving a *link margin* of 8.7 dB over the minimum CNR allowed of 14.0 dB. This link margin is available in clear air conditions, but will be reduced when there is rain in the slant path.

Heavy rain in the slant path can cause up to 1 dB of attenuation at 4 GHz when the satellite has a low elevation angle and the slant path through the rain is long, which reduces the received power by 1 dB and increases the noise temperature of the receiving system. Using the output noise model discussed in the previous section with a medium temperature of 273 K, and a total path loss for clear air plus rain of 1.2 dB (ratio of 1.32), the sky noise temperature in rain is

 $T_{\rm sky} = 273 \times (1 - 1/1.32) = 66 \, {\rm K}$ 

In clear air the sky noise temperature is approximately 13 K, the result of 0.2 dB of clear air attenuation. The system noise temperature with rain in the downlink path is  $T_{s rain}$  where

 $T_{s rain} = 45 + 66 = 111 K$ 

The clear air system noise temperature is 58 K. The increase in system noise temperature results in a corresponding increase in receiver noise power given by  $\Delta N$  where

 $\Delta N = 10 \log_{10} (111/58) = 2.8 \text{ dB}$ 

Note that the CNR in the C-band earth station receiver is affected much more by the increase in sky noise temperature than by the rain attenuation. In making this calculation, it is important to remember that clear air attenuation, 0.2 dB, in this case, is always present and must be added to the rain attenuation to give the total path attenuation before calculating the system noise temperature in rain. (You still want to be able to breathe when it rains.)

We can now adjust the link budget very easily to account for heavy rain in the slant path without having to recalculate the CNR from the beginning. The received carrier power is reduced by 1 dB because of the rain attenuation and the system noise temperature is increased by 2.8 dB. Table 4.5b shows the new downlink budget in rain.

CNR in the receiver in heavy rain is

$$CNR = P_{r rain} - N_{rain} = -114.5 \text{ dBW} - (-132.7 \text{ dBW}) = 18.2 \text{ dB}$$

The CNR in rain has a downlink margin of 4.2 dB over the minimum permissible CNR of 14.0 dB. The excess CNR margin will translate into lower BER, and can be traded off against earth station antenna gain to allow the use of smaller (and therefore lower cost)

antenna. We will examine how the 4.2 dB of link margin can be traded against other parameters in the system.

A reduction in earth station antenna gain of 4.2 dB is a reduction in the gain value, as a ratio, of 2.63. Antenna gain is proportional to diameter squared, so the diameter of the earth station antenna can be reduced by a factor of  $\sqrt{2.63} = 1.62$ , from 9 to 5.6 m to lower the cost of the earth station.

## Example 4.8 4 GHz Downlink With Regional Beam

Global beam antennas are not widely used, although most satellites with international coverage carry them to serve outlying earth stations that are not within the coverage of regional beams. The low gain and broad coverage of a global beam results in poor utilization of transponder power, since most of the transmitted power is lost over the oceans, as seen in Figure 4.8, and global beams have been referred to derisively as *fish warmers*. Regional TV signal distribution is much more common, so the C-band link in Tables 4.5a and 4.5b is more likely to use a regional antenna, serving a continent or a group of countries. The United States, for example, can be covered with a 6° by 3° beam, as illustrated in Figure 4.9. Additional beams may be needed to cover Alaska and Hawaii, or a more complex antenna with a *shaped beam* can be used.

The gain of a typical satellite antenna providing coverage of the 48 contiguous states (CONUS) is 32.2 dB on axis (calculated from  $G = 30000/[\theta_1 \times \theta_2]$ ), which is 12.2 dB higher than the on-axis gain of a global beam. Using the link budget in Tables 4.5a and 4.5b, we can trade the extra 12.2 dB gain (ratio 16.7) of a regional coverage satellite antenna for a reduction in earth station antenna dimensions. For the example of a 9.0 m earth station antenna in Example 4.7, we could reduce the antenna diameter by a factor of  $\sqrt{16.7} \approx 4.1$  m to a diameter of 2.2 m (approximately 7 ft 3 in.). The cost of antennas increases approximately as the diameter of the antenna to the power 2.7 for antennas larger than 2 m. (See Appendix B for details.) Reducing the diameter of the



Figure 4.9 GEO satellite at 100°W longitude with elliptical regional beam antenna serving the continental United States.

earth station antenna from 9.0 to 2.2 m reduces its cost by a factor of approximately 45, a very significant cost saving.

Table 4.5b	C-band	downlink	budget in rain
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Receiver noise power in rain	$N_{ m rain}$	-132.7 dBW
Increase in noise power due to rain	$\Delta N_{\rm rain}$	2.8 dB
Receiver noise power in clear air	$N_{\rm ca}$	-135.5 dBW
Received power at earth station in rain	$P_{ m rrain}$	-114.5 dBW
Rain attenuation	Α	1.0 dB
Received power at earth station in clear air	$P_{ m rca}$	-113.5 dBW

In the 1970s and 1980s, television programming was distributed to cable TV *head ends* by C-band regional satellites, and later by Ku-band satellites. The C-band signals were transmitted as one video channel per 36 MHz transponder using frequency modulation (FM) and at first were not encrypted. An industry grew up supplying 8 and 10 ft diameter dishes to home owners, equipped with receivers with a 100 K LNA, allowing reception of cable TV channels without payment. The threshold for successful demodulation of the FM video signal was 11 dB. Comparing these parameters to the example in Tables 4.5a and 4.5b shows that a link margin of approximately 2 dB was available with the C-band home satellite TV system. Eventually the video signals of the popular TV programs were encrypted and users of the satellite receiving systems were forced to pay a subscription to receive those cable TV channels.

The above examples show how the link budget can be used to study different combinations of system parameters. Most satellite link analyses do not yield the wanted result at the first try, and the designer or analyst must use the link budget to adjust system parameters until an acceptable result is achieved. More examples of link budgets are included later in this chapter.

### 4.5.2 Effect of Rain on Direct to Home Satellite TV Ku-Band Downlink

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The first step is to determine the total path attenuation,  $A_{\text{total}}$  in dB, which is the sum of the clear sky path attenuation due to atmospheric gaseous absorption,  $A_{\text{ca}}$  and attenuation due to rain,  $A_{\text{rain}}$ 

$$A_{\text{total}} = A_{\text{ca}} + A_{\text{rain}} \, \mathrm{dB} \tag{4.29}$$

The sky noise temperature resulting from a path attenuation  $A_{\rm total}$  dB is found from the output noise model of Section 4.3 using an assumed medium temperature of 270 K for the rain.

$$T_{\rm sky} = 270 \times (1 - 10^{-A/10}) \,\mathrm{K}$$
 (4.30)

The antenna noise temperature may be assumed to be equal to the sky noise temperature, although in practice not all of the incident noise energy from the sky is output by the antenna, and a coupling coefficient,  $\eta_{c_{i}}$  of 90–95% is sometimes used when calculating antenna noise temperature in rain. Thus antenna noise temperature may be calculated as

$$T_{\rm A} = \eta_{\rm c} \times T_{\rm sky} \, {\rm K} \tag{4.31}$$

Almost all satellite receivers use a high gain LNA as the first element in the receiver front end. This makes the contribution of all later parts of the receiver to the system noise temperature negligible. System noise temperature is then given by  $T_{s rain}$  where

$$T_{\rm s \ rain} = T_{\rm LNA} + T_{\rm A \ rain} \ {\rm K} \tag{4.32}$$

In Eq. (4.32), the LNA is assumed to be placed right at the feed horn so that there is no waveguide or coaxial cable run between the feed horn of the antenna and the LNA. We will assume that there are no feed losses. The increase in noise power,  $\Delta N_{rain}$  dB, caused by the increase in sky noise temperature is given by

$$\Delta N_{\rm rain} = 10 \, \log_{10} \left[ \frac{kT_{\rm s \ rain} B_{\rm n}}{kT_{\rm sca} B_{\rm n}} \right] = 10 \, \log_{10} \left[ \frac{T_{\rm s \ rain}}{T_{\rm sca}} \right] \, \rm dB \tag{4.33}$$

where  $T_{\rm sca}$  is the system noise temperature in clear sky conditions.

The received power is reduced by the attenuation caused by the rain in the slant path, so in rain the value of carrier power is reduced from  $C_{ca}$  to  $C_{rain}$  where

$$C_{\rm rain} = C_{\rm ca} - A_{\rm rain} \, \rm dB \tag{4.34}$$

The resulting (CNR)<sub>dn rain</sub> value when rain intersects the downlink is given by

$$(CNR)_{dn rain} = (CNR)_{dn ca} - A_{rain} - \Delta N_{rain} dB$$
(4.35)

where (CNR)<sub>dn ca</sub> is the downlink CNR in clear sky conditions.

If a linear (bent pipe) transponder is used, the  $(CNR)_{up}$  must be combined with  $(CNR)_{dn rain}$  to yield the overall  $(CNR)_o$  ratio for the link. However, the transmitting earth stations for DBS-TV service use large antennas and high power transmitters with

uplink power control (UPC) to ensure that the uplink CNR is always at least 20 dB higher than the downlink CNR. The contribution of satellite noise to overall CNR can be ignored when this is the case. Some digital systems use regenerative transponders that provide constant output power regardless of uplink attenuation provided that the received CNR at the satellite is above the threshold of the onboard processing demodulator. In this case the value of (CNR)<sub>dn rain</sub> will be used as the overall (CNR)<sub>o</sub> value in rain for the link.

#### Example 4.9 Calculation of Rain Attenuation Margin

In the example of a DBS-TV system in Table 4.6a, a link margin of 5.7 dB is available before the  $(CNR)_{o}$  threshold of 8.3 dB is reached. This example shows how the link margin can be distributed between rain attenuation and an increase in receiver system noise power caused by an increase in sky noise. It is very difficult to write a set of equations that solve this problem, because the combination of linear and decibel arithmetic leads to a transcendental equation. Instead, an iterative calculation is used to find the exact rain attenuation, which causes the receiver CNR to equal the threshold value. We will begin by calculating the increase in system noise temperature that results from an estimated 3 dB rain attenuation in the downlink path to determine the increase in noise power and thus the value of  $(CNR)_{dn rain}$ .

The clear sky attenuation is given in Table 4.6a as 0.4 dB. This must be added to the rain attenuation – the atmosphere does not go away when it rains! Thus total path attenuation is 3.4 dB, and the sky noise temperature in rain will be, from Eq. (4.30)

$$T_{\rm sky\ rain} = 270 \times (1 - 10^{-3.4/10}) = 147 \,\rm K$$

We will assume 100% coupling between the sky noise temperature and the antenna temperature in this example. In clear sky conditions the sky noise temperature is

$$T_{\rm ca} = 270 \times (1 - 10^{-0.04}) = 24 \,\rm K$$

The sky temperature has increased from 24 K in clear sky conditions to 147 K when 3 dB rain attenuation occurs in the downlink. We must calculate the new system noise temperature when rain is present in the slant path. The LNA of the system in Table 4.6a has a noise temperature of 86 K and the clear sky system noise temperature is  $T_{\text{sky ca}} = 110$  K. With 3 dB of rain attenuation in the downlink, the system noise temperature, given by Eq. (4.32), increases to

$$T_{\rm s\ rain} = T_{\rm LNA} + T_{\rm A} = 86 + 147 = 233 \ {\rm K}$$

The increase in receiver noise power referred to the receiver input is given by Eq. (4.33)

$$\Delta N_{\text{rain}} = 10 \log_{10}(233/110) = 3.3 \text{ dB}$$

From Eq. (4.35)

$$(CNR)_{dn rain} = 14.0 - 3.0 - 3.3 = 7.8 \text{ dB}$$

The receiver CNR is below the threshold value of 8.3 dB by 0.5 dB, indicating that the maximum downlink rain attenuation is less than the estimated 3.0 dB. The previous procedure needs to be repeated with a revised estimate of the rain attenuation until the closest value of rain attenuation that gives 8.3 dB for (CNR)<sub>dn rain</sub> is found; that value is 2.7 dB in this case. In the southeastern United States, in states such as Florida and

Louisiana where very heavy rain occurs more often than in other parts of the United States, link availability may be slightly less than 99.7%. Shaping of the satellite beam to direct more power to these parts of the United States helps to reduce the number of outages experienced in that region.

Receivers located within the -1 dB contour of the satellite antenna beam have shorter path lengths giving 2.3 dB higher CNR than the receiver used in the example shown in Tables 4.6a and 4.6b, so they have a clear air downlink CNR of 16.3 dB and a downlink margin of 8.0 dB. The calculation of the availability of these receivers requires some care, because we cannot just add the extra 2.3 dB of link margin to the rain attenuation. Antenna noise increases with every decibel of extra rain attenuation, reducing the received power level, *C*, and increasing the system noise power *N*. The iterative procedure must be used again to find the combination of reduction in *C* and increase in *N* that leads to an additional 2.3 dB degradation in the overall CNR value. We will guess that increasing the rain attenuation from 3 to 4.1 dB gives the required result. With 4.1 dB rain attenuation, the path attenuation is 4.5 dB and system noise temperature is

$$T_{\rm s \ rain} = 86 + 270(1 - 0.355) = 260 \, {\rm K}$$

The increase in noise power from the clear sky condition is

$$\Delta N = 10 \log_{10} \left(\frac{260}{110}\right) = 3.7 \text{ dB}$$

Hence the decrease in  $(CNR)_{dn rain}$  for 4.1 dB of rain attenuation is 4.1 + 3.7 = 7.8 dB, and

so we are above the 8.3 dB threshold by 0.2 dB. Another trial is needed to determine the exact downlink attenuation that reduces the signal to the threshold level. A rain attenuation margin of 4.1 dB at Ku-band would give an availability of 99.85% or better over the central region of the United States. This example demonstrates that the increase in noise temperature of a low noise DBS-TV Ku-band receiving system is a significant factor when rain attenuation is present in the downlink path. Rain attenuation alone cannot be equated to link margin.

It is worth noting that rain causes significant attenuation on Ku-band downlinks for less than 2% of an average year over most of the United States. For 98% of the year these links are operating in near clear sky conditions with downlink CNR values in the 14–17 dB range. With half rate FEC applied to the data stream, the bit error rate will be below 10<sup>-16</sup>, and the error rate for a live MPEG-2 compressed video stream will be effectively zero. Thus the video transmission is essentially error free for all but a few tens of hours each year.